Core 3 Jan 2008 Answers

1 i) $g(1) = 2\times1-5 = -3$ 1

$fg(1) = (-3)^3+4 = -23$ 1

ii) $x \to \text{cube} \to +4 = f(x)$

$f^{-1}(x) = \sqrt[3]{x-4}$ 2

$f^{-1}(12) = \sqrt[3]{8} = 2$ 1

$\boxed{5}$

2 $x_1 = 3$ $x_2 = 2.864327$ 1

$x_3 = 2.878042$ $x_4 = 2.877$ 2

use $\sqrt[3]{31-\frac{5}{2}\text{Ans}}$ on calculator

ii) $x = \sqrt[3]{31-\frac{5}{2}x}$ remove subscripts 1

$x^3 = 31-\frac{5}{2}x$ cube b.s

$2x^3+5x-62=0$ × by 2 2

$\boxed{6}$

3) a) $\sec\frac{1}{2}\alpha = 4 \Rightarrow \cos\frac{1}{2}\alpha = \frac{1}{4}$ 1

$\frac{1}{2}\alpha = \cos^{-1}\frac{1}{4} = 75.52^\circ$

$\alpha = 151^\circ$ to 3sf 2

b) $\tan\beta = 7\cot\beta$ 1

$\tan\beta = \frac{7}{\tan\beta}$

$\tan^2\beta = 7$

$\beta = \tan^{-1}\pm\sqrt{7}$ 1

$= 69.3$ or -69.3

$= 69.3$ or 111 to 3sf 2

($+180^\circ$ to obtain more solns)

$\boxed{7}$

4) i) $\frac{dV}{dh} = \frac{1}{2}(h^6+16)^{-\frac{1}{2}}\times 6h^5$

$= 3h^5(h^6+16)^{-\frac{1}{2}}$ 2

if $h=2$ $\frac{dV}{dh} = 10.7$ 1

ii) $\frac{dh}{dt} = \frac{dh}{dV}\times\frac{dV}{dt} = \frac{1}{10.7}\times 8$ 1

$= 0.75$ m/hr 2

$\boxed{6}$

5) a) $I = \frac{(3x+7)^{10}}{10\times3} = \frac{1}{30}(3x+7)^{10}+c$ 3

b) $V = \pi\int\left(\frac{1}{2\sqrt{x}}\right)^2 dx = \pi\int\frac{1}{4x}\,dx$ 1

$= \frac{\pi}{4}\ln x$ 1

$V_6 = \frac{\pi}{4}\ln 6$ $V_3 = \frac{\pi}{4}\ln 3$

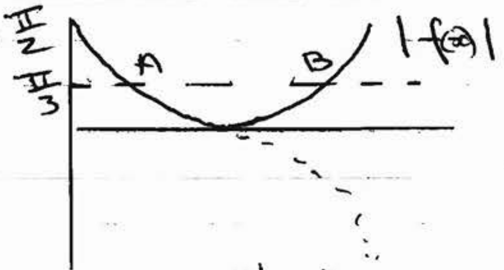
$V = \frac{\pi}{4}(\ln 6-\ln 3) = \frac{\pi}{4}\ln(2)$ 2

