4723 Core Mathematics 3

1 (i) Show correct process for composition of functions   **M1**   numerical or algebraic; the right way round

Obtain (–3 and hence) –23   **A1 2**

(ii) <u>Either</u>:   State or imply $x^3 + 4 = 12$ **B1**

Attempt solution of equation involving x^3 **M1**   as far as $x = \ldots$
Obtain 2   **A1 3**   and no other value

<u>Or</u>:   Attempt expression for f^{-1} **M1**   involving x or y; involving cube root
Obtain $\sqrt[3]{x-4}$ or $\sqrt[3]{y-4}$ **A1**
Obtain 2   **A1 (3)** and no other value

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2 (i) Obtain correct first iterate 2.864   **B1**   or greater accuracy 2.864327…; condone 2 dp here and in working

Carry out correct iteration process   **M1**   to find at least 3 iterates in all
Obtain 2.877   **A1 3**   after at least 4 steps; answer required to exactly 3 dp

$$[3 \rightarrow 2.864327 \rightarrow 2.878042 \rightarrow 2.876661 \rightarrow 2.876800]$$

(ii) State or imply $x = \sqrt[3]{31 - \frac{5}{2}x}$ **B1**

Attempt rearrangement of equation in x **M1**   involving cubing and grouping non-zero terms on LHS

Obtain equation $2x^3 + 5x - 62 = 0$ **A1 3**   or equiv with integers

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3 (a) State correct equation involving $\cos\frac{1}{2}\alpha$ **B1**   such as $\cos\frac{1}{2}\alpha = \frac{1}{4}$ or $\dfrac{1}{\cos\frac{1}{2}\alpha} = 4$

or …
Attempt to find value of α **M1**   using correct order for the steps
Obtain 151   **A1 3**   or greater accuracy; and no other values between 0 and 180

(b) State or imply $\cot\beta = \dfrac{1}{\tan\beta}$ **B1**

Rearrange to the form $\tan\beta = k$ **M1**   or equiv involving $\sin\beta$ only or $\cos\beta$ only; allow missing $\pm$

Obtain 69.3   **A1**
Obtain 111   **A1 4**   or greater accuracy; and no others between 0 and 180

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4 (i) Obtain derivative of form $kh^5(h^6 + 16)^n$ **M1**   any constant k; any $n < \frac{1}{2}$; allow if -4 term retained

Obtain correct $3h^5(h^6 + 16)^{-\frac{1}{2}}$ **A1**   or (unsimplified) equiv; no –4 now
Substitute to obtain 10.7   **A1 3**   or greater accuracy or exact equiv

(ii) Attempt multn or divn using 8 and answer from (i)   **M1**
Attempt 8 divided by answer from (i)   **M1**
Obtain 0.75   **A1√ 3** or greater accuracy; allow 0.75 ± 0.01; following their answer from (i)

5 (a) Obtain integral of form $k(3x + 7)^{10}$ **M1**   any constant k

Obtain (unsimplified) $\frac{1}{10} \times \frac{1}{3}(3x + 7)^{10}$ **A1**   or equiv

Obtain (simplified) $\frac{1}{30}(3x + 7)^{10} + c$ **A1 3**

(b) State $\int \pi \left(\frac{1}{2\sqrt{x}}\right)^2 dx$ **B1**   or equiv involving x; condone no dx

Integrate to obtain $k \ln x$ **M1**   any constant k involving π or not;

                                         or equiv such as $k \ln 4x$ or $k \ln 2x$

Obtain $\frac{1}{4}\pi \ln x$ or $\frac{1}{4}\ln x$ or $\frac{1}{4}\pi \ln 4x$ or $\frac{1}{4}\ln 4x$ **A1**

Show use of the log a – log b property   **M1**   not dependent on earlier marks

Obtain $\frac{1}{4}\pi \ln 2$ **A1 5**   or similarly simplified equiv

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6 (i) <u>Either</u>:   Refer to translation and reflection   **B1**   in either order; allow clear equivs

            State translation by 1 in negative x-direction   **B1**   or equiv but now using correct terminology

            State reflection in x-axis   **B1 3**   using correct terminology

         <u>Or</u>:   Refer to translation and reflection   **B1**   in either order; allow clear equivs

