C3 June 2007

1) $y = x^3(x+1)^5 = uv$

$u' = 3x^2 \qquad v' = 5(x+1)^4$

$y' = x^3\,5(x+1)^4 + (x+1)^5\,3x^2$

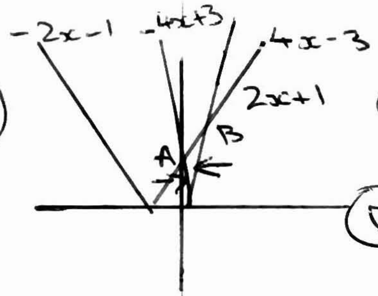
$= (x+1)^4 x^2\left(5x + 3(x+1)\right)$

$= x^2(x+1)^4(8x+3)$

b) $y = (3x^4+1)^{\frac{1}{2}} \qquad y' = \frac{1}{2}(3x^4+1)^{-\frac{1}{2}}\,12x^3$

$= 6x^3(3x^4+1)^{-\frac{1}{2}}$

2)



(A) $2x+1 = -4x+3$

$6x = 2 \quad x = \frac{1}{3}$

(B) $2x+1 = 4x-3$

$2x = 4 \quad x = 2$

Hence $x < 2$ + $x > \frac{1}{3}$

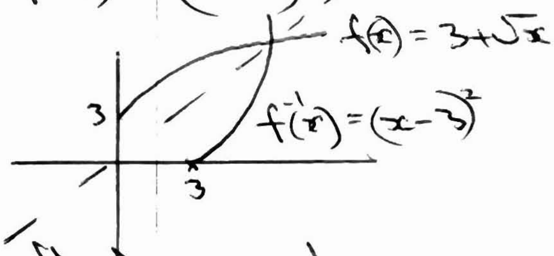
or $\frac{1}{3} < x < 2$

3) i) $f(x) = 3 + \sqrt{x} \qquad f(169) = 3 + 13 = 16$

$f(16) = 3 + 4 = 7$

ii) $x \to \sqrt{\ } \to +3 = f(x)$

$f^{-1}(x) = (x-3)^2, \quad x > 3$



reflection in line $y = x$

4) $\int_0^{13} (2x+1)^{\frac{1}{3}}\,dx = \dfrac{(2x+1)^{\frac{4}{3}}}{2 \times \frac{4}{3}}$

$I(13) = \dfrac{27^{\frac{4}{3}}}{\frac{8}{3}} = \frac{3}{8} \times 81$

$I(0) = \dfrac{1}{\frac{8}{3}} = \frac{3}{8}$

$I = \frac{3}{8} \times 81 - \frac{3}{8} = 30$

4ii)

x	0	6.5	13
y	1	$14^{\frac{1}{3}}$ (2.4101)	3

$I = \frac{1}{3} \times 6.5\left(1 + 3 + 4 \times 2.4101\right)$

$= 29.6$ to 3sf

5) i) $M = 240e^{-0.04t}$

if $t = 0$ $M = 240$ ie initial mass

if $M = 120 \Rightarrow 120 = 240e^{-0.04t}$

$\ln 0.5 = -0.04t$

$t = 17.3$ yrs to 3sf

ii) $\dfrac{dM}{dt} = -240 \times 0.04\,e^{-0.04t} = -2.1$ (given)

so $e^{-0.04t} = \dfrac{-2.1}{-240 \times 0.04} = .21875$

$t = \dfrac{\ln .21875}{-0.04} = 38$ yrs to 3sf

6) $\int 6e^{2x} + x\,dx = 3e^{2x} + \dfrac{x^2}{2}$

$I(a) = 3e^{2a} + \dfrac{a^2}{2} \qquad I(0) = 3$

so $3e^{2a} + \dfrac{a^2}{2} - 3 = 42$

$3e^{2a} = 45 - \dfrac{a^2}{2}$

$e^{2a} = 15 - \dfrac{a^2}{6}$

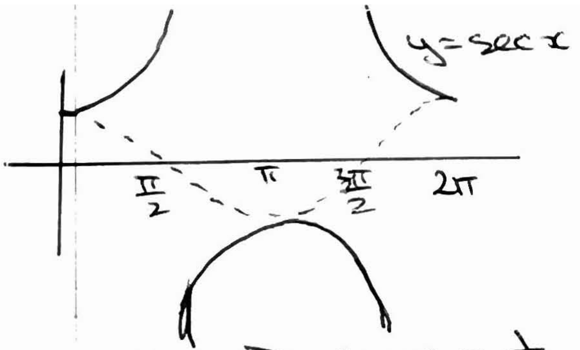
$2a = \ln\left(15 - \dfrac{a^2}{6}\right) \qquad a = \dfrac{1}{2}\ln\left(15 - \dfrac{a^2}{6}\right)$

