Core 3 Jan 2010

1) $\int 10(2x-7)^{-2}\,dx = \frac{10(2x-7)^{-1}}{2x-1}$

$= -5(2x-7)^{-1} = \underline{\frac{-5}{2x-7}}$

2) i) $6\sin 2\theta = 5\cos\theta$

$12\sin\theta\cos\theta - 5\cos\theta = 0$

$\cos\theta(12\sin\theta - 5) = 0$

$\cos\theta = 0 \quad \theta = 90°$ impossible

$\sin\theta = \frac{5}{12}$

ii) $8\cot\theta\,\mathrm{cosec}^2\theta = 3$

$\frac{8\cos\theta}{\sin^2\theta} = 3$

$8\cos\theta = 3\sin^2\theta = 3(1-\cos^2\theta)$

$3\cos^2\theta + 8\cos\theta - 3 = 0$

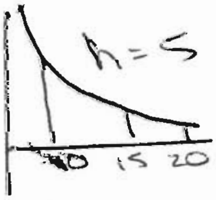
$(3\cos\theta - 1)(\cos\theta + 3) = 0$

$\cos\theta = \frac{1}{3}$ or -3 (impossible)

3) i) $\int_{10}^{20} \frac{60}{x}\,dx = [60\ln x]_{10}^{20}$

$I = 60(\ln 20 - \ln 10)$

$= \underline{60\ln 2}$

ii) 

x	y	
10	6	y_0
15	4	y_1
20	3	y_2

$\int_{10}^{20} \frac{60}{x}\,dx = \frac{5}{3}(6 + 4\times4 + 3)$

$= \underline{\frac{125}{3}}$

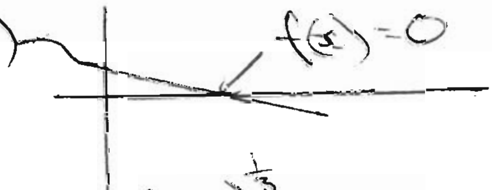
3 iii) $60\ln 2 \approx \frac{125}{3}$

$\ln 2 \approx \frac{125}{180} = \underline{\frac{25}{36}}$

4) i) $f(x) = 2 - (x+1)^{\frac{1}{3}}$

$f(-126) = 2 - (-125)^{\frac{1}{3}} = 2 - -5$
$= 7$

$f(7) = 2 - (7+1)^{\frac{1}{3}} = \underline{0}$

ii) 

$2 - (x+1)^{\frac{1}{3}} = 0$

$(x+1)^{\frac{1}{3}} = 2 \quad x+1 = 8 \quad x = 7$

$f(x) = |f(x)|$ if $\underline{x \leq 7}$

iii) $x \to +1 \to p^{\frac{1}{3}} \to x-1 \to +2 = f(x)$

$f^{-1}(x) = (-(x-2))^3 - 1$

$= \underline{(2-x)^3 - 1}$

iv) reflection in line $y = x$

4) i) $y = (x^2+1)^8$

$\frac{dy}{dx} = 8(x^2+1)^7 \times 2x = 16x(x^2+1)^7$

for a TP $16x(x^2+1)^7 = 0$

either $\underline{x = 0}$ or $x^2+1 = 0$
impossible

ii) $\frac{d^2y}{dx^2} = 16x\,7(x^2+1)^6 \times 2x$
$+ 16(x^2+1)^7$

$= 16(x^2+1)^6(14x^2 + x^2 + 1)$

$= 16(x^2+1)^6(15x^2+1)$

If $x = 0$ $\frac{d^2y}{dx^2} = \underline{16}$

6) $\int k e^{3x} + (k-2)e^{-\frac{1}{2}x}\, dx$

$I = \left[\frac{1}{3}k e^{3x} - 2(k-2)e^{-\frac{1}{2}x}\right]_0^{\ln 4}$

$I_{\ln 4} = \frac{1}{3}k e^{3\ln 4} - 2(k-2)e^{-\frac{1}{2}\ln 4}$

$= \frac{1}{3}k\, 4^3 - 2(k-2)\frac{1}{2}$

$= \frac{64k}{3} - k + 2$

$I_0 = \frac{1}{3}k - 2(k-2)$

$= \frac{1}{3}k - 2k + 4 = -\frac{5}{3}k + 4$

so $\left(\frac{64}{3}k - k + 2\right) - \left(-\frac{5}{3}k + 4\right) = 185$

$\frac{61k}{3} + \frac{5k}{3} + 2 - 4 = 185$

$\frac{66k}{3} - 2 = 185 \quad \underline{\underline{k = 8.5}}$

7) a) $\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt} \qquad A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$\frac{dr}{dt} = \frac{1}{2\pi r} \times 250$

