RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4723:** Core Mathematics 3

Mark Scheme for January 2013

Oxford Cambridge and RSA Examinations

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners’ meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates’ scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
ft or √	Follow through

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(i)		<u>Either</u> Attempt use of quotient rule	M1	allow numerator wrong way round but needs minus sign in numerator and both terms in numerator involving x; for M1 condone minor errors such as absence of square in denominator, absence of brackets, …
			Obtain $\frac{3(2x+1)-6x}{(2x+1)^2}$ or equiv	A1	give A0 if necessary brackets absent unless subsequent calculation indicates their 'presence'
			Substitute 2 to obtain $\frac{3}{25}$ or 0.12	A1	or simplified equiv but A0 for final $\frac{3}{5^2}$
				[3]	
			<u>Or</u> Attempt use of product rule for $3x(2x+1)^{-1}$	M1	allow sign error; condone no use of chain rule
			Obtain $3(2x+1)^{-1}-6x(2x+1)^{-2}$ or equiv	A1	
			Substitute 2 to obtain $\frac{3}{25}$ or 0.12	A1	or simplified equiv
1	(ii)		Differentiate to obtain form $kx(4x^2+9)^n$	M1	any non-zero constants k and n (including 1 or $\frac{1}{2}$ for n)
			Obtain $4x(4x^2+9)^{-\frac{1}{2}}$	A1	or (unsimplified) equiv
			Substitute 2 to obtain $\frac{8}{5}$ or 1.6	A1	or simplified equiv but A0 for final $\frac{8}{\sqrt{25}}$
				[3]	
2	(i)		<u>Either</u> Attempt to find exact value of $\sin A$	M1	using right-angled triangle or identity or …
			Obtain $\frac{1}{2}\sqrt{5}$ or $\sqrt{\frac{5}{4}}$ or exact equiv	A1	final $\pm\frac{1}{2}\sqrt{5}$ is A0; correct answer only earns M1A1
				[2]	
			<u>Or</u> Attempt use of identity $1+\cot^2 A=\operatorname{cosec}^2 A$	M1	using $\cot A=\frac{1}{2}$; allow sign error in attempt at identity
			Obtain $\frac{1}{2}\sqrt{5}$ or $\sqrt{\frac{5}{4}}$ or exact equiv	A1	final $\pm\frac{1}{2}\sqrt{5}$ is A0; correct answer only earns M1A1
2	(ii)		State or imply $\frac{2+\tan B}{1-2\tan B}=3$	B1	
			Attempt solution of equation of form $\frac{\text{linear in } t}{\text{linear in } t}=3$	M1	by sound process at least as far as $k\tan B=c$
			Obtain $\tan B=\frac{1}{7}$	A1	answer must be exact; ignore subsequent attempt to find angle B
				[3]	

Question			Answer	Marks	Guidance
3	(a)		Substitute $t=3$ in $\|2t-1\|$ and obtain value 5	B1	not awarded for final $\|5\|$ nor for ± 5
			Substitute $t=-3$ in $\|2t-1\|$ and apply modulus correctly to any negative value to obtain a positive value	M1	with no modulus signs remaining
			Obtain value 7 as final answer	A1	not awarded for final $\|7\|$ nor for ± 7 NB: substitutions in $\|2t+1\|$ will give 5 and 7 – this is 0/3, not MR; a further step to $5 < t < 7$ – B1 M1 A0; answers $\pm 5, \pm 7$ – this is B0 M0 A0
				[3]	
3	(b)		Either Attempt solution of linear equation or inequality with signs of x different	M1	
			Obtain critical value $-\sqrt{2}$	A1	or equiv (exact or decimal approximation)
			Or 1 Attempt to square both sides	M1	obtaining at least 3 terms on each side
			Obtain $x^2 - 2\sqrt{2}\,x + 2 > x^2 + 6\sqrt{2}\,x + 18$	A1	or equiv; or equation; condone $>$ here
			Or 2 Attempt sketches of $y = \|x - \sqrt{2}\|$, $y = \|x + 3\sqrt{2}\|$	M1	
			Obtain $x = -\sqrt{2}$ at point of intersection	A1	or equiv
			Conclude with inequality of one of the following types: $x < k\sqrt{2},\quad x > k\sqrt{2},\quad x < \dfrac{k}{\sqrt{2}},\quad x > \dfrac{k}{\sqrt{2}}$	M1	any integer k
			Obtain $x < -\sqrt{2}$ or $-\sqrt{2} > x$ as final answer	A1	final answer $x < -\frac{2}{\sqrt{2}}$ (or similar unsimplified version) is A0
				[4]	

