GCE Examinations

Advanced Subsidiary

Core Mathematics C3

Paper B

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has seven questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

1. *(a)* Simplify

$$\frac{x^2 + 7x + 12}{2x^2 + 9x + 4}.$$ **(3)**

(b) Solve the equation

$$\ln(x^2 + 7x + 12) - 1 = \ln(2x^2 + 9x + 4),$$

giving your answer in terms of e. **(4)**

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2. A curve has the equation $y = \sqrt{3x + 11}$.

The point P on the curve has x-coordinate 3.

(a) Show that the tangent to the curve at P has the equation

$$3x - 4\sqrt{5}\,y + 31 = 0.$$ **(6)**

The normal to the curve at P crosses the y-axis at Q.

(b) Find the y-coordinate of Q in the form $k\sqrt{5}$. **(3)**

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3. *(a)* Use the identities for $\sin(A + B)$ and $\sin(A - B)$ to prove that

$$\sin P + \sin Q \equiv 2 \sin \frac{P + Q}{2} \cos \frac{P - Q}{2}.$$ **(4)**

(b) Find, in terms of π, the solutions of the equation

$$\sin 5x + \sin x = 0,$$

for x in the interval $0 \leq x < \pi$. **(5)**

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4. The curve with equation $y = x^{\frac{5}{2}} \ln \frac{x}{4}$, $x > 0$ crosses the x-axis at the point P.

(a) Write down the coordinates of P. **(1)**

The normal to the curve at P crosses the y-axis at the point Q.

(b) Find the area of triangle OPQ where O is the origin. **(6)**

The curve has a stationary point at R.

(c) Find the x-coordinate of R in exact form. **(3)**

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5. $$f(x) \equiv 2x^2 + 4x + 2, \quad x \in \mathbb{R}, \quad x \geq -1.$$

   (a)   Express f(x) in the form $a(x + b)^2 + c$.   **(2)**

   (b)   Describe fully two transformations that would map the graph of $y = x^2, \ x \geq 0$ onto the graph of $y = f(x)$.   **(3)**

   (c)   Find an expression for $f^{-1}(x)$ and state its domain.   **(4)**

   (d)   Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram and state the relationship between them.   **(4)**

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6. $$f(x) = e^{3x + 1} - 2, \quad x \in \mathbb{R}.$$

   (a)   State the range of f.   **(1)**

The curve $y = f(x)$ meets the y-axis at the point P and the x-axis at the point Q.

   (b)   Find the exact coordinates of P and Q.   **(4)**

   (c)   Show that the tangent to the curve at P has the equation

$$y = 3ex + e - 2.$$ **(4)**

   (d)   Find to 3 significant figures the x-coordinate of the point where the tangent to the curve at P meets the tangent to the curve at Q.   **(4)**

Turn over

7. *(a)* Solve the equation

$$\pi - 3\arccos\theta = 0.$$ **(2)**

(b) Sketch on the same diagram the curves $y = \arccos(x-1)$, $0 \le x \le 2$ and $y = \sqrt{x+2}$, $x \ge -2$. **(5)**

Given that α is the root of the equation

$$\arccos(x-1) = \sqrt{x+2},$$

(c) show that $0 < \alpha < 1$, **(3)**

(d) use the iterative formula

$$x_{n+1} = 1 + \cos\sqrt{x_n + 2}$$

with $x_0 = 1$ to find α correct to 3 decimal places. **(4)**

END

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