if $A = 1900 = \pi r^2$

$r = \sqrt{\frac{1900}{\pi}}$

$\frac{dr}{dt} = \frac{1}{2\pi\sqrt{\frac{1900}{\pi}}} \times 250 = \underline{\underline{1.6\,\text{m/hr}}}$

b) $m = 150e^{-kt}$

$\frac{dm}{dt} = -150ke^{-kt}$

$-150ke^{-kt} = -3$

$e^{-kt} = \frac{1}{50k}$

$e^{kt} = 50$

$kt = \ln(50k)$

$\underline{\underline{t = \frac{1}{k}\ln(50k)}}$

8) i) $y = \sqrt{2x+3} = (2x+3)^{\frac{1}{2}}$

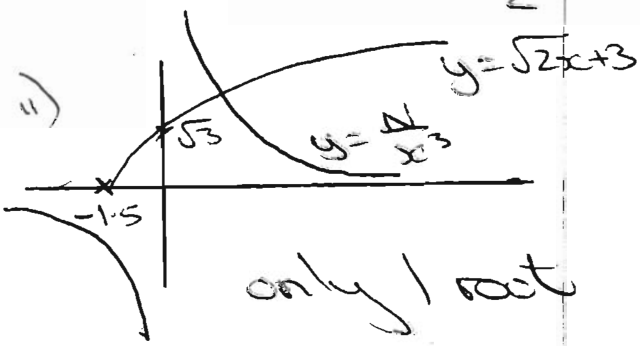
$= \left(2\left(x+\frac{3}{2}\right)\right)^{\frac{1}{2}}$

$= 2^{\frac{1}{2}}\left(x+\frac{3}{2}\right)^{\frac{1}{2}}$

↑ stretch factor $2^{\frac{1}{2}}$ ($\sqrt{2}$)

→ translation $-\frac{3}{2}$

ii)



only 1 root

iii) a) $\sqrt{2x+3} = \frac{N}{x^3}$

$x_{n+1} = N^{\frac{1}{3}}(2x_n+3)^{-\frac{1}{6}}$

if converges to 1.9037

then $x_{n+1} = x_n = 1.9037$

$1.9037 = N^{\frac{1}{3}}(2 \times 1.9037 + 3)^{-\frac{1}{6}}$

$= N^{\frac{1}{3}}\ 0.726389$

$N^{\frac{1}{3}} = \frac{1.9037}{0.726389} = 2.62076$

$N = (2.62076\ldots)^3$

$N = 18$

8b) $x_3 = 2.6022$

$x_4 = 2.6282$

$$2.6282 = N^{\frac{1}{3}}(2\times 2.6022+3)^{-\frac{1}{6}}$$
$$= N^{\frac{1}{3}} \times 0.70414$$

$$N^{\frac{1}{3}} = \frac{2.6282}{0.70414} = 3.73249$$

$$N = 3.73249^3$$
$$= 52$$

9) $\tan 55 = \tan(45+10)$

$$= \frac{\tan 45 + \tan 10}{1 - \tan 45 \tan 10}$$

$p = \tan 10$

$$\tan 55 = \underline{\underline{\frac{1+p}{1-p}}}$$

ii) $\tan 10 = \dfrac{2\tan 5}{1-\tan^2 5}$

$$(1-\tan^2 5)p = 2\tan 5$$

$$p\tan^2 5 + 2\tan 5 - p = 0$$

quadratic

$$\tan 5 = \frac{-2 \pm \sqrt{4+4p^2}}{2p}$$

$$= \frac{-2 \pm 2\sqrt{1+p^2}}{2p}$$

$$= \frac{-1 \pm \sqrt{1+p^2}}{p}$$

9(iii)

$$3\sin(\theta+10) = 7\cos(\theta-10)$$

$$3(\sin\theta\cos 10 + \cos\theta\sin 10) =$$
$$7(\cos\theta\cos 10 + \sin\theta\sin 10)$$

divide by $\cos\theta\cos 10$

$$3(\tan\theta + \tan 10) = 7(1+\tan\theta\tan 10)$$

$$3(\tan\theta + p) = 7(1+p\tan\theta)$$

$$3\tan\theta - 7p\tan\theta = 7 - 3p$$

$$\tan\theta = \frac{7-3p}{3-7p}$$