C3 Paper A – Marking Guide

1.	$= [\frac{3}{4}(2x-3)^{\frac{2}{3}}]_2^{15}$		M1 A1	
	$= \frac{3}{4}(27^{\frac{2}{3}} - 1)$		M1	
	$= \frac{3}{4}(9 - 1) = 6$		M1 A1	**(5)**
2.	$= \pi\int_1^3 \frac{(3x+1)^2}{x}\ dx$		M1	
	$= \pi\int_1^3 \frac{9x^2+6x+1}{x}\ dx = \int_1^3 (9x + 6 + \frac{1}{x})\ dx$		A1	
	$= \pi[\frac{9}{2}x^2 + 6x + \ln\lvert x\rvert]_1^3$		M1 A1	
	$= \pi\{(\frac{81}{2} + 18 + \ln 3) - (\frac{9}{2} + 6 + 0)\}$		M1	
	$= \pi(48 + \ln 3)$		A1	**(6)**
3.	*(i)*	$\frac{dy}{dx} = 3(3x-5)^2 \times 3 = 9(3x-5)^2$	M1	
		grad = 9	A1	
		$\therefore\ y - 1 = 9(x - 2)$ $[\ y = 9x - 17\ ]$	M1 A1	
	(ii)	$9(3x-5)^2 = 9,\quad 3x - 5 = \pm 1$	M1	
		$x = 2$ (at P), $\frac{4}{3}$ $\therefore\ Q\,(\frac{4}{3}, -1)$	A2	**(7)**
4.	$e^{2y} - x + 2 = 0 \Rightarrow e^{2y} = x - 2$			
	$2y = \ln(x-2)$		M1	
	sub. $\Rightarrow \ln(x+3) - \ln(x-2) - 1 = 0$		A1	
	$\ln\frac{x+3}{x-2} = 1$		M1	
	$\frac{x+3}{x-2} = e$		A1	
	$x + 3 = e(x-2),\quad 3 + 2e = x(e-1)$		M1	
	$x = \frac{2e+3}{e-1} = 4.91$ (2dp), $y = \frac{1}{2}\ln(\frac{2e+3}{e-1} - 2) = 0.53$ (2dp)		A2	**(7)**
5.	*(i)*	$\tan^{-1}(x-2) = -\frac{\pi}{3}$	M1	
		$x - 2 = \tan(-\frac{\pi}{3}) = -\sqrt{3}$	M1	
		$x = 2 - \sqrt{3}$	A1	
	(ii)	$1 - 2\sin^2\theta - \sin\theta - 1 = 0$	M1	
		$2\sin^2\theta + \sin\theta = 0,\quad \sin\theta(2\sin\theta + 1) = 0$	M1	
		$\sin\theta = 0$ or $-\frac{1}{2}$	A1	
		$\theta = 0$ or $-\frac{\pi}{6}, -\pi + \frac{\pi}{6}$		
		$\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, 0$	A2	**(8)**
6.	*(i)*	$= f(\frac{1}{2}) = -\frac{5}{2}$	M1 A1	
	(ii)	$gf(x) = \frac{2}{(3x-4)+3} = \frac{2}{3x-1}$	M1 A1	
		$\therefore\ \frac{2}{3x-1} = 6,\quad 2 = 6(3x-1)$	M1	
		$x = \frac{4}{9}$	A1	
	(iii)	$y = \frac{2}{x+3},\quad x + 3 = \frac{2}{y},\quad x = \frac{2}{y} - 3$	M1	
		$\therefore\ g^{-1}(x) = \frac{2}{x} - 3$	A1	**(8)**

7. (i) $2 \sin x - 3 \cos x = R \sin x \cos \alpha - R \cos x \sin \alpha$

$R \cos \alpha = 2, \ R \sin \alpha = 3$ — M1

$\therefore \ R = \sqrt{2^2 + 3^2} = \sqrt{13}$ — A1

$\tan \alpha = \frac{3}{2}, \ \alpha = 56.3$ (3sf) — A1

$\therefore \ 2 \sin x° - 3 \cos x° = \sqrt{13} \sin (x - 56.3)°$

(ii) $\operatorname{cosec} x° + 3 \cot x° = 2 \ \Rightarrow \ \frac{1}{\sin x} + \frac{3\cos x}{\sin x} = 2$

$\Rightarrow \ 1 + 3 \cos x = 2 \sin x$

$\Rightarrow \ 2 \sin x° - 3 \cos x° = 1$ — B1

(iii) $\sqrt{13} \sin (x - 56.31) = 1$

$\sin (x - 56.31) = \frac{1}{\sqrt{13}}$ — M1

$x - 56.31 = 16.10, \ 180 - 16.10 = 16.10, \ 163.90$ — M1

$x = 72.4, \ 220.2$ (1dp) — A2 — **(8)**

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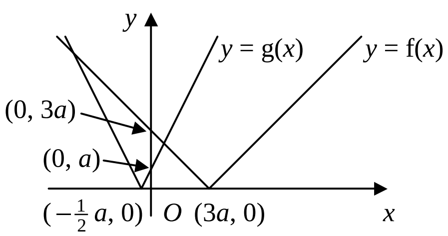
8. (i) $= \mathrm{f}(3a) = 0$ — M1 A1

(ii)



B4

(iii) $(x - 3a)^2 = (2x + a)^2$ — M1

$3x^2 + 10ax - 8a^2 = 0$ — A1

$(3x - 2a)(x + 4a) = 0$ — M1

$x = -4a, \ \frac{2}{3}a$ — A1 — **(10)**

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9. (i) $\frac{\mathrm{d}y}{\mathrm{d}x} = 2 - \frac{3}{2x+5} \times 2 = 2 - \frac{6}{2x+5}$ — M1

grad $= -4$, grad of normal $= \frac{1}{4}$ — A1

$\therefore \ y + 4 = \frac{1}{4}(x + 2) \qquad [\ y = \frac{1}{4}x - \frac{7}{2}\ ]$ — M1 A1

(ii) $\frac{1}{4}x - \frac{7}{2} = 2x - 3 \ln (2x + 5)$ — M1

$\frac{7}{4}x + \frac{7}{2} - 3 \ln (2x + 5) = 0$

let $\mathrm{f}(x) = \frac{7}{4}x + \frac{7}{2} - 3 \ln (2x + 5)$

$\mathrm{f}(1) = -0.59, \ \mathrm{f}(2) = 0.41$ — M1

sign change, $\mathrm{f}(x)$ continuous $\therefore$ root — A1

(iii) $\frac{7}{4}x + \frac{7}{2} - 3 \ln (2x + 5) = 0$

$7x + 14 - 12 \ln (2x + 5) = 0$

$7x = 12 \ln (2x + 5) - 14$ — M1

$x = \frac{12}{7} \ln (2x + 5) - 2$ — A1

(iv) $x_{n+1} = \frac{12}{7} \ln (2x_n + 5) - 2, \ x_0 = 1.5$

$x_1 = 1.5648, \ x_2 = 1.5923, \ x_3 = 1.6039, \ x_4 = 1.6087, \ x_5 = 1.6107$ — M1

$q = 1.61$ (3sf) — A1

$\mathrm{f}(1.605) = -0.0073, \ \mathrm{f}(1.615) = 0.0029$ — M1

sign change, $\mathrm{f}(x)$ continuous $\therefore$ root $\therefore \ q = 1.61$ (3sf) — A1 — **(13)**

Total **(72)**