

Thursday 13 June 2013 – Morning

A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4723/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Find

(i) $\int (4 - 3x)^7 dx,$

(ii) $\int (4 - 3x)^{-1} dx.$

[5]

2 Using an appropriate identity in each case, find the possible values of

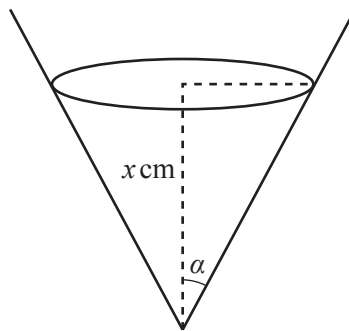
(i) $\sin \alpha$ given that $4 \cos 2\alpha = \sin^2 \alpha,$

[3]

(ii) $\sec \beta$ given that $2 \tan^2 \beta = 3 + 9 \sec \beta.$

[4]

3



The diagram shows a container in the form of a right circular cone. The angle between the axis and the slant height is α , where $\alpha = \tan^{-1}(\frac{1}{2})$. Initially the container is empty, and then liquid is added at the rate of 14 cm^3 per minute. The depth of liquid in the container at time t minutes is x cm.

(i) Show that the volume, $V \text{ cm}^3$, of liquid in the container when the depth is x cm is given by

$$V = \frac{1}{12} \pi x^3.$$

[The volume of a cone is $\frac{1}{3} \pi r^2 h.$]

[2]

(ii) Find the rate at which the depth of the liquid is increasing at the instant when the depth is 8 cm. Give your answer in cm per minute correct to 2 decimal places.

[3]

4 Find the exact value of the gradient of the curve

$$y = \sqrt{4x - 7} + \frac{4x}{2x + 1}$$

at the point for which $x = 4.$

[6]

5 (i) Give full details of a sequence of two transformations needed to transform the graph of $y = |x|$ to the graph of $y = |2(x + 3)|.$

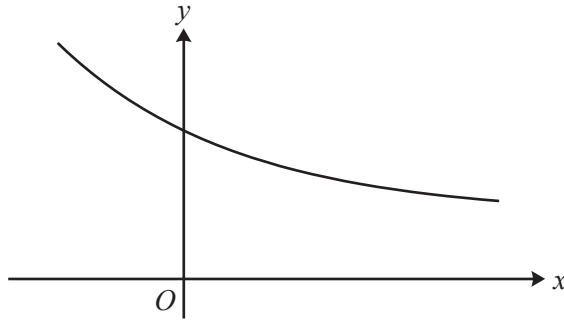
[3]

(ii) Solve the inequality $|x| > |2(x + 3)|$, showing all your working.

[5]

- 6 The value of $\int_0^8 \ln(3 + x^2) dx$ obtained by using Simpson's rule with four strips is denoted by A .
- (i) Find the value of A correct to 3 significant figures. [4]
- (ii) Explain why an approximate value of $\int_0^8 \ln(9 + 6x^2 + x^4) dx$ is $2A$. [2]
- (iii) Explain why an approximate value of $\int_0^8 \ln(3e + ex^2) dx$ is $A + 8$. [2]

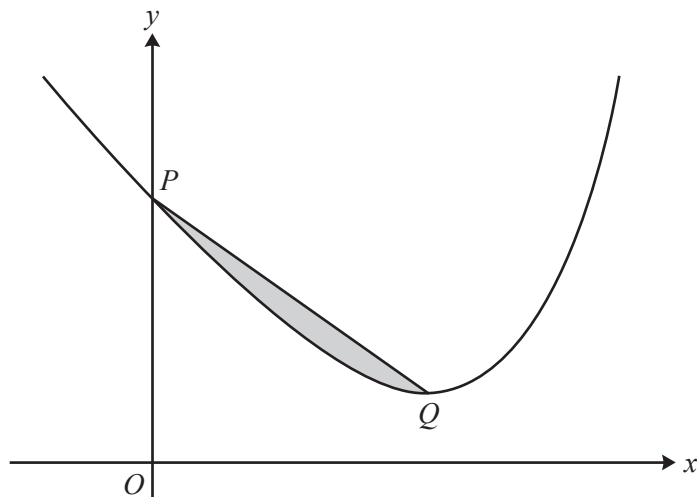
7



The diagram shows the curve $y = f(x)$, where f is the function defined for all real values of x by

$$f(x) = 3 + 4e^{-x}.$$

- (i) State the range of f . [1]
- (ii) Find an expression for $f^{-1}(x)$, and state the domain and range of f^{-1} . [4]
- (iii) The straight line $y = x$ meets the curve $y = f(x)$ at the point P . By using an iterative process based on the equation $x = f(x)$, with a starting value of 3, find the coordinates of the point P . Show all your working and give each coordinate correct to 3 decimal places. [4]
- (iv) How is the point P related to the curves $y = f(x)$ and $y = f^{-1}(x)$? [1]
- 8 (i) Express $4 \cos \theta - 2 \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]
- (ii) Hence
- (a) solve the equation $4 \cos \theta - 2 \sin \theta = 3$ for $0^\circ < \theta < 360^\circ$, [4]
- (b) determine the greatest and least values of
- $$25 - (4 \cos \theta - 2 \sin \theta)^2$$
- as θ varies, and, in each case, find the smallest positive value of θ for which that value occurs. [5]



The diagram shows the curve

$$y = e^{2x} - 18x + 15.$$

The curve crosses the y -axis at P and the minimum point is Q . The shaded region is bounded by the curve and the line PQ .

- (i) Show that the x -coordinate of Q is $\ln 3$. [3]
- (ii) Find the exact area of the shaded region. [8]

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.