Core 3 June 2006

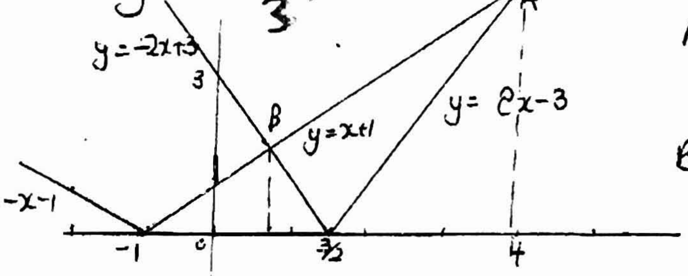
1) $y = \sqrt{4x+1} = (4x+1)^{\frac{1}{2}}$ $\quad \frac{dy}{dx} = \frac{1}{2}(4x+1)^{-\frac{1}{2}} \times 4 = \frac{2}{\sqrt{4x+1}}$

$(2,3)$ $x = 2$ $\quad \frac{dy}{dx} = \frac{2}{3}$ $\quad y - 3 = \frac{2}{3}(x-2)$

$y = \frac{2}{3}x + \frac{5}{3}$

$3y - 2x - 5 = 0$

2) $|2x-3| = |x+1|$



A $\quad 2x-3 = x+1$

$x = 4$

B $\quad -2x+3 = x+1$

$2 = 3x$

$x = \frac{2}{3}$

$|2x-3| < |x+1|$ when $\frac{2}{3} < x < 4$

3) $f(x) = 2x^3 + 4x - 35$. $\quad f(2) = 16 + 8 - 35 = -11$ $\quad f(3) = 54 + 12 - 35 = +31$

change of sign -ve to +ve between $x = 2$ and 3.

$x_{n+1} = \sqrt[3]{17.5 - 2x_n}$ $\quad x_1 = 2$ $\quad x_2 = \sqrt[3]{17.5 - 4} = 2.3811$ $\quad x_3 = \sqrt[3]{17.5 - 2 \times 2.3811} = 2.3354$

CALC. 2 ENTER $\quad (17.5-(2ANS))^{1/3}$ ENTER $\quad x_4 = 2.3410$ $\quad x_5 = 2.3403$ $\quad x_6 = 2.3404$ $\quad x_7 = 2.3404$

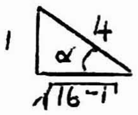
root is 2.34 (2dp).

4) $y = 5^{x-1}$ $\quad \ln y = \ln 5^{x-1}$ $\quad \ln y = (x-1)\ln 5$ $\quad \ln y + \ln 5 = x \ln 5$

$x = \frac{\ln y}{\ln 5} + 1$ $\quad \frac{dx}{dy} = \frac{1}{\ln 5} \times \frac{1}{y}$ $\quad$ gradient of curve $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = y \ln 5$

$\frac{dy}{dx} = 5^{x-1} \ln 5$. $\quad$ at $(3,25)$ $\quad \frac{dy}{dx} = 5^2 \ln 5 = 25 \ln 5$.

5) ii) $\sin 2\theta = 2 \sin\theta \cos\theta$ $\quad \sin\alpha = \frac{1}{4}$



$\cos\alpha = \frac{\sqrt{15}}{4}$.

$\sin 2 = 2 \times \frac{1}{4} \times \frac{\sqrt{15}}{4} = \frac{\sqrt{15}}{8}$

iii) $5 \times 2 \sin\beta \cos\beta \frac{1}{\cos\beta} = 3$ $\quad 10 \sin\beta \cos\beta - 3\cos\beta = 0$ $\quad \cos\beta(10\sin\beta - 3) = 0$

$0 < \beta < 90$ $\quad \cos\beta = 0$ not required solution $\quad \sin\beta = 0.3$ $\quad \beta = 17.5°$ (3sf).

6) i) $f(x) = 2 - x^2$ $\quad x \leq 0$ $\quad f(-3) = 2 - (-3)^2 = -7$ $\quad f(-7) = 2 - (-7)^2 = -47$

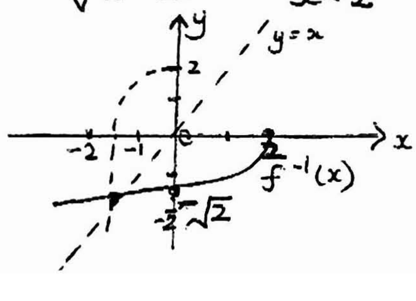
$ff(-3) = -47$.

ii) $x \longrightarrow \overset{x^2}{\wedge^2} \longrightarrow$ changesign $\overset{-x^2}{} \longrightarrow \overset{-x^2+2}{+2} \longrightarrow -x^2 + 2$

$x \longrightarrow \overset{x-2}{-2} \longrightarrow$ changesign $\overset{2-x}{} \longrightarrow \overset{\sqrt{2-x}}{\pm\sqrt{}} \longrightarrow \pm\sqrt{2-x}$. $\quad$ domain $f(x)$ $x \leq 0$

range $f(x)$ $x \leq 2$

$f^{-1}(x) = -\sqrt{2-x}$ $\quad x \leq 2$ $\quad$ (-ve square root as reflection of $f(x)$ in $y = x$)



graph meets axis $(2,0)$. $(0, -\sqrt{2})$