



ADVANCED GCE
MATHEMATICS
Core Mathematics 3

4723

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

None

Monday 1 June 2009
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1

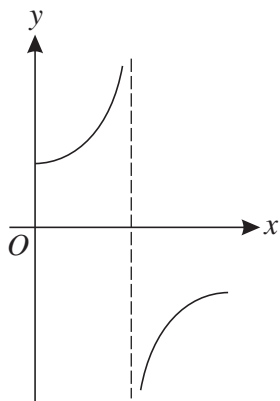


Fig. 1

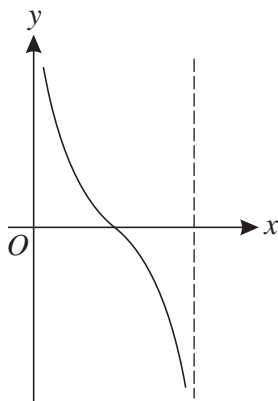


Fig. 2

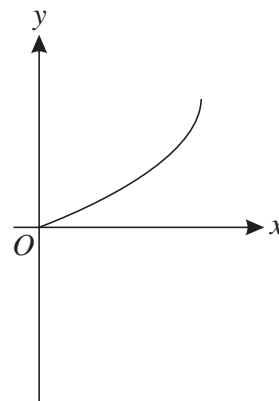


Fig. 3

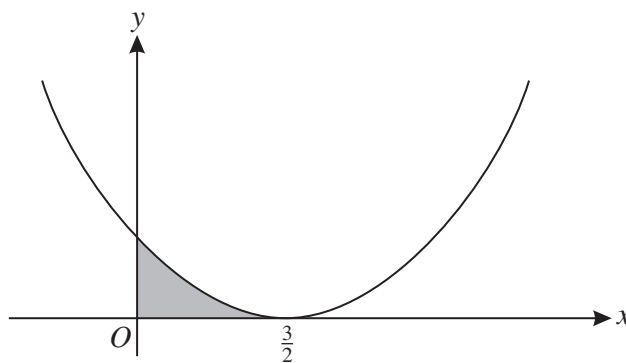
Each diagram above shows part of a curve, the equation of which is one of the following:

$$y = \sin^{-1} x, \quad y = \cos^{-1} x, \quad y = \tan^{-1} x, \quad y = \sec x, \quad y = \operatorname{cosec} x, \quad y = \cot x.$$

State which equation corresponds to

- (i) Fig. 1, [1]
- (ii) Fig. 2, [1]
- (iii) Fig. 3. [1]

2



The diagram shows the curve with equation $y = (2x - 3)^2$. The shaded region is bounded by the curve and the lines $x = 0$ and $y = 0$. Find the exact volume obtained when the shaded region is rotated completely about the x -axis. [5]

3 The angles α and β are such that

$$\tan \alpha = m + 2 \quad \text{and} \quad \tan \beta = m,$$

where m is a constant.

- (i) Given that $\sec^2 \alpha - \sec^2 \beta = 16$, find the value of m . [3]
- (ii) Hence find the exact value of $\tan(\alpha + \beta)$. [3]

4 It is given that $\int_a^{3a} (e^{3x} + e^x) dx = 100$, where a is a positive constant.

(i) Show that $a = \frac{1}{9} \ln(300 + 3e^a - 2e^{3a})$. [5]

(ii) Use an iterative process, based on the equation in part (i), to find the value of a correct to 4 decimal places. Use a starting value of 0.6 and show the result of each step of the process. [4]

5 The functions f and g are defined for all real values of x by

$$f(x) = 3x - 2 \quad \text{and} \quad g(x) = 3x + 7.$$

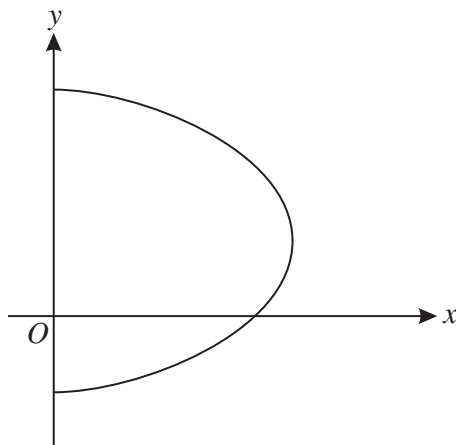
Find the exact coordinates of the point at which

(i) the graph of $y = fg(x)$ meets the x -axis, [3]

(ii) the graph of $y = g(x)$ meets the graph of $y = g^{-1}(x)$, [3]

(iii) the graph of $y = |f(x)|$ meets the graph of $y = |g(x)|$. [4]

6



The diagram shows the curve with equation $x = (37 + 10y - 2y^2)^{\frac{1}{2}}$.

(i) Find an expression for $\frac{dx}{dy}$ in terms of y . [2]

(ii) Hence find the equation of the tangent to the curve at the point $(7, 3)$, giving your answer in the form $y = mx + c$. [5]

7 (i) Express $8 \sin \theta - 6 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(ii) Hence

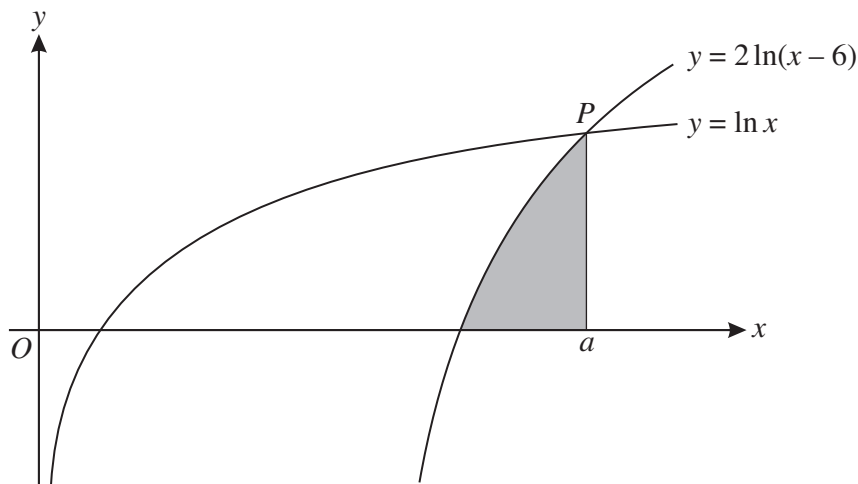
(a) solve, for $0^\circ < \theta < 360^\circ$, the equation $8 \sin \theta - 6 \cos \theta = 9$, [4]

(b) find the greatest possible value of

$$32 \sin x - 24 \cos x - (16 \sin y - 12 \cos y)$$

as the angles x and y vary. [3]

8



The diagram shows the curves $y = \ln x$ and $y = 2 \ln(x - 6)$. The curves meet at the point P which has x -coordinate a . The shaded region is bounded by the curve $y = 2 \ln(x - 6)$ and the lines $x = a$ and $y = 0$.

(i) Give details of the pair of transformations which transforms the curve $y = \ln x$ to the curve $y = 2 \ln(x - 6)$. [3]

(ii) Solve an equation to find the value of a . [4]

(iii) Use Simpson's rule with two strips to find an approximation to the area of the shaded region. [3]

9 (a) Show that, for all non-zero values of the constant k , the curve

$$y = \frac{kx^2 - 1}{kx^2 + 1}$$

has exactly one stationary point. [5]

(b) Show that, for all non-zero values of the constant m , the curve

$$y = e^{mx}(x^2 + mx)$$

has exactly two stationary points. [7]



Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1PB.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.