GCE

Mathematics

Advanced GCE

Unit **4723:** Core Mathematics 3

Mark Scheme for January 2011

Oxford Cambridge and RSA Examinations

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1	Either: Obtain $\frac{1}{3}a$	B1		condone $\lvert x\rvert = \frac{1}{3}a$
	Attempt solution of linear eqn	M1		with signs of $3x$ and $5a$ different; allow M1 only if a given particular value and no recovery occurs; allow M1 only if a in terms of x attempted; allow M1 only if double inequality attempted but with no recovery to state actual values of x
	Obtain $-3a$	A1	**3**	as final answer
	Or: Obtain $9x^2 + 24ax + 16a^2 = 25a^2$	B1		
	Attempt solution of 3-term quad eqn	M1		as far as substitution into correct quadratic formula or correct factorisation of their quadratic; allow M1 only if a given particular value
	Obtain $-3a$ and $\frac{1}{3}a$	A1	**(3)**	or equivs; as final answers; and no others
			3	
2	Draw graph showing reflection in a horizontal axis	M1		
	Draw graph showing translation	M1		parallel to x-axis, in either direction; independent of first M1; not earned if curve still passes through O but ignore other coordinates given at this stage
	Draw (more or less) correct graph which must at least reach the negative x-axis, if not cross it, at left end of curve	A1		but ignoring no or wrong stretch in y-dir'n; condone graph existing only for $x < 0$; consider shape of curve and ignore coordinates given
	State $(-5, 24)$ and $(-3, 0)$ wherever located	B1	**4**	or clearly implied by sketch; allow for coordinates whatever sketch looks like; allow if in solution with no sketch
			4	
3	Either: State or imply $8\pi r$ as derivative	B1		or equiv
	Attempt to connect 12 and their derivative	M1		numerical or algebraic; using multiplication or division
	Obtain $8\pi \times 150 \times 12$ and hence 45000 or 14400π or 14000π	A1	**3**	or equiv; or greater accuracy (45239); condone absence of units or use of wrong units
	Or: Use $r = 12t$ to show $S = 576\pi t^2$	B1		
	Attempt $\frac{\mathrm{d}S}{\mathrm{d}t}$ and substitute for t	M1		
	Obtain $1152\pi \times \frac{150}{12}$ and hence 45000 or 14400π or 14000π	A1	**(3)**	or equiv; or greater accuracy (45239); condone absence of units or use of wrong units
			3	

4	(i)	Obtain $R = 25$	B1		allow $\sqrt{625}$ or value rounding to 25
		Attempt to find value of α	M1		implied by correct answer or its complement; allow sin/cos muddles; allow use of radians for this mark; condone $\sin\alpha = 7$, $\cos\alpha = 24$ in the working
		Obtain $16.3°$	A1	3	or greater accuracy 16.260…; must be degrees now; allow $16°$ here
	(ii)	Show correct process for finding one answer	M1		even if leading to answer outside 0 to 360
		Obtain (28.69 – 16.26 and hence) $12.4°$	A1		or greater accuracy 12.425… or anything rounding to 12.4
		Show correct process for finding second answer	M1		even if further incorrect answers produced
		Obtain (151.31 – 16.26 and hence) $135°$ or $135.1°$	A1	4	or greater accuracy 135.054…; and no other between 0 and 360
		[SC: No working shown and 2 correct angles stated - B1 only in part (ii)]			
				7	

5	Integrate to obtain form $k(3x-2)^{\frac{1}{2}}$	M1		any non-zero constant k; or equiv involving substitution
	Obtain correct $4(3x-2)^{\frac{1}{2}}$	A1		or (unsimplified) equiv such as $\frac{6(3x-2)^{\frac{1}{2}}}{3\times\frac{1}{2}}$
	Apply limits and attempt solution for a	M1		assuming integral of form $k(3x-2)^n$; taking solution as far as removal of root; with subtraction the right way round; if sub'n used, limits must be appropriate
	Obtain $a = 9$	A1		(this answer written down with no working scores 0/4 so far but all subsequent marks are available)
	State or imply formula $\int \frac{36\pi}{3x-2}\,dx$	B1		or (unsimplified) equiv; condone absence of dx; allow B1 retroactively if π absent here but inserted later
	Integrate to obtain form $k\ln(3x-2)$	*M1		any constant k including π or not; condone absence of brackets
	Obtain $12\pi\ln(3x-2)$ or $12\ln(3x-2)$	A1√		following their integral of form $\int \frac{k}{3x-2}\,dx$
	Apply limits the correct way round	M1		dep *M; use of limit 1 is implied by absence of second term; allow use of limit a
	Obtain $12\pi\ln 25$ (or $24\pi\ln 5$)	A1	9	or exact equiv but not with $\ln 1$ remaining; condone answers such as $\pi 12\ln 25$ and $12\ln 25\pi$
			9	

