4723 Core Mathematics 3

1		Obtain integral of form $k(2x-7)^{-1}$	M1		any constant k
		Obtain correct $-5(2x-7)^{-1}$	A1		or equiv
		Include … + c	B1	**3**	at least once; following any integral
				3	
2	**(i)**	Use $\sin 2\theta = 2\sin\theta\cos\theta$	B1		
		Attempt value of $\sin\theta$ from $k\sin\theta\cos\theta = 5\cos\theta$	M1		any constant k; or equiv
		Obtain $\frac{5}{12}$	A1	**3**	or exact equiv; ignore subsequent work
	(ii)	Use $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ or $\operatorname{cosec}^2\theta = 1+\cot^2\theta$	B1		or equiv
		Attempt to produce equation involving $\cos\theta$ only	M1		using $\sin^2\theta = \pm1\pm\cos^2\theta$ or equiv
		Obtain $3\cos^2\theta + 8\cos\theta - 3 = 0$	A1		or equiv
		Attempt solution of 3-term quadratic equation	M1		using formula or factorisation or equiv
		Obtain $\frac{1}{3}$ as only final value of $\cos\theta$	A1	**5**	or exact equiv; ignore subsequent work
				8	
3	**(i)**	Obtain or clearly imply $60\ln x$	B1		
		Obtain ($60\ln 20 - 60\ln 10$ and hence) $60\ln 2$	B1	**2**	with no error seen
	(ii)	Attempt calculation of form $k(y_0 + 4y_1 + y_2)$	M1		any constant k; using y-value attempts
		Identify k as $\frac{5}{3}$	A1		
		Obtain $\frac{5}{3}(6+4\times4+3)$ and hence $\frac{125}{3}$ or 41.7	A1	**3**	or equiv
	(iii)	Equate answers to parts **(i)** and **(ii)**	M1		provided $\ln 2$ involved
		Obtain $60\ln 2 = \frac{125}{3}$ and hence $\frac{25}{36}$	A1	**2**	AG; necessary detail required including clear use of an exact value from **(ii)**
				7	
4	**(i)**	Attempt correct process for composition	M1		numerical or algebraic
		Obtain (7 and hence) 0	A1	**2**	
	(ii)	Attempt to find x-intercept	M1		
		Obtain $x \le 7$	A1	**2**	or equiv; condone use of <
	(iii)	Attempt correct process for finding inverse	M1		
		Obtain $\pm(2-y)^3 - 1$ or $\pm(2-x)^3 - 1$	A1		
		Obtain correct $(2-x)^3 - 1$	A1	**3**	or equiv in terms of x
	(iv)	Refer to reflection in $y = x$	B1	**1**	or clear equiv
				8	

5	**(i)**	Obtain derivative of form $kx(x^2+1)^7$	M1		any constant k
		Obtain $16x(x^2+1)^7$	A1		or equiv
		Equate first derivative to 0 and confirm $x=0$ or substitute $x=0$ and verify first derivative zero	M1		AG; allow for deriv of form $kx(x^2+1)^7$
		Refer, in some way, to $x^2+1=0$ having no root	A1	**4**	or equiv
	(ii)	Attempt use of product rule	*M1		obtaining … + … form
		Obtain $16(x^2+1)^7$ + …	A1√		follow their $kx(x^2+1)^7$
		Obtain … + $224x^2(x^2+1)^6$	A1√		follow their $kx(x^2+1)^7$; or unsimplified equiv
		Substitute 0 in attempt at second derivative	M1		dep *M
		Obtain 16	A1	**5**	from second derivative which is correct at some point
				9	
6		Integrate e^{3x} to obtain $\frac{1}{3}e^{3x}$ or $e^{-\frac{1}{2}x}$ to obtain $-2e^{-\frac{1}{2}x}$	B1		or both
		Obtain indefinite integral of form $m_1e^{3x}+m_2e^{-\frac{1}{2}x}$	M1		any constants m_1 and m_2
		Obtain correct $\frac{1}{3}ke^{3x}-2(k-2)e^{-\frac{1}{2}x}$	A1		or equiv
		Obtain $e^{3\ln 4}=64$ or $e^{-\frac{1}{2}\ln 4}=\frac{1}{2}$	B1		or both
		Apply limits and equate to 185	M1		including substitution of lower limit
		Obtain $\frac{64}{3}k-(k-2)-\frac{1}{3}k+2(k-2)=185$	A1		or equiv
		Obtain $\frac{17}{2}$	A1	**7**	or equiv
				7	
7	**(a)**	Either: State or imply either $\frac{dA}{dr}=2\pi r$ or $\frac{dA}{dt}=250$	B1		or both
		Attempt manipulation of derivatives to find $\frac{dr}{dt}$	M1		using multiplication / division
		Obtain correct $\frac{250}{2\pi r}$	A1		or equiv
		Obtain 1.6	A1	**4**	or equiv; allow greater accuracy
		Or: Attempt to express r in terms of t	M1		using $A=250t$
		Obtain $r=\sqrt{\frac{250t}{\pi}}$	A1		or equiv
		Differentiate $kt^{\frac{1}{2}}$ to produce $\frac{1}{2}kt^{-\frac{1}{2}}$	M1		any constant k
		Substitute $t=7.6$ to obtain 1.6	A1	**(4)**	allow greater accuracy

