OCR
RECOGNISING ACHIEVEMENT

Friday 20 January 2012 – Afternoon

A2 GCE MATHEMATICS

4723 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4723
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

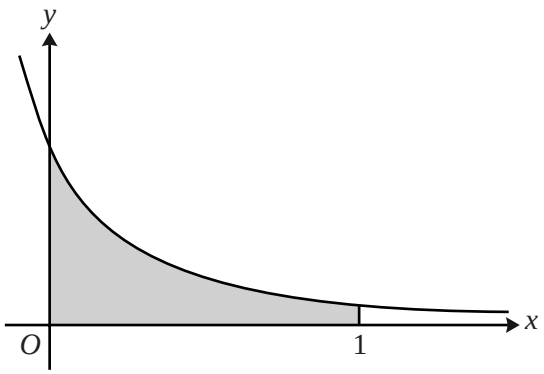
This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Show that $\int_{\sqrt{2}}^{\sqrt{6}} \frac{2}{x}\,\mathrm{d}x = \ln 3$. **[3]**

2



The diagram shows part of the curve $y = \frac{6}{(2x+1)^2}$. The shaded region is bounded by the curve and the lines $x = 0$, $x = 1$ and $y = 0$. Find the exact volume of the solid produced when this shaded region is rotated completely about the x-axis. **[5]**

3 Find the equation of the normal to the curve $y = \frac{x^2+4}{x+2}$ at the point $(1, \frac{5}{3})$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[7]**

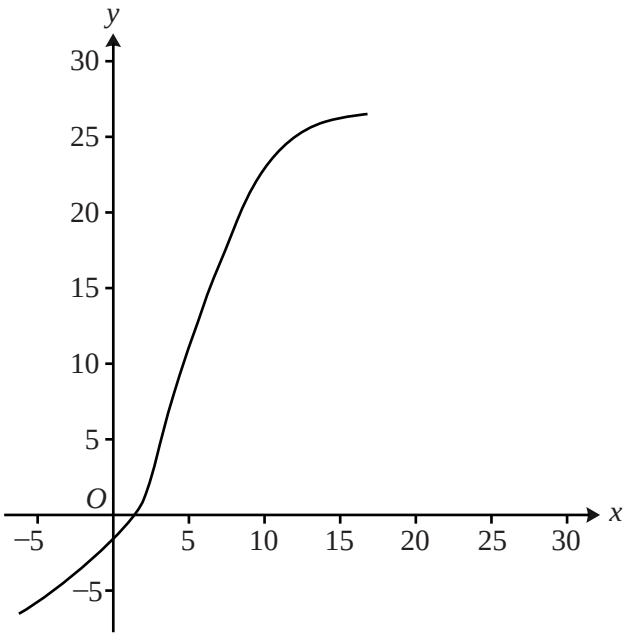
4 The acute angles α and β are such that

$$2\cot\alpha = 1 \quad \text{and} \quad 24 + \sec^2\beta = 10\tan\beta.$$

(i) State the value of $\tan\alpha$ and determine the value of $\tan\beta$. **[4]**

(ii) Hence find the exact value of $\tan(\alpha + \beta)$. **[3]**

5



It is given that f is a one-one function defined for all real values. The diagram shows the curve with equation $y = \mathrm{f}(x)$. The coordinates of certain points on the curve are shown in the following table.

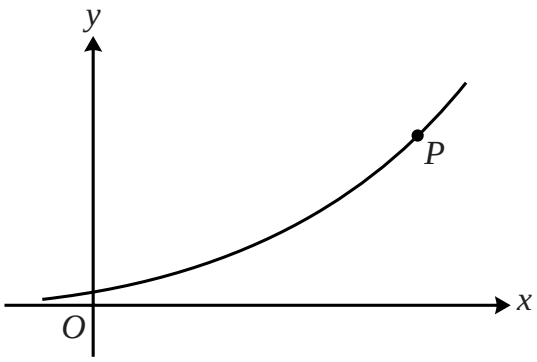
x	2	4	6	8	10	12	14
y	1	8	14	19	23	25	26

(i) State the value of $\mathrm{ff}(6)$ and the value of $\mathrm{f}^{-1}(8)$. **[2]**