6 **(i)**	Attempt use of quotient rule	M1		or equiv; allow numerator wrong way round but needs minus sign in numerator; for M1 condone 'minor' errors such as sign slips, absence of square in denominator, and absence of some brackets
	Obtain $\frac{3(x^3-4x^2+2)-(3x+4)(3x^2-8x)}{(x^3-4x^2+2)^2}$	A1		or equiv; allow A1 if brackets absent from $3x+4$ term or from $3x^2-8x$ term but not from both
	Equate numerator to 0 and attempt simplification	M1		at least as far as removing brackets, condoning sign or coeff slips; or equiv
	Obtain $-6x^3+32x+6=0$ or equiv and hence $x=\sqrt[3]{\frac{16}{3}x+1}$	A1	**4**	AG; necessary detail needed (i.e. at least one intermediate step) and following first derivative with correct numerator
(ii)	Obtain correct first iterate having used initial value 2.4	B1		showing at least 3 dp (2.398 or 2.399 or greater accuracy 2.39861…)
	Apply iterative process	M1		to obtain at least 3 iterates in all; implied by plausible, converging sequence of values; having started with any initial non-negative value
	Obtain at least 3 correct iterates from their starting point	A1		allowing recovery after error
	Obtain 2.398	A1		value required to exactly 3 dp
	Obtain –1.552	A1	**5**	value required to exactly 3 dp; allow if apparently obtained by substitution of 2.4; answers only with no iterates shown gets 0/5

[2.4 $\rightarrow$ 2.3986103 $\rightarrow$ 2.3981808 $\rightarrow$ 2.3980480]

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7	**(i)**	State $\ln(x^2+8)=8$	B1		or equiv such as $x^2+8=e^8$
		Attempt solution involving e^8	M1		by valid (exact) method at least as far as $x^2 = ...$
		Obtain $\sqrt{e^8-8}$	A1	**3**	or exact equiv; and no other answer
	(ii)	State f only	B1		
		State e^x or e^y	B1		or equiv; allow if g, or f and g, chosen
		Indicate domain is all real numbers	B1	**3**	however expressed
	(iii)	Attempt use of chain rule	M1		whether applied to gf or fg; or equiv such as use of product rule on $(\ln x)(\ln x)+8$
		Obtain $\frac{2\ln x}{x}$	A1		or equiv
		Obtain $6e^{-3}$	A1	**3**	or exact equiv but not including ln
	(iv)	Attempt evaluation using *y* attempts	M1		with coeffs 1, 4 and 2 occurring at least once each; whether fg or gf
		Obn $k(\ln 24+4\ln 12+2\ln 8+4\ln 12+\ln 24)$	A1		any constant *k*
		Use $k=\frac{2}{3}$ and obtain 20.3	A1	**3**	or greater accuracy (20.26…) but must round to 20.3

[Note that use of Simpson's rule between 0 and 4 with two strips, coeffs 1, 4, 1, followed by doubling of result is equiv;
SC: Use of Simpson's rule between 0 and 4 with four strips followed by doubling of result - allow 3/3 - answer is 20.2 (20.2327…)]

