C3 Paper B – Marking Guide

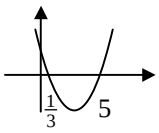
1.

$(2x-3)^2 > (x+2)^2$ M1

$3x^2 - 16x + 5 > 0$ A1

$(3x-1)(x-5) > 0$ M1

$x < \frac{1}{3}$ or $x > 5$ A2 **(5)**



2.

$3(\text{cosec}^2 x - 1) - 4\,\text{cosec}\,x + \text{cosec}^2 x = 0$ M1

$4\,\text{cosec}^2 x - 4\,\text{cosec}\,x - 3 = 0$

$(2\,\text{cosec}\,x + 1)(2\,\text{cosec}\,x - 3) = 0$ M1

$\text{cosec}\,x = -\frac{1}{2}$ or $\frac{3}{2}$ A1

$\sin x = -2$ (no solutions) or $\frac{2}{3}$ M1

$x = 0.73, \pi - 0.7297$

$x = 0.73, 2.41$ (2dp) A2 **(6)**

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3. *(i)* $\frac{dx}{dy} = 2y - \frac{3}{y} = \frac{2y^2-3}{y}$ M1 A1

$\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{y}{2y^2-3}$ A1

(ii) $y = \frac{1}{2}$, $x = \frac{1}{4}$, grad $= -\frac{1}{5}$ B1

$\therefore\ y - \frac{1}{2} = -\frac{1}{5}(x - \frac{1}{4})$ M1

$20y - 10 = -4x + 1$

$4x + 20y - 11 = 0$ A1 **(6)**

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4. *(i)*

x	0	0.25	0.5	0.75	1
xe^{2x}	0	0.4122	1.3591	3.3613	7.3891

M1

$I \approx \frac{1}{3} \times 0.25 \times [0 + 7.3891 + 4(0.4122 + 3.3613) + 2(1.3591)]$ M1

$= 2.10$ (3sf) A1

(ii) $= [-\frac{1}{2}e^{1-2x}]_{\frac{1}{2}}^{1}$ M1 A1

$= -\frac{1}{2}(e^{-1} - 1) = \frac{1}{2}(1 - e^{-1})$ M1 A1 **(7)**

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5. *(i)* $= \int_1^5 \frac{1}{\sqrt{3x+1}}\,dx = [\frac{2}{3}(3x+1)^{\frac{1}{2}}]_1^5$ M1 A1

$= \frac{2}{3}(4 - 2) = \frac{4}{3}$ M1 A1

(ii) $= \pi \int_1^5 \frac{1}{3x+1}\,dx$

$= \pi[\frac{1}{3}\ln|3x + 1|]_1^5$ M1 A1

$= \frac{1}{3}\pi(\ln 16 - \ln 4) = \frac{1}{3}\pi \ln 4 = \frac{2}{3}\pi \ln 2$ $[\,k = \frac{2}{3}\,]$ M1 A1 **(8)**

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6. *(a)* let radius = r, $\therefore \tan 30° = \frac{1}{\sqrt{3}} = \frac{r}{h}$, $r = \frac{h}{\sqrt{3}}$ M1

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h \times \frac{h^2}{3} = \frac{1}{9}\pi h^3$ A1

(b) *(i)* $\frac{dV}{dt} = 120$, $\frac{dV}{dh} = \frac{1}{3}\pi h^2$ B1

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$, $120 = \frac{1}{3}\pi h^2 \frac{dh}{dt}$, $\frac{dh}{dt} = \frac{360}{\pi h^2}$ M1 A1

when $h = 6$, $\frac{dh}{dt} = 3.18$ cm s^{-1} (2dp) M1 A1

(ii) $V = 8 \times 120 = 960 = \frac{1}{9}\pi h^3$ $\therefore h = \sqrt[3]{\frac{9\times960}{\pi}} = 14.011$ M1

$\therefore \frac{dh}{dt} = 0.58$ cm s^{-1} (2dp) A1 **(9)**

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7. *(i)* LHS $\equiv 2 \sin x \cos x - \frac{\sin x}{\cos x}$ M1

$\equiv \frac{2\sin x \cos^2 x - \sin x}{\cos x}$ M1 A1

$\equiv \frac{\sin x(2\cos^2 x - 1)}{\cos x} \equiv \frac{\sin x}{\cos x} \times \cos 2x \equiv \tan x \cos 2x \equiv$ RHS M1 A1

(ii) $\tan x \cos 2x = 2 \cos 2x$

$\cos 2x (\tan x - 2) = 0$ M1

$\cos 2x = 0$ or $\tan x = 2$ A1

$2x = 90, 270$ or $x = 63.4$

$x = 45°, 63.4°$ (3sf), $135°$ A2 **(9)**

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8. *(i)* $t = 0$, $m = 480$ B1

$\therefore$ $t = 10$, $m = 0.998 \times 480 = 479.04$ M1

$\therefore$ $479.04 = 400 + 80e^{-10k}$

$e^{-10k} = \frac{79.04}{80}$ A1

$k = -\frac{1}{10}\ln\frac{79.04}{80} = 0.00121$ (3sf) M1 A1

(ii) $475 = 400 + 80e^{-kt}$, $e^{-kt} = \frac{75}{80}$ M1

$t = -\frac{1}{k}\ln\frac{75}{80} = 53.5$ (3sf) A1

(iii) $\frac{dm}{dt} = -80ke^{-kt}$ M1 A1

$t = 100$, $\frac{dm}{dt} = -80ke^{-100k} = -0.0856$ M1

$\therefore$ decreasing at rate of 0.0856 g yr^{-1} (3sf) A1 **(11)**

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9. *(i)* $f(x) < 3$ B1

(ii) $= f(2) = 3 - e^4$ M1 A1

(iii) $y = 3 - e^{2x}$, $e^{2x} = 3 - y$, $2x = \ln(3 - y)$, $x = \frac{1}{2}\ln(3 - y)$ M1

$\therefore$ $f^{-1}(x) = \frac{1}{2}\ln(3 - x)$, $x \in \mathbb{R}$, $x < 3$ A2

(iv) e.g. $y = f^{-1}(x)$ is the reflection of $y = f(x)$ in the line $y = x$ so they intersect on the line $y = x$, hence $f^{-1}(x) = f(x) \Rightarrow f^{-1}(x) = x$ B2

(v) $x_1 = 0.4581$, $x_2 = 0.4664$, $x_3 = 0.4648$, $x_4 = 0.4651$, $x_5 = 0.4651$ M1 A1

$\therefore$ $\alpha = 0.465$ (3sf) A1 **(11)**

Total **(72)**