GCE

Mathematics

Advanced GCE

Unit **4723:** Core Mathematics 3

Mark Scheme for June 2013

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations

Annotation in scoris	Meaning
✓and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(i)		Obtain integral of form $k(4-3x)^8$	M1	any non-zero constant k; using substitution to obtain ku^8 earns M1
			Obtain $-\frac{1}{24}(4-3x)^8$	A1	or unsimplified equiv; must be in terms of x
1	(ii)		Obtain integral of form $k\ln(4-3x)$	M1	any non-zero constant k; allow M1 if brackets missing; using substitution to obtain $k\ln u$ earns M1; $\log(4-3x)$ with base e not specified is M1A0
			Obtain $-\frac{1}{3}\ln(4-3x)$	A1	now with either brackets or modulus signs; must be in terms of x; note that $-\frac{1}{3}\ln(x-\frac{4}{3})$ and $-\frac{1}{3}\ln(\frac{4}{3}-x)$ are correct alternatives
			Include + c or + k at least once	B1	anywhere in solution to question 1; this mark available even if no other marks earned
				[5]	
2	(i)		Use $2\cos^2\alpha-1$ or $\cos^2\alpha-\sin^2\alpha$ or $1-2\sin^2\alpha$	B1	
			Obtain equation in which $\sin^2\alpha$ appears once	M1	condoning sign slips or arithmetic slips; for solution which gives equation involving $\tan^2\alpha$, M1 is not earned until valid method for reaching $\sin\alpha$ is used; attempt involving $4(1-s^2)=s^2$ is M0
			Obtain $\pm\frac{2}{3}$	A1	both values needed; ± 0.667 is A0; $\pm\sqrt{\frac{4}{9}}$ is A0; ignore subsequent work to find angle(s)
				[3]	
2	(ii)		Either Attempt use of identity	M1	of form $\tan^2\beta = \pm\sec^2\beta \pm 1$
			Obtain $2\sec^2\beta - 9\sec\beta - 5 = 0$	A1	condone absence of $=0$
			Attempt solution of 3-term quadratic in $\sec\beta$ to obtain at least one value of $\sec\beta$	M1	if factorising, factors must be such that expansion gives their first and third terms; if using formula, this must be correct for their values
			Obtain 5 with no errors in solution	A1	and, finally, no other value; no need to justify rejection of $-\frac{1}{2}$
				[4]	
			Or Attempt to express equation in terms of $\cos\beta$	M1	using identities which are correct apart maybe for sign slips
			Obtain $5\cos^2\beta + 9\cos\beta - 2 = 0$	A1	condone absence of $=0$
			Attempt solution of 3-term quadratic and show switch at least once to a secant value	M1	if factorising, factors must be such that expansion gives their first and third terms; if using formula, this must be correct for their values
			Obtain 5 with no errors in solution	A1 [4]	and, finally, no other value; no need to justify rejection of $-\frac{1}{2}$

Question			Answer	Marks	Guidance
3	(i)		Use α (possibly implicitly) to state that radius of 'base' is $\frac{1}{2}x$	*B1	or to obtain equiv such as $2r = x$ or $\frac{r}{x} = \frac{1}{2}$ or $\frac{x}{r} = 2$
			Substitute into formula to obtain $\frac{1}{3}\pi(\frac{1}{2}x)^2 x$ or $\frac{1}{3}\pi\frac{1}{4}x^2 x$ and obtain $\frac{1}{12}\pi x^3$	B1	dep *B; AG; necessary detail needed Note: comparing formulae $\frac{1}{3}\pi r^2 h$ and $\frac{1}{12}\pi x^3$ to 'deduce' is B0B0
				[2]	
3	(ii)		Differentiate to obtain $\frac{1}{4}\pi x^2$ or equiv	B1	whatever they call it
			Attempt division involving 14 and their value of derivative when $x = 8$	M1	ie $14 \div$ deriv or deriv $\div 14$ with $x = 8$
			Obtain 0.28	A1	allow 0.279 but not greater accuracy Alternatives: 1. $14t = \frac{1}{12}\pi x^3$ Obtain $\frac{dt}{dx} = \frac{1}{56}\pi x^2$ B1 Sub 8 and invert M1 Ans A1 2. $x^3 = \frac{168t}{\pi}$ Obtain $3x^2 \frac{dx}{dt} = \frac{168}{\pi}$ B1 Sub 8 M1 Ans A1
				[3]	
4			Differentiate first term to obtain form $k(4x-7)^{-\frac{1}{2}}$	*M1	any non-zero constant k ; M0 if this differentiation is carried out in the midst of some incorrect involved expression
			Obtain $2(4x-7)^{-\frac{1}{2}}$	A1	or (unsimplified) equiv
			Attempt use of quotient rule or, after adjustment, product rule	*M1	for QR, allow numerator wrong way round but needs – sign in numerator; condone a single error such as absence of square in denominator, absence of brackets, …; for PR, condone no use of chain rule M0 if this differentiation is carried out in the midst of some incorrect involved expression
			Obtain $\frac{4(2x+1)-8x}{(2x+1)^2}$ or $4(2x+1)^{-1} - 8x(2x+1)^{-2}$	A1	or (unsimplified) equivs; give A0 if brackets absent unless subsequent calculation indicates their 'presence'
			Substitute 4 into expression for first derivative so that (initially at least) exactness is retained	M1	dep *M *M
			Obtain $\frac{58}{81}$	A1	answer must be exact Note: using $y = \sqrt{4x-7} + \frac{4}{2x+1}$: do not apply MR
				[6]	