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8 (a) (i) Draw at least two correctly shaped branches, one for $y > 0$, one for $y < 0$	M1		otherwise located anywhere including $x < 0$
Draw four correct branches	M1		now (more or less) correctly located;
Draw (more or less) correct graph	A1	**3**	with some indication of horiz scale (perhaps only 4π indicated); with asymptotic behaviour shown (but not too fussy about branch drifting slightly away from asymptotic value nor about branch touching asymptote) but branches must not obviously cross asymptotic value; with –1 and 1 shown (or implied by presence of sine curve or by presence of only one of them on a reasonably accurate sketch); no need for vertical (dotted) lines drawn to indicate asymptotic values
(ii) State expression of form $k\pi + \alpha$ or $k\pi - \alpha$ or $\alpha = k\pi + \beta$ or $\alpha = k\pi - \beta$	M1		any non-zero numerical value of *k*; M0 if degrees used
State $3\pi - \alpha$	A1	**2**	or unsimplified equiv
(b) (i) State $\dfrac{2\tan\theta}{1-\tan^2\theta}$	B1	**1**	or equiv such as $\dfrac{t+t}{1-t\times t}$ or $\dfrac{2\tan A}{1-\tan^2 A}$
(ii) State or imply $\tan\phi = \frac{1}{4}$	B1		or equiv such as $\frac{1}{\tan\phi} = 4$
Attempt to evaluate $\tan 2\phi$ or $\cot 2\phi$	M1		perhaps within attempt at complete expression but using correct identity
Obtain $\tan 2\phi = \frac{8}{15}$ or $\cot 2\phi = \frac{15}{8}$	A1		or (unsimplified) equiv; may be implied
Attempt to evaluate value of $\tan 4\phi$	M1		perhaps within attempt at complete expression; condone only minor slip(s) in use of relevant identity
Obtain $\frac{240}{161}$	A1		or (unsimplified) exact equiv; may be implied
Obtain final answer $\frac{225}{322}$	A1	**6**	or exact equiv

[SC – (use of calculator and little or no working)

State or imply $\tan\phi = \frac{1}{4}$ B1; Obtain $\tan 2\phi = \frac{8}{15}$ B1; Obtain $\frac{225}{322}$ B1 (max 3/6)

State or imply $\tan\phi = \frac{1}{4}$ B1; Obtain $\frac{225}{322}$ B2 (max 3/6)

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9	**(i)**	**(a)** Differentiate to obtain $k_1e^{2x} + k_2e^{-2x}$	M1		any constants k_1 and k_2 but derivative must be different from f(x); condone presence of + c
		Obtain $2e^{2x} + 6e^{-2x}$	A1		or unsimplified equiv; no + c now
		Refer to $e^{2x} > 0$ and $e^{-2x} > 0$ or to more general comment about exponential functions	A1	**3**	or equiv (which might be sketch of $y = f(x)$ with comment that gradient is positive or might be sketch of $y = f'(x)$ with comment that $y > 0$; AG
		(b) Differentiate to obtain $k_3e^{2x} + k_4e^{-2x}$	M1		any constants k_3 and k_4 but second derivative must be different from their first derivative; condone presence of + c
		Obtain $4e^{2x} - 12e^{-2x}$	A1		or unsimplified equiv; no + c now
		Attempt solution of $f''(x) > 0$ or of $f(x) > 0$ or of corresponding eqn	M1		at least as far as term involving e^{4x} or e^{-4x}
		Obtain $x > \frac{1}{4}\ln 3$	A1		
		Confirm both give same result	B1	**5**	AG; necessary detail needed; either by solving the other or by observing that same inequality involved (just noting that $f''(x) = 4f(x)$ is sufficient)
	(ii)	Differentiate to obtain $2e^{2x} - 2ke^{-2x}$	B1		or unsimplified equiv
		Attempt to find x-coordinate of stationary pt	M1		equating to 0 and reaching $e^{4x} = ...$ or equiv
		Obtain $e^{4x} = k$ and hence $\frac{1}{4}\ln k$ or equiv	A1		or equiv such as $e^{2x} = \sqrt{k}$
		Substitute and attempt simplification	M1		using valid processes but allow if only limited progress [note that question can be successfully concluded (without actually finding x) by substitution of $e^{2x} = \sqrt{k}$ and $e^{-2x} = \frac{1}{\sqrt{k}}$]
		Obtain $g(x) \geq 2\sqrt{k}$ or $y \geq 2\sqrt{k}$	A1	**5**	or similarly simplified equiv with ≥ not >
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