



**Monday 16 June 2014 – Morning**

**A2 GCE MATHEMATICS**

**4723/01** Core Mathematics 3

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4723/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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1 Given that  $y = 4x^2 \ln x$ , find the value of  $\frac{d^2y}{dx^2}$  when  $x = e^2$ . [5]

2 By first using appropriate identities, solve the equation

$$5 \cos 2\theta \operatorname{cosec} \theta = 2$$

for  $0^\circ < \theta < 180^\circ$ . [6]

3 (i) Use Simpson's rule with four strips to find an approximation to

$$\int_0^2 e^{\sqrt{x}} dx,$$

giving your answer correct to 3 significant figures. [4]

(ii) Deduce an approximation to  $\int_0^2 (1 + 10e^{\sqrt{x}}) dx$ . [2]

4 The functions  $f$  and  $g$  are defined for all real values of  $x$  by

$$f(x) = 2x^3 + 4 \quad \text{and} \quad g(x) = \sqrt[3]{x-10}.$$

(i) Evaluate  $f^{-1}(-50)$ . [2]

(ii) Show that  $fg(x) = 2x - 16$ . [2]

(iii) Differentiate  $gf(x)$  with respect to  $x$ . [3]

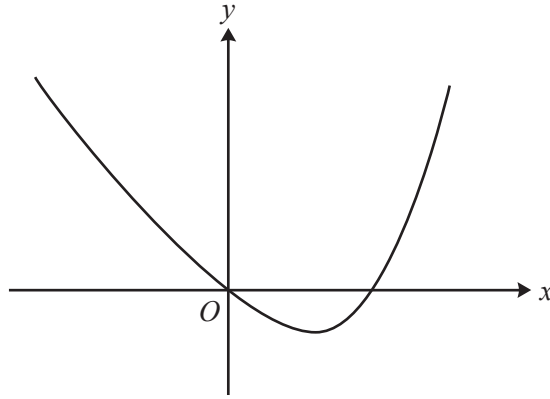
5 (a) The mass,  $M$  grams, of a substance at time  $t$  years is given by

$$M = 58e^{-0.33t}.$$

Find the rate at which the mass is decreasing at the instant when  $t = 4$ . Give your answer correct to 2 significant figures. [3]

(b) The mass of a second substance is increasing exponentially. The initial mass is 42.0 grams and, 6 years later, the mass is 51.8 grams. Find the mass at a time 24 years after the initial value. [4]

6



The diagram shows the curve  $y = x^4 - 8x$ .

- (i) By sketching a second curve on the copy of the diagram, show that the equation

$$x^4 + x^2 - 8x - 9 = 0$$

has two real roots. State the equation of the second curve. [2]

- (ii) The larger root of the equation  $x^4 + x^2 - 8x - 9 = 0$  is denoted by  $\alpha$ .

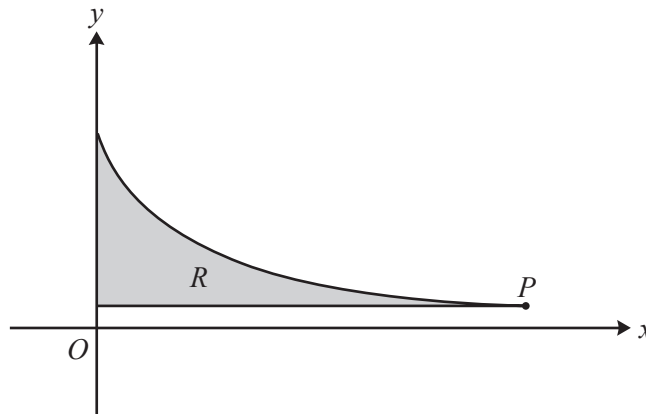
- (a) Show by calculation that  $2.1 < \alpha < 2.2$ . [2]

- (b) Use an iterative process based on the equation

$$x = \sqrt[4]{9 + 8x - x^2},$$

with a suitable starting value, to find  $\alpha$  correct to 3 decimal places. Give the result of each step of the iterative process. [4]

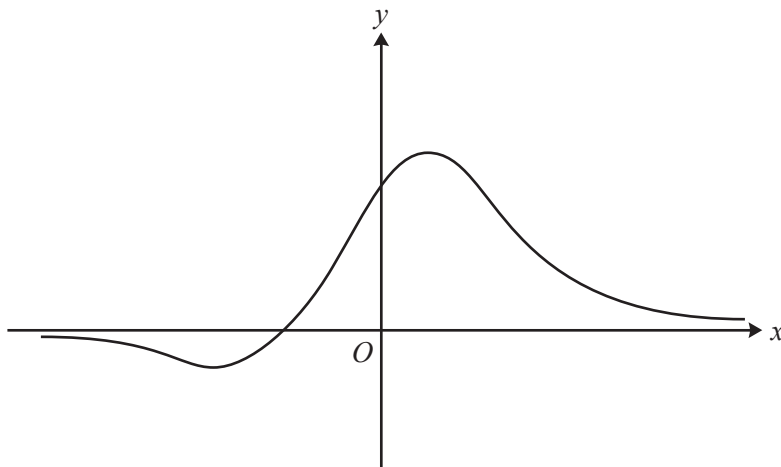
7



The diagram shows the curve  $y = \sqrt{\frac{3}{4x+1}}$  for  $0 \leq x \leq 20$ . The point  $P$  on the curve has coordinates  $(20, \frac{1}{9}\sqrt{3})$ . The shaded region  $R$  is enclosed by the curve and the lines  $x = 0$  and  $y = \frac{1}{9}\sqrt{3}$ .

- (i) Find the exact area of  $R$ . [4]

- (ii) Find the exact volume of the solid obtained when  $R$  is rotated completely about the  $x$ -axis. [6]



The diagram shows the curve  $y = \frac{2x+4}{x^2+5}$ .

(i) Find  $\frac{dy}{dx}$  and hence find the coordinates of the two stationary points. [6]

(ii) The function  $g$  is defined for all real values of  $x$  by

$$g(x) = \left| \frac{2x+4}{x^2+5} \right|.$$

(a) Sketch the curve  $y = g(x)$  and state the range of  $g$ . [3]

(b) It is given that the equation  $g(x) = k$ , where  $k$  is a constant, has exactly two distinct real roots. Write down the set of possible values of  $k$ . [2]

9 (i) Express  $5 \cos(\theta - 60^\circ) + 3 \cos \theta$  in the form  $R \sin(\theta + \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . [4]

(ii) Hence

(a) give details of the transformations needed to transform the curve  $y = 5 \cos(\theta - 60^\circ) + 3 \cos \theta$  to the curve  $y = \sin \theta$ , [3]

(b) find the smallest positive value of  $\beta$  satisfying the equation

$$5 \cos\left(\frac{1}{3}\beta - 40^\circ\right) + 3 \cos\left(\frac{1}{3}\beta + 20^\circ\right) = 3. \quad [5]$$

**END OF QUESTION PAPER**



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