Question			Answer	Marks	Guidance
5	(i)		Refer to translation and stretch	M1	in either order; ignore details here; allow any equiv wording (such as move or shift for translation) to describe geometrical transformation but not statements such as add 3 to *x*
			Either State translation in negative *x*-direction by 3	A1	or state translation by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$; accept horizontal to indicate direction; term 'translate' or 'translation' needed for award of A1
			State stretch by factor 2 in *y*-direction	A1	or parallel to *y*-axis or vertically; term 'stretch' needed for award of A1; these two transformations can be given in either order SC: if M0 but details of one transformation correct, award B1 for 1/3 (in Either, Or 1, Or 2 cases)
				[3]	
			Or 1 State stretch by factor $\frac{1}{2}$ in *x*-direction	A1	or parallel to *x*-axis; term 'stretch' needed for award of A1
			State translation in negative *x*-direction by 3	A1 [3]	or state translation by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$; term 'translate' or 'translation' needed for award of A1; these two transformations must be in this order – if details correct for M1A1A1 but order wrong, award M1A1A0
			Or 2 State translation in negative *x*-direction by 6	A1	or state translation by $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$; term 'translate' or 'translation' needed for award of A1
			State stretch by factor $\frac{1}{2}$ in *x*-direction	A1 [3]	or parallel to *x*-axis; term 'stretch' needed for award of A1; these two transformations must be in this order – if details correct for M1A1A1 but order wrong, award M1A1A0
5	(ii)		Either Solve linear eqn/ineq to obtain critical value –6	B1	
			Attempt solution of linear eqn/ineq where signs of *x* and 2*x* are different	M1	
			Obtain critical value –2	A1	
			Attempt solution of inequality	M1	using table, sketch, …; implied by correct answer or answer of form $a < x < b$ or of form $x < a, x > b$ (where $a < b$); allow $\leq$ here
			Obtain $-6 < x < -2$	A1	as final answer; must be < not $\leq$; allow "$x > -6$ and $x < -2$"
				[5]	

Question			Answer	Marks	Guidance
			Or Square both sides to obtain $x^2 > 4(x^2+6x+9)$	B1	or equiv
			Attempt solution of 3-term quadratic eqn/ineq	M1	with same guidelines as in Q2(ii) for factorising and formula
			Obtain critical values –6 and –2	A1	
			Attempt solution of inequality	M1	using table, sketch, …; implied by correct answer or answer of form $a<x<b$ or of form $x<a, x>b$ (where $a<b$); allow $\le$ here
			Obtain $-6<x<-2$	A1 [5]	as final answer; must be < not $\le$; allow ' $x>-6$ and $x<-2$ '
6	(i)		Attempt evaluation involving *y* values	M1	with coefficients 1, 4 and 2 each occurring at least once; allow for wrong *y*-values; solution must include sufficient evidence of method
			Obtain $k(\ln 3+4\ln 7+2\ln 19+4\ln 39+\ln 67)$	A1	any constant *k*; or decimal equivs; correct use of brackets required unless subsequent working shows their 'presence'
			Identify value of *k* as $\frac{2}{3}$	A1	as factor for their complete expression
			Obtain 22.4	A1 **[4]**	allow any value rounding to 22.4; answer only is 0/4
6	(ii)		State $9+6x^2+x^4 = (3+x^2)^2$	B1	or, if proceeding numerically, demonstrate in at least three cases that $\ln 9=\ln 3^2, \ln 49=\ln 7^2, \ln 361=\ln 19^2, ...$
			Show relevant property $\ln(3+x^2)^2 = 2\ln(3+x^2)$ and conclude with value $2A$	B1 **[2]**	AG; necessary detail needed; if proceeding numerically, needs all five cases with relevant property Note: using Simpson's rule again here is B0B0
6	(iii)		Recognise $\ln(3\mathrm{e}+\mathrm{e}x^2)$ as $1+\ln(3+x^2)$	B1	
			Indicate in some way that $\int_0^8 1\,\mathrm{d}x$ is 8 and conclude with value $A+8$	B1 **[2]**	AG; necessary detail needed Note: using Simpson's rule again here is B0B0
7	(i)		State $y>3$ or $\mathrm{f}(x)>3$ or $\mathrm{f}>3$ or 'greater than 3'	B1 **[1]**	must be > not $\ge$; allow $3<y<\infty$