            State reflection in y-axis   **B1**

            State translation by 1 in positive x-direction   **B1 (3)**   with order reflection then translation clearly intended

(ii) Show sketch with attempt at reflection of 'negative' part in x-axis   **M1**   and curve for $0<x<1$ unchanged

Show (more or less) correct sketch   **A1 2**   with correct curvature

(iii) Attempt correct process for finding at least one value   **M1**   as far as $x = \ldots$; accept decimal equivs (degrees or radians) or expressions involving $\sin(\frac{1}{3}\pi)$

Obtain $1 - \frac{1}{2}\sqrt{3}$ **A1**   or exact equiv

Obtain $1 + \frac{1}{2}\sqrt{3}$ **A1 3**   or exact equiv; give **A1A0** if extra incorrect solution(s) provided

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7 (i) Attempt use of product rule for $x e^{2x}$ **M1**   obtaining $\ldots + \ldots$

Obtain $e^{2x} + 2x e^{2x}$ **A1**   or equiv; maybe within QR attempt

Attempt use of quotient rule   **M1**   with or without product rule

Obtain unsimplified $\dfrac{(x + k)(e^{2x} + 2x e^{2x}) - x e^{2x}}{(x + k)^2}$ **A1**

Obtain $\dfrac{e^{2x}(2x^2 + 2kx + k)}{(x + k)^2}$ **A1 5**   **AG**; necessary detail required

(ii) Attempt use of discriminant   **M1**   or equiv

Obtain $4k^2 - 8k = 0$ or equiv and hence $k = 2$ **A1**

Attempt solution of $2x^2 + 2kx + k = 0$ **M1**   using their numerical value of k or solving in terms of k using correct formula

Obtain $x = -1$ **A1**

Obtain $-e^{-2}$ **A1 5**   or exact equiv

8 (i) State or imply $h = 1$ **B1**

Attempt calculation involving attempts at y values   **M1**   addition with each of coefficients 1, 2, 4 occurring at least once; involving at least 5 y values

Obtain $a(1 + 4 \times 2 + 2 \times 4 + 4 \times 8 + 2 \times 16 + 4 \times 32 + 64)$**A1**   any constant a

Obtain 91   **A1 4**

(ii) State $e^{x \ln 2}$ or $k = \ln 2$ **B1**   allow decimal equiv such as $e^{0.69x}$

Integrate e^{kx} to obtain $\frac{1}{k} e^{kx}$ **M1**   any constant k or in terms of general k

Obtain $\frac{1}{\ln 2}(e^{6 \ln 2} - e^0)$ **A1**   or exact equiv

Simplify to obtain $\frac{63}{\ln 2}$ **A1 4**   allow if simplification in part (iii)

(iii) Equate answers to (i) and (ii)   **M1**   provided ln 2 involved other than in power of e

Obtain $\frac{63}{91}$ and hence $\frac{9}{13}$ **A1 2**   **AG**; necessary correct detail required

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9 (i) State at least one of $\cos\theta \cos 60 - \sin\theta \sin 60$
   and $\cos\theta \cos 30 - \sin\theta \sin 30$ **B1**

Attempt complete multiplication of identities of form
   $\pm \cos \cos \pm \sin \sin$ **M1**   with values $\frac{1}{2}\sqrt{3}$, $\frac{1}{2}$ involved

Use $\cos^2\theta + \sin^2\theta = 1$ and $2\sin\theta \cos\theta = \sin 2\theta$ **M1**

Obtain $\sqrt{3} - 2\sin 2\theta$ **A1 4**   **AG**; necessary detail required

(ii) Attempt use of 22.5 in right-hand side   **M1**

Obtain $\sqrt{3} - \sqrt{2}$ **A1 2**   or exact equiv

(iii) Obtain 10.7   **B1**   or greater accuracy; allow ± 0.1

Attempt correct process to find two angles   **M1**   from values of 2θ between 0 and 180

Obtain 79.3   **A1 3**   or greater accuracy and no others between 0 and 90; allow ± 0.1

(iv) Indicate or imply that critical values of
   $\sin 2\theta$ are -1 and 1   **M1**

Obtain both of $k > \sqrt{3} + 2$, $k < \sqrt{3} - 2$ **A1**   condoning decimal equivs, $\leq \geq$ signs

Obtain complete correct solution   **A1 3**   now with exact values and unambiguously stated