GCE

Mathematics

Advanced GCE

Unit **4723:** Core Mathematics 3

Mark Scheme for June 2011

Oxford Cambridge and RSA Examinations

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1 **(i)**	Obtain integral of form ke^{2x+1}	M1		any non-zero constant k different from 6; using substitution $u = 2x+1$ to obtain ke^u earns M1 (but answer to be in terms of x)
	Obtain correct $3e^{2x+1}$	A1		or equiv such as $\frac{6}{2}e^{2x+1}$
(ii)	Obtain integral of form $k_1 \ln(2x+1)$	M1		any non-zero constant k_1; allow if brackets absent; $k_1 \ln u$ (after sub'n) earns M1
	Obtain correct $5\ln(2x+1)$	A1		or equiv such as $\frac{10}{2}\ln(2x+1)$; condone brackets rather than modulus signs but brackets or modulus signs must be present (so that $5\ln 2x+1$ earns A0)
	Include … + c at least once	B1	5	anywhere in the whole of question 1; this mark available even if no marks awarded for integration
			$\boxed{5}$	
2	Apply one of the transformations correctly to their equation	B1		
	Obtain correct $-3\ln x + \ln 4$	B1		or equiv
	Show at least one logarithm property	M1		correctly applied to their equation of resulting curve (even if errors have been made earlier)
	Obtain $y = \ln(4x^{-3})$	A1	4	or equiv of required form; $\ln 4x^{-3}$ earns A1; correct answer only earns 4/4; condone absence of $y =$
			$\boxed{4}$	
3 **(a)**	State $14\sin\alpha\cos\alpha = 3\sin\alpha$	B1		or unsimplified equiv such as $7(2\sin\alpha\cos\alpha) = 3\sin\alpha$
	Attempt to find value of $\cos\alpha$	M1		by valid process; may be implied
	Obtain $\frac{3}{14}$	A1	3	exact answer required; ignore subsequent work to find angle
(b)	Attempt use of identity for $\cos 2\beta$	M1		of form $\pm 2\cos^2\beta \pm 1$; initial use of $\cos^2\beta - \sin^2\beta$ needs attempt to express $\sin^2\beta$ in terms of $\cos^2\beta$ to earn M1
	Obtain $6\cos^2\beta + 19\cos\beta + 10$	A1		or unsimplified equiv or equiv involving $\sec\beta$
	Attempt solution of 3-term quadratic eqn	M1		for $\cos\beta$ or (after adjustment) for $\sec\beta$
	Use $\sec\beta = \dfrac{1}{\cos\beta}$ at some stage	M1		or equiv
	Obtain $-\frac{3}{2}$	A1	5	or equiv; and (finally) no other answer
			$\boxed{8}$	

4	(i)	Draw sketch of $y=(x-2)^4$	*B1		touching positive x-axis and extending at least as far as the y-axis; no need for 2 or 16 to be marked; ignore wrong intercepts
		Draw straight line with positive gradient	*B1		at least in first quadrant and reaching positive y-axis; assess the two graphs independently of each other
		Indicate two roots	B1	3	AG; dep *B *B and two correct graphs which meet on the y-axis; indicated in words or by marks on sketch
		[SC: Draw sketch of $y=(x-2)^4-x-16$ and indicate the two roots : B1 (i.e. max 1 mark)]			
	(ii)	State 0 or $x=0$	B1	1	not merely for coordinates (0, 16)
	(iii)	Obtain correct first iterate	B1		to at least 3 dp; any starting value (> –16)
		Show correct iteration process	M1		producing at least 3 iterates in all; may be implied by plausible converging values
		Obtain at least 3 correct iterates	A1		allowing recovery after error; iterates given to only 3 d.p. acceptable; values may be rounded or truncated
		Obtain 4.118	A1	4	answer required to exactly 3 dp; A0 here if number of iterates is not enough to justify 4.118; attempt consisting of answer only earns 0/4

$$
\begin{array}{l}
[\,0 \to 4 \to 4.114743 \to 4.117769 \to 4.117849\,; \\
1 \to 4.030543 \to 4.115549 \to 4.117790 \to 4.117849\,; \\
2 \to 4.059767 \to 4.116321 \to 4.117811 \to 4.117850\,; \\
3 \to 4.087798 \to 4.117060 \to 4.117830 \to 4.117850\,; \\
4 \to 4.114743 \to 4.117769 \to 4.117849 \to 4.117851\,; \\
5 \to 4.140695 \to 4.118452 \to 4.117867 \to 4.117851\,]
\end{array}
$$

