

GCE Examinations
Advanced Subsidiary

Core Mathematics C3

Paper H

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

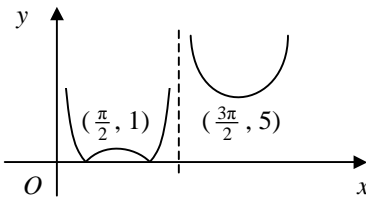


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C3 Paper H – Marking Guide

1. (a) $= f(2) = -2$ M1 A1
- (b) $gf(x) = g(2 - x^2) = \frac{3(2 - x^2)}{2(2 - x^2) - 1} = \frac{6 - 3x^2}{3 - 2x^2}$ M1 A1
- $\therefore \frac{6 - 3x^2}{3 - 2x^2} = \frac{1}{2}, \quad 2(6 - 3x^2) = 3 - 2x^2$
- $x^2 = \frac{9}{4}, \quad x = \pm \frac{3}{2}$ M1 A1 (7)
-
2. $5(\sec^2 2\theta - 1) - 13 \sec 2\theta = 1$ M1
- $5 \sec^2 2\theta - 13 \sec 2\theta - 6 = 0$
- $(5 \sec 2\theta + 2)(\sec 2\theta - 3) = 0$ M1
- $\sec 2\theta = -\frac{2}{5}$ or 3 A1
- $\cos 2\theta = -\frac{5}{2}$ (no solutions) or $\frac{1}{3}$
- $2\theta = 70.529, 360 - 70.529, 360 + 70.529, 720 - 70.529$ B1 M1
- $= 70.529, 289.471, 430.529, 649.471$
- $\theta = 35.3^\circ, 144.7^\circ, 215.3^\circ, 324.7^\circ$ (1dp) A2 (7)
-
3. (a) $= \frac{(2x-3)(x+3)}{(2x-3)(x-2)} = \frac{x+3}{x-2}$ M1 A2
- (b) $\ln(2x^2 + 3x - 9) - \ln(2x^2 - 7x + 6) = 2, \quad \ln \frac{2x^2 + 3x - 9}{2x^2 - 7x + 6} = 2$ M1
- $\ln \frac{x+3}{x-2} = 2, \quad \frac{x+3}{x-2} = e^2$ A1
- $x + 3 = e^2(x - 2)$
- $3 + 2e^2 = x(e^2 - 1)$ M1
- $x = \frac{2e^2 + 3}{e^2 - 1}$ A1 (7)
-
4. (a)  B3
- (b) $(\frac{\pi}{2}, -1) \Rightarrow -1 = a + b$
- $(\frac{3\pi}{2}, -5) \Rightarrow -5 = a - b$ B1
- adding, $-6 = 2a \therefore a = -3, b = 2$ M1 A1
- (c) $-3 + 2 \operatorname{cosec} x = 0, \quad \operatorname{cosec} x = \frac{3}{2}, \quad \sin x = \frac{2}{3}$ M1
- $x = 0.73, \pi - 0.7297, \quad x = 0.73, 2.41$ (2dp) A2 (9)
-
5. (a) $t = 3, N = 18\,000 \Rightarrow 18\,000 = 2000e^{3k}, \quad e^{3k} = 9$ M1
- $k = \frac{1}{3} \ln 9 = 0.732$ (3sf) M1 A1
- (b) $4000 = 2000e^{0.7324t}$ M1
- $t = \frac{1}{0.7324} \ln 2 = 0.9464$ hours M1 A1
- \therefore doubles in 57 minutes (nearest minute) A1
- (c) $N = 2000e^{0.7324t}, \quad \frac{dN}{dt} = 0.7324 \times 2000e^{0.7324t} = 1465e^{0.7324t}$ M1 A1
- when $t = 3, \frac{dN}{dt} = 13\,200 \therefore$ increasing at rate of 13 200 per hour (3sf) A1 (10)

6. (a) $\frac{d}{dx}(\sec x) = \frac{d}{dx}[(\cos x)^{-1}]$
 $= -(\cos x)^{-2} \times (-\sin x)$ M1 A1
 $= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$ M1
 $= \sec x \tan x$ A1
- (b) $\frac{dy}{dx} = 2e^{2x} \times \sec x + e^{2x} \times \sec x \tan x = e^{2x} \sec x (2 + \tan x)$ M1 A1
 $x = 0, y = 1, \text{ grad} = 2$ M1
 $\therefore y = 2x + 1$ A1
- (c) SP: $e^{2x} \sec x (2 + \tan x) = 0$ M1
 $\tan x = -2$ M1
 $x = -1.11$ (2dp) A1 (11)

7. (a) $f(x) = (x - 1)^2 - 1 + 5 = (x - 1)^2 + 4$ M1 A1
- (b) $f(x) \geq 4$ B1
- (c) $y = (x - 1)^2 + 4$
 $(x - 1)^2 = y - 4$
 $x - 1 = \pm \sqrt{y - 4}$ M1
 $x = 1 \pm \sqrt{y - 4}$
 $f^{-1}(x) = 1 + \sqrt{x - 4}$ M1 A1
- (d) translation by 4 units in negative x direction
translation by 1 unit in negative y direction (either first) B2
- (e) $\frac{dy}{dx} = \frac{1}{2}(x - 4)^{-\frac{1}{2}}$ M1
 $x = 8, y = 3, \text{ grad} = \frac{1}{4}$ A1
 $\therefore \text{ grad of normal} = -4$
 $\therefore y - 3 = -4(x - 8)$ [$y = 35 - 4x$] M1 A1 (12)

8. (a) $\frac{dy}{dx} = -e^2 x^{-2} + e^x$ M1 A1
- (b) SP: $-e^2 x^{-2} + e^x = 0$ M1
let $f(x) = -e^2 x^{-2} + e^x$
 $f(1.3) = -0.70, f(1.4) = 0.29$ M1
sign change, $f(x)$ continuous \therefore root A1
- (c) $x = 2, y = \frac{3}{2}e^2, \text{ grad} = \frac{3}{4}e^2$ M1
 $\therefore y - \frac{3}{2}e^2 = \frac{3}{4}e^2(x - 2)$ M1 A1
 $y = \frac{3}{4}e^2 x$
 $\therefore x = 0 \Rightarrow y = 0$ so passes through origin A1
- (d) $x_1 = -1.125589, x_2 = -1.125803, x_3 = -1.125804$ (7sf) M1 A2
 $\therefore x$ -coordinate of $B = -1.1258$ (5sf) A1 (13)

Total (75)

