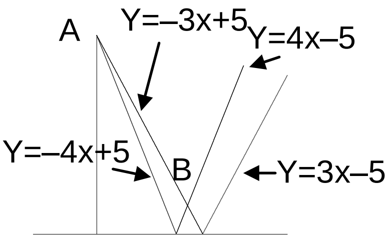
Core 3 June 2008 Answers

1) At A

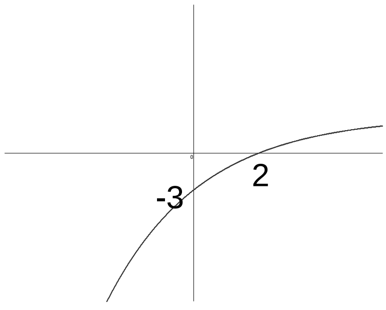
$$-4x+5=-3x+5$$
$$x=0$$

At B

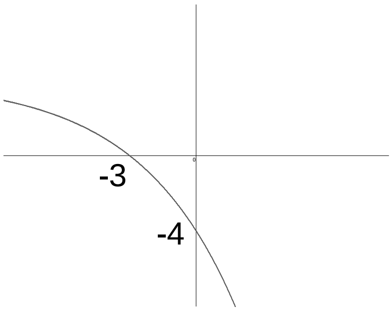
$$4x-5=-3x+5$$
$$7x=10$$
$$x=\frac{10}{7}$$



2) i)



ii)



3) $m=\frac{dy}{dx}=2x\ln x+x^2\times\frac{1}{x}=2x\ln x+x$

$x=e \quad y=e^2 \qquad m=2e\ln e+e=3e$

$$y-e^2=3e(x-e)$$
$$y=3ex-2e^2$$

4) i) $\frac{dy}{dx}=\frac{5}{2}(2x^2+9)^{\frac{3}{2}}\times 4x=10x(2x^2+9)^{\frac{3}{2}}$

$$100=10x(2x^2+9)^{\frac{3}{2}}$$
$$10=x(2x^2+9)^{\frac{3}{2}}$$
$$\therefore x=10(2x^2+9)^{-\frac{3}{2}}$$

ii) $0=10(2x^2+9)^{-\frac{3}{2}}-x$

$x=0.3 \qquad 10(2\times 0.3^2+9)^{-\frac{3}{2}}-0.3=0.0595$ Sign change and continuous function

$x=0.4 \qquad 10(2\times 0.4^2+9)^{-\frac{3}{2}}-0.4=-0.365$ therefore root between 0.3 and 0.4

iii) $x_1=0.3 \qquad x_{n+1}=10(2x^2+9)^{-\frac{3}{2}}$

$x_2=0.359530698 \quad x_3=0.35496646 \quad x_4=0.35534188 \quad x_5=0.355311157$

$x_6=0.355313672$ Therefore $x=0.3553$ to 4dp

5) a) $\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

$\tan 2\alpha \tan\alpha = 8$

$\frac{2\tan\alpha}{1-\tan^2\alpha}\tan\alpha = 8$

$2\tan^2\alpha = 8(1-\tan^2\alpha)$

$2\tan^2\alpha = 8 - 8\tan^2\alpha$

$10\tan^2\alpha = 8$

$\tan^2\alpha = 0.8$

$\tan\alpha = \pm\sqrt{0.8}$

$\alpha = 41.8, \quad 138$

b) i) $\operatorname{cosec}\beta = \frac{7}{6}$

ii) $\cot^2\beta = \operatorname{cosec}^2\beta - 1 = \left(\frac{7}{6}\right)^2 - 1 = \frac{13}{36}$

6) Volume $= \pi\int_0^{\frac{1}{2}} (e^{3x})^2 - ((2x-1)^4)^2\,dx = \pi\int_0^{\frac{1}{2}} e^{6x} - (2x-1)^8\,dx$

$$= \pi\left[\frac{e^{6x}}{6} - \frac{(2x-1)^9}{2\times 9}\right]_0^{\frac{1}{2}} = \pi\left\{\left(\frac{e^3}{6} - 0\right) - \left(\frac{e^0}{6} - \frac{-1}{18}\right)\right\} = \pi\left(\frac{e^3}{6} - \frac{2}{9}\right)$$

7) N=42 when t=0 and N=84 when t=9

i) Subst t=0 N=42 into $N = A\times 2^{kt}$ $\quad 42 = A\times 2^0$

Subst t=9 N=84 into $N = 42\times 2^{kt}$ $\quad 84 = 42\times 2^{9k}$ $\quad 2^{9k} = 2$ $\quad k = \frac{1}{9}$

Subst t=9 N=84 into $N = 42\times e^{mt}$ $\quad 84 = 42\times e^{9m}$ $\quad 2 = e^{9m}$ $\quad m = \frac{1}{9}\ln 2$

ii) N=100 use $N = 42\times e^{\frac{1}{9}t\ln 2}$ $\quad 100 = 42\times e^{\frac{1}{9}t\ln 2}$ $\quad t = \frac{9\ln(100\div 42)}{\ln 2} = 11.3$years

iii) $\frac{dN}{dt} = 42\times\frac{1}{9}\ln 2\times e^{\frac{1}{9}t\ln 2}$

t=35 $\quad \frac{dN}{dt} = 42\times\frac{1}{9}\ln 2\times e^{\frac{1}{9}\times 35\ln 2} = 47.9$

8) i) $T(\theta) = 3\cos(\theta-60) + 2\cos(\theta+60)$ $\quad$ Use $\cos(A\pm B)$

$T(\theta) = 3(\cos\theta\cos 60 + \sin\theta\sin 60) + 2(\cos\theta\cos 60 - \sin\theta\sin 60)$

$T(\theta) = 3\left(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right) + 2\left(\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right)$

$T(\theta) = \frac{5}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta$

ii) $T(\theta) = \frac{5}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta$

$= R\sin(\theta - \alpha) = R\cos\alpha\sin\theta + R\sin\alpha\cos\theta$

Matching up $\quad R\cos\alpha = \frac{\sqrt{3}}{2} \quad R\sin\alpha = \frac{5}{2}$

$R = \sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{7} \qquad \alpha = \tan^{-1}\left(\frac{5/2}{\sqrt{3}/2}\right) = 70.9°$

So $T(\theta) = \sqrt{7}\sin(\theta + 70.9)$

iii) $\sqrt{7}\sin(\theta + 70.9) + 1 = 0$

$\sin(\theta + 70.9) = -\frac{1}{\sqrt{7}}$

$\theta + 70.9 = (-22.2),\ 202$

$\theta = 131$

9) i) $f'(x) = \frac{(x^2+5)\times 15 - 15x \times 2x}{(x^2+5)^2} = \frac{75 - 15x^2}{(x^2+5)^2}$

$f'(x) = 0 \qquad \therefore 75 - 15x^2 = 0 \qquad x^2 = 5 \qquad \therefore x = \pm\sqrt{5}$

Subst to find y. $y = \frac{15\sqrt{5}}{(\sqrt{5})^2 + 5} = \frac{3\sqrt{5}}{2}$ So Range $0 \le f(x) \le \frac{3\sqrt{5}}{2}$

ii) $k = \sqrt{5}$

iii)

$-1 = \frac{75 - 15x^2}{(x^2+5)^2}$

$-(x^2+5)^2 = 75 - 15x^2$

$x^4 + 10x^2 + 25 = 15x^2 - 75$

$x^4 - 5x^2 + 100 = 0$

$b^2 - 4ac = (-5)^2 - 4 \times 1 \times 100 = -375 < 0$

therefore no solutions for x^2 which implies the gradient $\neq -1$