$\boxed{8}$

6) i) horizontal translation of -1 ie $\begin{pmatrix}-1\\0\end{pmatrix}$ 1

x axis reflection 1

ii) 2



iii) at A $-\sin^{-1}(x-1) = \frac{\pi}{3}$

$\sin^{-1}(x-1) = -\frac{\pi}{3}$

$x-1 = \sin\left(-\frac{\pi}{3}\right)$

$= -\frac{\sqrt{3}}{2}$

$x = 1-\frac{\sqrt{3}}{2}$

at B $\sin^{-1}(x-1) = \frac{\pi}{3}$

$x-1 = \sin\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

$x = 1+\frac{\sqrt{3}}{2}$ 3

$\boxed{8}$

7 $z = xe^{2x}$

$\frac{dz}{dx} = 2xe^{2x} + e^{2x}$ 2

$\frac{dy}{dx} = \frac{(x+k)(2xe^{2x}+e^{2x}) - xe^{2x}}{(x+k)^2}$

$= \frac{e^{2x}(2x^2+x+2kx+k-x)}{(x+k)^2}$ 1

$= \frac{e^{2x}(2x^2+2kx+k)}{(x+k)^2}$ 2

ii) for TP $\frac{dy}{dx} = 0$

so $2x^2+2kx+k=0$

for 1 root $b^2-4ac=0$

$4k^2-4\times2\times k=0$ 1

$4k^2-8k=0$

$4k(k-2)=0$ so $k=2$ 2

so $2x^2+4x+2=0$

$x^2+2x+1=0$

$(x+1)(x+1)=0 \Rightarrow x=-1$ 1

If $x=-1$ $y = \frac{-1e^{-2}}{-1+2} = -e^{-2}$ 1

TP $(-1,-e^{-2})$ $\boxed{10}$

8)

x	0	1	2	3	4	5	6
$y=2^x$	1	2	4	8	16	32	64

$A = \frac{1}{3}(1+4(2+8+32)+2(4+16)+64)$ 1 1

$= 91$ 1

ii) let $y = 2^x$

$\ln_e y = \ln 2^x = x\ln 2$

so $y = e^{x\ln 2}$ 1

$\int_0^6 e^{x\ln2}\,dx = \left[\frac{1}{\ln 2}e^{x\ln 2}\right]_0^6$ 2

$A = \frac{1}{\ln 2}(e^{6\ln 2}-1) = \frac{1}{\ln 2}(e^{\ln 64}-1)$

$I = \frac{1}{\ln 2}(64-1) = \frac{63}{\ln 2}$ 1

iii) so using (i) + (ii) $91 = \frac{63}{\ln 2}$

$\ln 2 = \frac{63}{91} = \frac{9}{13}$ 2 $\boxed{10}$

9) $4\cos(\theta+60^\circ) = 4(\cos\theta\cos60 - \sin\theta\sin60)$

$= 4\cos\theta\times\frac{1}{2} - 4\frac{\sqrt{3}}{2}\sin\theta$

$= 2\cos\theta - 2\sqrt{3}\sin\theta$ 1

$\cos(\theta+30^\circ) = \cos\theta\cos30 - \sin\theta\sin30$

$= \cos\theta\frac{\sqrt{3}}{2} - \sin\theta\frac{1}{2}$ 1

so LHS $= (2\cos\theta - 2\sqrt{3}\sin\theta)(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta)$

$= \sqrt{3}\cos^2\theta - \cos\theta\sin\theta - 3\cos\theta\sin\theta + \sqrt{3}\sin^2\theta$

$= \sqrt{3} - 4\cos\theta\sin\theta$ $(c^2+s^2=1)$

$= \sqrt{3} - 2\sin 2\theta$ 2 $(2sc = \sin 2\theta)$

ii) subst $\theta = 22.5^\circ$

so RHS $= \sqrt{3} - 2\sin45^\circ$ 2

$= \sqrt{3} - \frac{2}{\sqrt{2}} = \sqrt{3} - \sqrt{2}$

iii) LHS $= \sqrt{3} - 2\sin2\theta = 1$

$\sin 2\theta = \frac{\sqrt{3}-1}{2}$

$2\theta = 21.47, 158.53$

$\theta = 10.7^\circ, 79.3^\circ$ 3

iv) max $\sin 2\theta = 1$ 1

min $\sin 2\theta = -1$

so min $f(\theta) = \sqrt{3}-2$

max $f(\theta) = \sqrt{3}+2$

3

so $f(\theta)$ k>√3+2 k<√3 -2
If f(θ) ≠ k

$\boxed{12}$