ADVANCED GCE UNIT

4723/01

MATHEMATICS

Core Mathematics 3

MONDAY 11 JUNE 2007

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 Differentiate each of the following with respect to x.

(i) $x^3(x+1)^5$ [2]

(ii) $\sqrt{3x^4+1}$ [3]

2 Solve the inequality $|4x-3| < |2x+1|$. [5]

3 The function f is defined for all non-negative values of x by

$$\mathrm{f}(x) = 3 + \sqrt{x}.$$

(i) Evaluate $\mathrm{ff}(169)$. [2]

(ii) Find an expression for $\mathrm{f}^{-1}(x)$ in terms of x. [2]

(iii) On a single diagram sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, indicating how the two graphs are related. [3]

4 The integral I is defined by

$$I = \int_0^{13} (2x+1)^{\frac{1}{3}}\,\mathrm{d}x.$$

(i) Use integration to find the exact value of I. [4]

(ii) Use Simpson's rule with two strips to find an approximate value for I. Give your answer correct to 3 significant figures. [3]

5 A substance is decaying in such a way that its mass, m kg, at a time t years from now is given by the formula

$$m = 240\mathrm{e}^{-0.04t}.$$

(i) Find the time taken for the substance to halve its mass. [3]

(ii) Find the value of t for which the mass is decreasing at a rate of 2.1 kg per year. [4]

6 **(i)** Given that $\int_0^a (6\mathrm{e}^{2x} + x)\,\mathrm{d}x = 42$, show that $a = \frac{1}{2}\ln\left(15 - \frac{1}{6}a^2\right)$. [5]

(ii) Use an iterative formula, based on the equation in part **(i)**, to find the value of a correct to 3 decimal places. Use a starting value of 1 and show the result of each iteration. [4]

7 **(i)** Sketch the graph of $y = \sec x$ for $0 \leqslant x \leqslant 2\pi$. [2]

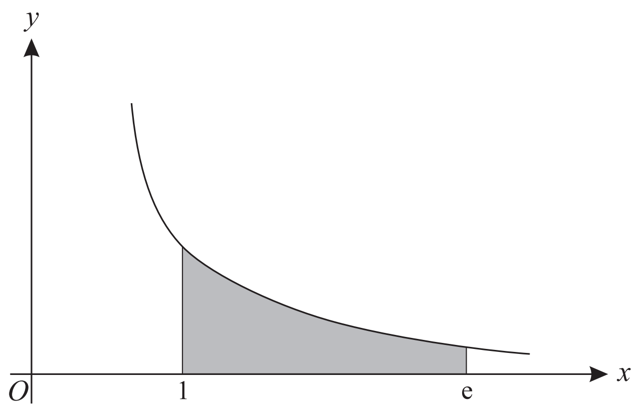
(ii) Solve the equation $\sec x = 3$ for $0 \leqslant x \leqslant 2\pi$, giving the roots correct to 3 significant figures. [3]

(iii) Solve the equation $\sec\theta = 5\operatorname{cosec}\theta$ for $0 \leqslant \theta \leqslant 2\pi$, giving the roots correct to 3 significant figures. [4]

8 **(i)** Given that $y = \dfrac{4\ln x - 3}{4\ln x + 3}$, show that $\dfrac{dy}{dx} = \dfrac{24}{x(4\ln x + 3)^2}$. [3]

(ii) Find the exact value of the gradient of the curve $y = \dfrac{4\ln x - 3}{4\ln x + 3}$ at the point where it crosses the x-axis. [4]

(iii)



The diagram shows part of the curve with equation

$$y = \frac{2}{x^{\frac{1}{2}}(4\ln x + 3)}.$$

The region shaded in the diagram is bounded by the curve and the lines $x = 1$, $x = \mathrm{e}$ and $y = 0$. Find the exact volume of the solid produced when this shaded region is rotated completely about the x-axis. [4]

9 **(i)** Prove the identity

$$\tan(\theta + 60^\circ)\tan(\theta - 60^\circ) \equiv \frac{\tan^2\theta - 3}{1 - 3\tan^2\theta}.$$ [4]

(ii) Solve, for $0^\circ < \theta < 180^\circ$, the equation

$$\tan(\theta + 60^\circ)\tan(\theta - 60^\circ) = 4\sec^2\theta - 3,$$

giving your answers correct to the nearest 0.1°. [5]

(iii) Show that, for all values of the constant k, the equation

$$\tan(\theta + 60^\circ)\tan(\theta - 60^\circ) = k^2$$

has two roots in the interval $0^\circ < \theta < 180^\circ$. [3]