Question			Answer	Marks	Guidance
7	(ii)		Obtain expression or eqn involving $\ln(\frac{y-3}{4})$ or $\ln(\frac{x-3}{4})$ Obtain $\ln(\frac{4}{x-3})$ or $-\ln(\frac{x-3}{4})$ State domain is $x > 3$ or equiv State range is all real numbers or equiv	M1 A1 B1FT B1 **[4]**	or equivs such as $\ln(\frac{4}{y-3})$ or $\ln(\frac{4}{x-3})$ or equiv following answer to part **(i)** **(**but with adjustment so that reference is to *x*)
7	(iii)		Obtain correct first iterate Show correct iteration process Obtain at least 3 correct iterates Obtain (3.168, 3.168) [3 → 3.199148.. → 3.163187.. → 3.169162.. → 3.168155.. → 3.168324..]	B1 M1 A1 A1 **[4]**	showing at least 3 dp; B0 if initial value not 3 but then M1A1A1 available showing at least 3 iterates in all; may be implied by plausible converging values; M1available if based on equation with just a slip in $x = f(x)$ but M0 if based on clearly different equation allowing recovery after error; iterates to only 3 dp acceptable; values may be rounded or truncated each coordinate required to exactly 3 dp; award A0 if fewer than 4 iterates shown; part (iii) consisting of answer only gets 0 out of 4
7	(iv)		State *P* is point where the curves meet	B1 **[1]**	or equiv
8	(i)		Obtain $R = \sqrt{20}$ or $R = 4.47$ Attempt to find value of α Obtain 26.6	B1 M1 A1 **[3]**	 implied by correct value or its complement; allow sin/cos muddles; allow use of radians for M1; condone use of $\cos\alpha = 4, \sin\alpha = 2$ here but not for A1 or greater accuracy 26.565…; with no wrong working seen
8	(ii)	(a)	Show correct process for finding one answer Obtain 21.3 Show correct process for finding second answer Obtain 286 or 285.6	M1 A1FT M1 A1FT **[4]**	allowing for case where the answer is negative or greater accuracy 21.3045…; or anything rounding to 21.3 with no obvious error; following a wrong value of α but not wrong *R* ie attempting fourth quadrant value minus α value or greater accuracy 285.5653…; or anything rounding to 286 with no obvious error; following a wrong value of α but not wrong *R* ;and no others between 0° and 360°

Question			Answer	Marks	Guidance
8	(ii)	(b)	State greatest value is 25	B1	allow if α incorrect
			Obtain value 63.4 clearly associated with correct greatest value	B1FT	or greater accuracy 63.4349…; following a wrong value of α
			State least value is 5	B1	allow if α incorrect
			Attempt to find θ from $\cos(\theta + \text{their}\,\alpha) = -1$	M1	and clearly associated with correct least value
			Obtain 153 or 153.4	A1FT	or greater accuracy 153.4349…; following a wrong value of α
				[5]	
9	(i)		Differentiate to obtain $2e^{2x} - 18$	B1	
			Equate first derivative to zero and use legitimate method to reach equation without e involved	M1	
			Confirm $x = \ln 3$	A1	AG; necessary detail needed (in particular, for solutions concluding $x = \frac{1}{2}\ln 9 = \ln 3$ or equiv award A0)
				[3]	
9	(ii)		Attempt integration	*M1	confirmed by at least one correct term
			Obtain $\frac{1}{2}e^{2x} - 9x^2 + 15x$	A1	or equiv
			Apply limits 0 and $\ln 3$ to obtain exact unsimplified expression	M1	dep *M
			Obtain $4 - 9(\ln 3)^2 + 15\ln 3$	A1	or exact (maybe unsimplified) equiv perhaps still involving e
			Attempt area of trapezium or equiv, retaining exactness throughout	M1	using $\frac{1}{2}\ln 3 \times (y_1 + y_2)$ where y_1 is 15 or 16 and y_2 is attempt at y-coordinate of Q; if using alternative approach involving rectangle and triangle, complete attempt needs to be seen for M1; another alternative approach involves equation of PQ ($y = \frac{8-18\ln 3}{\ln 3}x + 16$) with integration: M1 for attempting equation and integration, A1 for correct answer
			Obtain $\frac{1}{2}\ln 3 \times (16 + 24 - 18\ln 3)$	A1	or equiv perhaps still including e
			Subtract areas the right way round, retaining exactness	M1	dep on award of all three M marks
			Obtain $5\ln 3 - 4$	A1	or similarly simplified exact equiv
				[8]	

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