OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4723

Core Mathematics 3

Specimen Paper

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF 1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

[Turn over

1 Solve the inequality $|2x+1| > |x-1|$. [5]

2 **(i)** Prove the identity

$$\sin(x+30°)+(\sqrt{3})\cos(x+30°) \equiv 2\cos x,$$

where x is measured in degrees. [4]

(ii) Hence express $\cos 15°$ in surd form. [2]

3 The sequence defined by the iterative formula

$$x_{n+1} = \sqrt[3]{(17-5x_n)},$$

with $x_1 = 2$, converges to α.

(i) Use the iterative formula to find α correct to 2 decimal places. You should show the result of each iteration. [3]

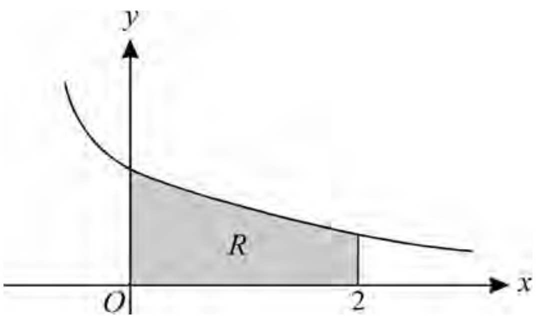
(ii) Find a cubic equation of the form

$$x^3 + cx + d = 0$$

which has α as a root. [2]

(iii) Does this cubic equation have any other real roots? Justify your answer. [2]

4



The diagram shows the curve

$$y = \frac{1}{\sqrt{(4x+1)}}.$$

The region R (shaded in the diagram) is enclosed by the curve, the axes and the line $x = 2$.

(i) Show that the exact area of R is 1. [4]

(ii) The region R is rotated completely about the x-axis. Find the exact volume of the solid formed. [4]

5 At time t minutes after an oven is switched on, its temperature θ°C is given by

$$\theta = 200 - 180\,e^{-0.1t}\,.$$

(i) State the value which the oven's temperature approaches after a long time. [1]

(ii) Find the time taken for the oven's temperature to reach 150°C. [3]

(iii) Find the rate at which the temperature is increasing at the instant when the temperature reaches 150°C. [4]

6 The function f is defined by

$$\mathrm{f} : x \mapsto 1 + \sqrt{x} \qquad \text{for } x \geqslant 0\,.$$

(i) State the domain and range of the inverse function f^{-1}. [2]

(ii) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

(iii) By considering the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, show that the solution to the equation

$$\mathrm{f}(x) = \mathrm{f}^{-1}(x)$$

is $x = \frac{1}{2}(3 + \sqrt{5})$. [4]

7 **(i)** Write down the formula for $\tan 2x$ in terms of $\tan x$. [1]

(ii) By letting $\tan x = t$, show that the equation

$$4\tan 2x + 3\cot x \sec^2 x = 0$$

becomes

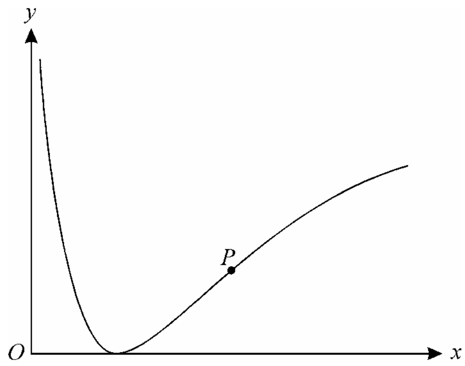
$$3t^4 - 8t^2 - 3 = 0\,.$$ [4]

(iii) Hence find all the solutions of the equation

$$4\tan 2x + 3\cot x \sec^2 x = 0$$

which lie in the interval $0 \leqslant x \leqslant 2\pi$. [4]

8

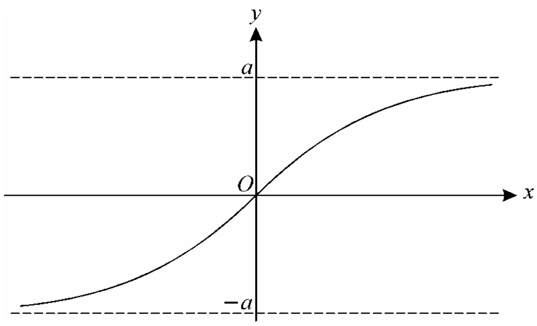


The diagram shows the curve $y = (\ln x)^2$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(ii) The point P on the curve is the point at which the gradient takes its maximum value. Show that the tangent at P passes through the point $(0, -1)$. [6]

9



The diagram shows the curve $y = \tan^{-1} x$ and its asymptotes $y = \pm a$.

(i) State the exact value of a. [1]

(ii) Find the value of x for which $\tan^{-1} x = \frac{1}{2}a$. [2]

The equation of another curve is $y = 2\tan^{-1}(x-1)$.

(iii) Sketch this curve on a copy of the diagram, and state the equations of its asymptotes in terms of a. [3]

(iv) Verify by calculation that the value of x at the point of intersection of the two curves is 1.54, correct to 2 decimal places. [2]

Another curve (which you are *not* asked to sketch) has equation $y = \left(\tan^{-1} x\right)^2$.

(v) Use Simpson's rule, with 4 strips, to find an approximate value for $\int_0^1 \left(\tan^{-1} x\right)^2 \, dx$. [3]