8

5	Attempt use of product rule	*M1		to produce $k_1 x\ln(4x-3)+\dfrac{k_2x^2}{4x-3}$ form
	Obtain $2x\ln(4x-3)$	A1		
	Obtain $\ldots+\dfrac{4x^2}{4x-3}$	A1		or equiv
	Attempt second use of product rule	*M1		
	Attempt use of quotient (or product) rule	*M1		allow numerator the wrong way round
	Obtain $2\ln(4x-3)+\dfrac{8x}{4x-3}+\dfrac{8x(4x-3)-16x^2}{(4x-3)^2}$	A1		or equiv
	Substitute 2 into attempt at second deriv	M1		dep *M *M *M
	Obtain $2\ln 5+\frac{96}{25}$	A1	8	or exact equiv consisting of two terms

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6 Method 1: (Differentiation; assume value $\frac{10}{3}$; eqn of tangent; through origin)

Differentiate to obtain $k(3x-5)^{-\frac{1}{2}}$	M1	any constant k
Obtain $\frac{3}{2}(3x-5)^{-\frac{1}{2}}$	A1	or equiv
Attempt to find equation of tangent at P and attempt to show tangent passing through origin	M1	assuming value $\frac{10}{3}$; or equiv
Obtain $y=\frac{3}{2\sqrt{5}}x$ and confirm that tangent passes through O	A1	AG; necessary detail needed

Method 2: (Differentiation; equate $\frac{y\text{ change}}{x\text{ change}}$ to deriv; solve for x)

Differentiate to obtain $k(3x-5)^{-\frac{1}{2}}$	M1	any constant k
Obtain $\frac{3}{2}(3x-5)^{-\frac{1}{2}}$	A1	or equiv
Equate $\frac{y\text{ change}}{x\text{ change}}$ to deriv and attempt solution	M1	
Obtain $\frac{\sqrt{3x-5}}{x}=\frac{3}{2}(3x-5)^{-\frac{1}{2}}$ and solve to obtain $\frac{10}{3}$ only	A1	

Method 3: (Differentiation; find x from $y=\mathrm{f}'(x)\,x$ and $y=\sqrt{3x-5}$)

Differentiate to obtain $k(3x-5)^{-\frac{1}{2}}$	M1	any constant k
Obtain $\frac{3}{2}(3x-5)^{-\frac{1}{2}}$	A1	or equiv
State $y=\frac{3}{2}(3x-5)^{-\frac{1}{2}}x$, $y=\sqrt{3x-5}$, eliminate y and attempt solution	M1	condone this attempt at 'eqn of tangent'
Obtain $\frac{10}{3}$ only	A1	

Method 4: (No differentiation; general line through origin to meet curve at one point only)

Eliminate y from equations $y=kx$ and $y=\sqrt{3x-5}$ and attempt formation of quadratic eqn	M1	
Obtain $k^2x^2-3x+5=0$	A1	or equiv
Equate discriminant to zero to find k	M1	
Obtain $k=\frac{3}{2\sqrt{5}}$ or equiv and confirm $x=\frac{10}{3}$	A1	

Method 5: (No differentiation; use coords of P to find eqn of OP; confirm meets curve once)

Use coordinates $(\frac{10}{3},\sqrt{5})$ to obtain $y=\frac{3\sqrt{5}}{10}x$ or equiv as equation of OP	B1	
Eliminate y from this eqn and eqn of curve and attempt quadratic eqn	M1	should be $9x^2-60x+100=0$ or equiv
Attempt solution or attempt discriminant	M1	
Confirm $\frac{10}{3}$ only or discriminant = 0	A1	

		Either:			
		Integrate to obtain $k(3x-5)^{\frac{3}{2}}$	*M1		any constant k
		Obtain correct $\frac{2}{9}(3x-5)^{\frac{3}{2}}$	A1		
		Apply limits $\frac{5}{3}$ and $\frac{10}{3}$	M1		dep *M; the right way round
		Make sound attempt at triangle area and calculate (triangle area) minus (their area under curve)	M1		or equiv
		Obtain $\frac{10}{6}\sqrt{5}-\frac{10}{9}\sqrt{5}$ and hence $\frac{5}{9}\sqrt{5}$	A1	**9**	or exact equiv involving single term
		Or:			
		Arrange to $x = \dots$ and integrate to obtain $k_1y^3+k_2y$ form	*M1		
		Obtain $\frac{1}{9}y^3+\frac{5}{3}y$	A1		
		Apply limits 0 and $\sqrt{5}$	M1		dep *M; the right way round
		Make sound attempt at triangle area and calculate (their area from integration) minus (triangle area)	M1		
		Obtain $\frac{20}{9}\sqrt{5}-\frac{5}{3}\sqrt{5}$ and hence $\frac{5}{9}\sqrt{5}$	A1	(**9**)	or exact equiv involving single term
				$\boxed{9}$	
7	**(i)**	Either: Attempt solution of at least one linear eq'n of form $ax+b=12$	M1		
		Obtain $\frac{1}{3}$	A2	**3**	and (finally) no other answer
		Or: Attempt solution of 3-term quadratic eq'n obtained by squaring attempt at $\mathrm{g}(x+2)$ on LHS and squaring 12 or –12 on RHS	M1		
		Obtain $\frac{1}{3}$	A2	(**3**)	and (finally) no other answer
	(ii)	Either: Obtain $3(3x+5)+5$ for h	B1		
		Attempt to find inverse function	M1		of function of form $ax+b$
		Obtain $\frac{1}{9}(x-20)$	A1	**3**	or equiv in terms of x
		Or: State or imply g^{-1} is $\frac{1}{3}(x-5)$	B1		
		Attempt composition of g^{-1} with g^{-1}	M1		
		Obtain $\frac{1}{9}(x-5)-\frac{5}{3}$	A1	(**3**)	or more simplified equiv in terms of x
	(iii)	State $x \le 0$	B2	**2**	give B1 for answer $x<0$
				$\boxed{8}$	