	(b)	State $\frac{dm}{dt} = -150ke^{-kt}$	B1	
		Equate to $(\pm)3$ and attempt value for t	M1	using valid process; condone sign confusion
		Obtain $-\frac{1}{k}\ln(\frac{1}{50k})$ or $\frac{1}{k}\ln(50k)$ or $\frac{\ln 50 + \ln k}{k}$	A1 **3**	or equiv but with correct treatment of signs
			7	
8	**(i)**	State scale factor is $\sqrt{2}$	B1	allow 1.4
		State translation is in negative x-direction …	B1	or clear equiv
		… by $\frac{3}{2}$ units	B1 **3**	
	(ii)	Draw (more or less) correct sketch of $y = \sqrt{2x+3}$	B1	'starting' at point on negative x-axis
		Draw (more or less) correct sketch of $y = \frac{N}{x^3}$	B1	showing both branches
		Indicate one point of intersection	B1 **3**	with both sketches correct
		[SC: if neither sketch complete or correct but diagram correct for both in first quadrant B1]		
	(iii)	**(a)** Substitute 1.9037 into $x = N^{\frac{1}{3}}(2x+3)^{-\frac{1}{6}}$	M1	or into equation $\sqrt{2x+3} = \frac{N}{x^3}$; or equiv
		Obtain 18 or value rounding to 18	A1 **2**	with no error seen
		(b) State or imply $2.6282 = N^{\frac{1}{3}}(2 \times 2.6022 + 3)^{-\frac{1}{6}}$	B1	
		Attempt solution for N	M1	using correct process
		Obtain 52	A1 **3**	concluding with integer value
			11	
9	**(i)**	Identify $\tan 55°$ as $\tan(45° + 10°)$	B1	or equiv
		Use correct angle sum formula for $\tan(A+B)$	M1	or equiv
		Obtain $\frac{1+p}{1-p}$	A1 **3**	with $\tan 45°$ replaced by 1
	(ii)	<u>Either</u>: Attempt use of identity for $\tan 2A$	*M1	linking $10°$ and $5°$
		Obtain $p = \frac{2t}{1-t^2}$	A1	
		Attempt solution for t of quadratic equation	M1	dep *M
		Obtain $\frac{-1+\sqrt{1+p^2}}{p}$	A1 **4**	or equiv; and no second expression
		<u>Or (1)</u>: Attempt expansion of $\tan(60° - 55°)$	*M1	
		Obtain $\frac{\sqrt{3} - \frac{1+p}{1-p}}{1+\sqrt{3}\,\frac{1+p}{1-p}}$	A1√	follow their answer from **(i)**
		Attempt simplification to remove denominators	M1	dep *M
		Obtain $\frac{\sqrt{3}(1-p)-(1+p)}{1-p+\sqrt{3}(1+p)}$	A1 **(4)**	or equiv

	Or (2): State or imply $\tan 15° = 2-\sqrt{3}$	B1	
	Attempt expansion of $\tan(15°-10°)$	M1	with exact attempt for $\tan 15°$
	Obtain $\dfrac{2-\sqrt{3}-p}{1+p(2-\sqrt{3})}$	A2 **(4)**	
	Or (3): State or imply $\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1}$	B1	or exact equiv
	Attempt expansion of $\tan(15°-10°)$	M1	with exact attempt for $\tan 15°$
	Obtain $\dfrac{\sqrt{3}-1-p\sqrt{3}-p}{\sqrt{3}+1+p\sqrt{3}-p}$	A2 **(4)**	or equiv
	Or (4): Attempt expansion of $\tan(10°-5°)$	*M1	
	Obtain $t = \dfrac{p-t}{1+pt}$	A1	
	Attempt solution for t of quadratic equation	M1	dep *M
	Obtain $\dfrac{-2+\sqrt{4+4p^2}}{2p}$	A1 **(4)**	or equiv; and no second expression
(iii)	Attempt expansion of both sides	M1	
	Obtain $3\sin\theta\cos 10° + 3\cos\theta\sin 10° = 7\cos\theta\cos 10° + 7\sin\theta\sin 10°$	A1	or equiv
	Attempt division throughout by $\cos\theta\cos 10°$	M1	or by $\cos\theta$ (or $\cos 10°$) only
	Obtain $3t + 3p = 7 + 7pt$	A1	or equiv
	Obtain $\dfrac{3p-7}{7p-3}$	A1 **5**	or equiv
		12	