Question			Answer	Marks	Guidance
4	(i)		Attempt process involving logarithm to solve $e^{0.021t} = 2$	M1	with t the only variable; at least as far as $0.021t = \ln 2$; must be $\ldots = 2$
			Obtain 33	A1	or greater accuracy; ignore absence of, or wrong, units; final answer $\frac{\ln 2}{0.021}$ is A0
			State (or calculate separately to obtain) 99	B1√	following previous answer; no need to include units
				[3]	
4	(ii)		Differentiate to obtain $ke^{0.021t}$	M1	where k is any constant not equal to 250
			Obtain $250 \times 0.021\,e^{0.021t}$	A1	or simplified equiv $5.25e^{0.021t}$
			Substitute to obtain 8.4 or $\frac{42}{5}$	A1	or value rounding to 8.4 with no obvious error
				[3]	
5	(i)		Integrate to obtain form $k(3x+1)^{\frac{1}{2}}$	*M1	any non-zero constant k
			Obtain $4(3x+1)^{\frac{1}{2}}$	A1	or (unsimplified) equiv; or $4u^{\frac{1}{2}}$ following substitution
			Apply the limits and subtract the right way round	M1	dep *M
			Obtain $4\sqrt{28} - 4\sqrt{7}$ and show at least one intermediate step in confirming $4\sqrt{7}$	A1	AG; necessary detail required; decimal verification is A0; $[...]_2^9 = 4\sqrt{28} - 4\sqrt{7} = 4\sqrt{7}$ is A0; $[...]_2^9 = 8\sqrt{7} - 4\sqrt{7} = 4\sqrt{7}$ is A0
				[4]	
5	(ii)		State or imply volume is $\int \pi \left(\frac{6}{\sqrt{3x+1}}\right)^2 \mathrm{d}x$ or equiv	B1	merely stating $\int \pi y^2 \,\mathrm{d}x$ not enough; condone absence of dx; no need for limits yet; π may be implied by its later appearance
			Integrate to obtain $k\ln(3x+1)$	M1	any non-zero constant with or without π
			Obtain $12\pi\ln(3x+1)$ or $12\ln(3x+1)$	A1	or unsimplified equiv
			Substitute limits correct way round and show each logarithm property correctly applied	M1	allowing correct applications to incorrect result of integration providing natural logarithm involved; evidence of $\ln 28 - \ln 7 = \frac{\ln 28}{\ln 7}$ error means M0
			Obtain $24\pi\ln 2$	A1	no need for explicit statement of value of k
				[5]	

Question			Answer	Marks	Guidance
6	(i)		Sketch more or less correct $y = \ln x$	B1	existing for positive and negative y; no need to indicate (1, 0); ignore any scales given on axes; condone graph touching y-axis but B0 if it crosses y-axis
			Sketch more or less correct $y = 8 - 2x^2$	B1	(roughly) symmetrical about y-axis; extending, if minimally, into quadrants for which $y < 0$; no need to indicate (± 2, 0), (0, 8); assess each curve separately
			Indicate intersection by some mark on diagram (just a 'blob' sufficient) of by statement in words away from diagram	B1	needs each curve to be (more or less) correct in the first quadrant and on curves being related to each other correctly there
				[3]	
6	(ii)		Refer, in some way, to graphs crossing x-axis at $x = 1$ and $x = 2$ and that intersection is between these values	B1	AG; the values 1 and 2 may be assumed from part (i) if clearly marked there; dependent on curves being (more or less) correct in first quadrant; carrying out the sign-change routine is B0
				[1]	
6	(iii)		Obtain correct first iterate	B1	to at least 3 dp (except in the case of starting value 1 leading to 2)
			Show correct iterative process	M1	involving at least 3 iterates in all; may be implied by plausible converging values
			Obtain at least 3 correct iterates	A1	allowing recovery after error; iterates given to at least 3 dp; values may be rounded or truncated
			Conclude with 1.917	A1	answer required to exactly 3 dp; answer only with no evidence of process is 0/4
				[4]	
			$1 \to 2 \to 1.91139 \to 1.91731... \to 1.91690... \to 1.91693...$ $1.5 \to 1.94865... \to 1.91479... \to 1.91707... \to 1.91692...$ $2 \to 1.91139... \to 1.91731... \to 1.91690... \to 1.91693...$		
6	(iv)		Obtain 3.92 or greater accuracy	B1√	following their answer to part (iii)
			Attempt $4 \times \ln(\text{part (iii) answer})$	M1	
			Obtain y-coordinate 2.60	A1	value required to exactly 2 dp (so A0 for 2.6 and 2.603)
				[3]	