(ii) On the copy of the diagram, sketch the curve $y = \mathrm{f}^{-1}(x)$, indicating how the curves $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ are related. **[2]**

(iii) Use Simpson's rule with 6 strips to find an approximation to $\int_2^{14} \mathrm{f}(x)\,\mathrm{d}x$. **[4]**

6



The diagram shows the curve with equation $x = \ln(y^3 + 2y)$. At the point P on the curve, the gradient is 4 and it is given that P is close to the point with coordinates (7.5, 12).

(i) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y. **[2]**

(ii) Show that the y-coordinate of P satisfies the equation

$$y = \frac{12y^2 + 8}{y^2 + 2}.$$ **[3]**

(iii) By first using an iterative process based on the equation in part **(ii)**, find the coordinates of P, giving each coordinate correct to 3 decimal places. **[5]**

7 **(i)** Substance A is decaying exponentially and its mass is recorded at regular intervals. At time t years, the mass, M grams, of substance A is given by

$$M = 40\mathrm{e}^{-0.132t}.$$

(a) Find the time taken for the mass of substance A to decrease to 25% of its value when $t = 0$. **[3]**

(b) Find the rate at which the mass of substance A is decreasing when $t = 5$. **[3]**

(ii) Substance B is also decaying exponentially. Initially its mass was 40 grams and, two years later, its mass is 31.4 grams. Find the mass of substance B after a further year. **[3]**

8 **(i)** Express $\cos 4\theta$ in terms of $\sin 2\theta$ and hence show that $\cos 4\theta$ can be expressed in the form $1 - k\sin^2\theta\cos^2\theta$, where k is a constant to be determined. **[3]**

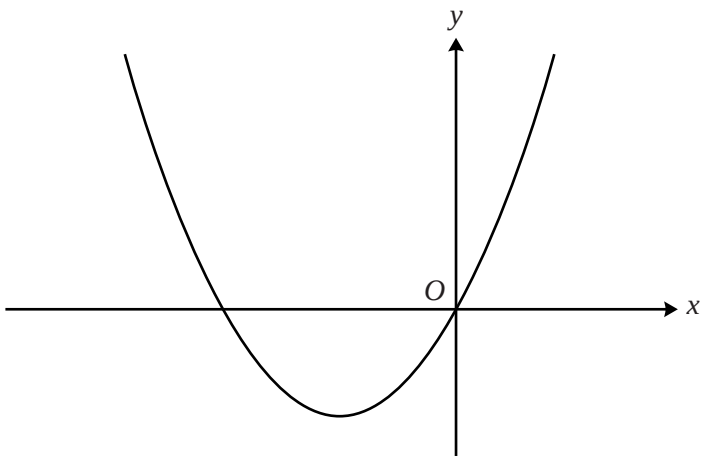
(ii) Hence find the exact value of $\sin^2(\frac{1}{24}\pi)\cos^2(\frac{1}{24}\pi)$. **[2]**

(iii) By expressing $2\cos^2 2\theta - \frac{8}{3}\sin^2\theta\cos^2\theta$ in terms of $\cos 4\theta$, find the greatest and least possible values of

$$2\cos^2 2\theta - \tfrac{8}{3}\sin^2\theta\cos^2\theta$$

as θ varies. **[5]**

9



The function f is defined for all real values of x by

$$f(x) = k(x^2 + 4x),$$

where k is a positive constant. The diagram shows the curve with equation $y = f(x)$.

(i) The curve $y = x^2$ can be transformed to the curve $y = f(x)$ by the following sequence of transformations:
a translation parallel to the x-axis,
a translation parallel to the y-axis,
a stretch.
Give details, in terms of k where appropriate, of these transformations. **[5]**

(ii) Find the range of f in terms of k. **[2]**

(iii) It is given that there are three distinct values of x which satisfy the equation $|f(x)| = 20$. Find the value of k and determine exactly the three values of x which satisfy the equation in this case. **[6]**

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