8	**(i)**	Differentiate to obtain form $ke^{-0.014t}$	M1		any constant k different from 400
		Obtain $5.6e^{-0.014t}$ or $-5.6e^{-0.014t}$	A1		or (unsimplified) equiv
		Obtain 4.9 or –4.9 or 4.87 or –4.87	A1	**3**	but not greater accuracy; allow if final statement seems contradictory; answer only earns 0/3 – differentiation is needed
	(ii)	Either: State or imply $M_2 = 75e^{kt}$	B1		or equiv
		Attempt to find formula for M_2	M1		
		Obtain $M_2 = 75e^{0.047t}$	A1		or equiv such as $75e^{(\frac{1}{10}\ln\frac{8}{5})t}$
		Equate masses and attempt rearrangement	M1		as far as equation with e appearing once
		Obtain $e^{0.061t} = \frac{16}{3}$	A1	**5**	or equiv of required form which might involve 5.33 or greater accuracy on RHS; final two marks might be earned in part iii
		Or: State or imply $M_2 = 75 \times r^{0.1t}$	B1		for positive value r
		Obtain $75 \times 1.6^{0.1t}$	B1		
		Attempt to find M_2 in terms of e	M1		
		Equate masses and attempt rearrangement	M1		
		Obtain $e^{0.061t} = \frac{16}{3}$	A1	**5**	or equiv of required form which might involve 5.33 or greater accuracy on RHS; final two marks might be earned in part iii
	(iii)	Attempt solution involving logarithm of any equation of form $e^{mt} = c_1$	M1		whether the conclusion of part ii or not
		Obtain 27.4	A1	**2**	or greater accuracy 27.4422…; correct answer only earns both marks
				$\boxed{10}$	

9	**(i)**	Use at least one identity correctly	B1		angle-sum or angle-difference identity
		Attempt use of relevant identities in single rational expression	M1		not earned if identities used in expression where step equiv to $\frac{A+B+C}{D+E+F}=\frac{A}{D}+\frac{B}{E}+\frac{C}{F}$ or similar has been carried out; condone (for M1A1) if signs of identities apparently switched (so that, for example, denominator appears as $\cos\theta\cos\alpha-\sin\theta\sin\alpha+3\cos\theta+\cos\theta\cos\alpha+\sin\theta\sin\alpha$)
		Obtain $\frac{2\sin\theta\cos\alpha+3\sin\theta}{2\cos\theta\cos\alpha+3\cos\theta}$	A1		or equiv but with the other two terms from each of num'r and den'r absent
		Attempt factorisation of num'r and den'r	M1		
		Obtain $\frac{\sin\theta}{\cos\theta}$ and hence $\tan\theta$	A1	**5**	AG; necessary detail needed
	(ii)	State or imply form $k\tan 150°$	M1		obtained without any wrong method seen
		State or imply $\frac{4}{3}\tan 150°$	A1		or equiv such as $\frac{12\sin 150°}{9\cos 150°}$
		Obtain $-\frac{4}{9}\sqrt{3}$	A1	**3**	or exact equiv (such as $-\frac{4}{3\sqrt{3}}$); correct answer only earns 3/3
	(iii)	State or imply $\tan 6\theta = k$	B1		
		State $\frac{1}{6}\tan^{-1}k$	B1		
		Attempt second value of θ	M1		using $6\theta=\tan^{-1}k+$ (multiple of 180)
		Obtain $\frac{1}{6}\tan^{-1}k+30°$	A1	**4**	and no other value
				$\boxed{12}$	

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