Question			Answer	Marks	Guidance
7	(i)		Attempt use of product rule	M1	to produce expression of form $(\text{something non-zero})\ln(2y+3) + \dfrac{\text{linear in } y}{\text{linear in } y}$; ignore what they call their derivative
			Obtain $\ln(2y+3)\ \ldots$	A1	with brackets included
			Obtain $\ldots + \dfrac{2(y+4)}{2y+3}$	A1	with brackets included as necessary
				[3]	
7	(ii)		Substitute $y=0$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1	
			Obtain 0.27 for gradient at A	A1	or greater accuracy 0.26558…; beware of 'correct' answer coming from incorrect version $\ln(2y+3)+\frac{8}{3}$ of answer in part (i)
			Attempt to find value of y for which $x=0$	M1	allowing process leading only to $y=-4$
			Substitute $y=-1$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1	
			Obtain 0.17 or $\frac{1}{6}$ for gradient at B	A1	or greater accuracy 0.16666…; value following from correct working
				[5]	
8	(i)		Attempt completion of square at least as far as $(x+2a)^2$ or differentiation to find stationary point at least as far as linear equation involving two terms	*M1	or equiv but a must be present
			Obtain $(x+2a)^2-3a^2$ or $(-2a,-3a^2)$	A1	
			Attempt inequality involving appropriate y-value	M1	dep *M; allow <, > or $\le$ here; allow use of x; or unsimplified equiv
			State $y \ge -3a^2$ or $\mathrm{f}(x) \ge -3a^2$	A1	now with $\ge$; here $x \ge -3a^2$ is A0
				[4]	

Question			Answer	Marks	Guidance
8	(ii)		Attempt composition of f and g the right way round	*M1	algebraic or (part) numerical; need to see $4x-2a$ replacing x at least once
			Obtain or imply $16x^2-3a^2$ or $144-3a^2$	A1	or less simplified equiv but with at least the brackets expanded correctly
			Attempt to find a from $\text{fg}(3)=69$	M1	dep *M
			Obtain at least $a=5$	A1	
			Attempt to solve $4x-10=x$ or $\frac{1}{4}(x+10)=x$ or $4x-10=\frac{1}{4}(x+10)$	M1	for their a; must be linear equation in one variable; condone sign slip in finding inverse of g
			Obtain $\frac{10}{3}$	A1	and no other answer
				[6]	
9	(i)		State $\cos\theta\cos 45-\sin\theta\sin 45$	B1	or equiv including use of decimal approximation for $\frac{1}{\sqrt{2}}$
			Use correct identity for $\sin 2\theta$ or $\cos 2\theta$	B1	must be used; not earned for just a separate statement
			Attempt complete simplification of left-hand side	M1	with relevant identities but allowing sign errors, and showing two terms involving $\sin\theta\cos\theta$
			Obtain $\sin^2\theta$	A1	AG; necessary detail needed
				[4]	
9	(ii)		Use identity to produce equation of form $\sin\frac{1}{2}\theta=c$	M1	condoning single value of constant c here (including values outside the range –1 to 1); M0 for $\sin\theta=c$ unless value(s) are subsequently doubled
			Obtain 70.5 or 70.6	A1	or greater accuracy 70.528…
			Obtain –70.5 or –70.6	A1√	or greater accuracy –70.528…; following first answer; and no other answer between –90 and 90; answer(s) only : 0/3
				[3]	
9	(iii)		State or imply $6\sin^2\frac{1}{3}\theta=k$	B1	
			Attempt to relate k to at least $6\sin^2 30°$	M1	
			Obtain $0<k<\frac{3}{2}$	A1	condone use of $\le$
				[3]	

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