ii) $a_{n+1} = \dfrac{1}{2}\ln\left(15 - \dfrac{a_n^2}{6}\right)$

$a_0 = 1 \quad a_1 = 1.348$

$a_2 = 1.3438 \quad a_3 = 1.3439$

so $a = 1.344$ to 3dp

7) 

ii) $\sec x = 3 \Rightarrow \cos x = \frac{1}{3}$

$x = 70.5^\circ + 289.5^\circ$

$= 1.23 + 5.05$ rads

iii) $\sec\theta = 5\operatorname{cosec}\theta$

$\frac{1}{\cos\theta} = \frac{5}{\sin\theta} \Rightarrow \sin\theta = 5\cos\theta$

$\tan\theta = 5 \quad \theta = 78.7^\circ + 258.7^\circ$

$= 1.373 + 4.515$ rads

Ans 1.37 + 4.52 rads to 3sf

8) $y = \frac{4\ln x - 3}{4\ln x + 3} = \frac{u}{v} \quad u' = \frac{4}{x} \quad v' = \frac{4}{x}$

$\frac{dy}{dx} = \frac{(4\ln x + 3)\frac{4}{x} - (4\ln x - 3)\frac{4}{x}}{(4\ln x + 3)^2}$

$= \frac{\frac{24}{x}}{(4\ln x + 3)^2} = \frac{24}{x(4\ln x + 3)^2}$

ii) if $y = 0 \quad 4\ln x - 3 = 0$

$x = e^{\frac{3}{4}}$

$\frac{dy}{dx} = \frac{24}{e^{\frac{3}{4}}(4\ln e^{\frac{3}{4}} + 3)^2}$

$= \frac{24}{e^{\frac{3}{4}} \times 36} = \frac{2}{3e^{\frac{3}{4}}}$ or $\frac{2}{3}e^{-\frac{3}{4}}$

iii) $V = \pi\int y^2\,dx = \pi\int_e^1 \frac{4}{x(4\ln x + 3)^2}\,dx$

$= \frac{\pi}{6}\int_e^1 \frac{24}{x(4\ln x + 3)^2}\,dx$

$= \frac{\pi}{6}\left(\frac{4\ln x - 3}{4\ln x + 3}\right)$ from part i)

$V(e) = \frac{\pi}{6}\left(\frac{4\ln e - 3}{4\ln e + 3}\right) = \frac{\pi}{42}$

$V(1) = -\frac{\pi}{6} \quad V = \frac{\pi}{42} + \frac{\pi}{6} = \frac{8\pi}{42}$

$= \frac{4\pi}{21}$

9) i) Use $\tan(A+B)$ + $\tan(A-B)$

$LHS = \left(\frac{\tan\theta + \sqrt{3}}{1 - \sqrt{3}\tan\theta}\right)\left(\frac{\tan\theta - \sqrt{3}}{1 + \sqrt{3}\tan\theta}\right)$

$= \frac{\tan^2\theta - 3}{1 - 3\tan^2\theta} = RHS$

ii) Use RHS of part i) + $1 + t^2 = \sec^2$

$\frac{\tan^2\theta - 3}{1 - 3\tan^2\theta} = 4(1 + \tan^2\theta) - 3$

$t^2 - 3 = (1 + 4t^2)(1 - 3t^2)$

$t^2 - 3 = 1 + t^2 - 12t^4$

$12t^4 - 4 = 0 \Rightarrow 3t^4 = 1$

$\tan\theta = \pm\frac{1}{\sqrt[4]{3}}$

$\tan\theta = \frac{\pm 1}{\sqrt[4]{3}} \quad \theta = 37.2^\circ$ or 142.8°

iii using part i)

$\frac{t^2 - 3}{1 - 3t^2} = k^2$

$t^2 - 3 = k^2(1 - 3t^2)$

$t^2(1 + 3k^2) - 3 - k^2 = 0$

discriminant $= b^2 - 4ac$

$= 0 - 4(1 + 3k^2)(-3 - k^2)$

$= (1 + 3k^2)(12 + 4k^2)$

This expression is > 0 for all values of k as $k^2 > 0$ so there are 2 roots