Jan 2007 C3

1) $y = \frac{u}{v}$ $y' = \frac{(3x-1)2-(2x+1)3}{(3x-1)^2}$ (2)

[5] $y' = \frac{-5}{(3x-1)^2}$

if $x=1$ $y' = \frac{-5}{4} = m$ (1)

$y - \frac{3}{2} = -\frac{5}{4}(x-1)$

$5x+4y-11=0$ (2)

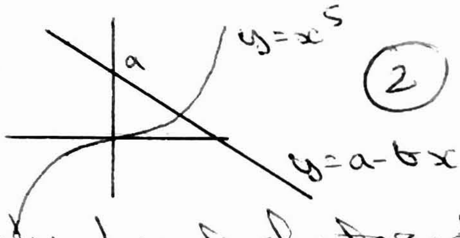
2) $\sin\theta = \frac{12}{13}$

$\cot\theta = \frac{5}{12}$ (2)

[5] $\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2\times\frac{144}{169}$ (1)

$= -\frac{119}{169}$ (2)

3) a)



(2)

only 1 point of intersection as grad line is -ve + both fns are 1 to 1 (1)

[7] so $x^5 = a - bx$

$x^5 + bx - a = 0$ has root

ii) start $x=0$ $x_1 = 2.21236$

$x_2 = 2.17412$

$x_3 = 2.1748$

or start $x=1$ $x_1 = 2.19540$ (4)

$x_2 = 2.17442$

$x_3 = 2.1748$

soln $x = 2.175$ to 3dp

4) $x = (4t+9)^{\frac{1}{2}}$ $\frac{dx}{dt} = \frac{1}{2}(4t+9)^{-\frac{1}{2}} \times 4$ (2)

$= 2(4t+9)^{-\frac{1}{2}}$

[7] $y = 6e^{\frac{1}{2}x+1}$ $\frac{dy}{dx} = 3e^{\frac{1}{2}x+1}$ (2)

(1) (1)

4ii) $\frac{dy}{dt} = 3e^{\frac{1}{2}x+1} \times 2(4t+9)^{-\frac{1}{2}}$

subst $t=4$ $x=5$ $\frac{dy}{dt} = 39.7$ (1)

5) i) $4\cos\theta - \sin\theta = R(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$

$R\cos\alpha = 4$ $\tan\alpha = \frac{1}{4}$

[8] $R\sin\alpha = 1$ $\alpha = 14^\circ$ (2)

$R = \sqrt{4^2+1^2} = \sqrt{17}$ (1)

ii) $\sqrt{17}\cos(\theta+14) = 2$ (2)

$\theta + 14 = \cos^{-1}\frac{2}{\sqrt{17}} = 61^\circ, -61^\circ$

$\theta = 47^\circ$ or -75° (3)

6) i) $A = \int_0^2 (3x+2)^{-\frac{1}{2}}\,dx$

[9] $= \frac{2}{3}(3x+2)^{\frac{1}{2}}$ (2)

$A(2) = \frac{2}{3}\times 8^{\frac{1}{2}}$ $A(0) = \frac{2}{3}\times 2^{\frac{1}{2}}$

$\text{Area} = \frac{2}{3}(\sqrt{8} - \sqrt{2})$ (2) $= \frac{2\sqrt{2}}{3}$

ii) $V = \pi\int_0^2 \frac{1}{3x+2}\,dx$ (1)

$= \pi\,\frac{1}{3}\ln(3x+2)$ (2)

$V(2) = \frac{\pi}{3}\ln 8$ $V(0) = \frac{\pi}{3}\ln 2$ (1)

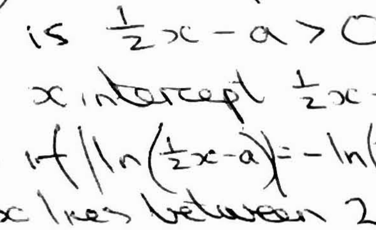
$\text{Vol} = \frac{\pi}{3}(\ln 8 - \ln 2) = \frac{\pi}{3}\ln 4$ (1)

$= \frac{2\pi}{3}\ln 2$

7) i) translation $\begin{bmatrix} a \\ 0 \end{bmatrix}$ (2)

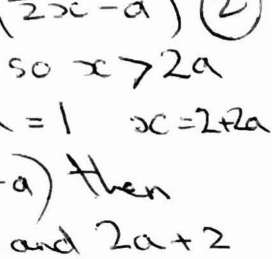
hor. stretch factor 2

ii)



[8] (2)

iii)



(2)

iv) domain for $\ln(\frac{1}{2}x - a)$ (2)

is $\frac{1}{2}x - a > 0$ so $x > 2a$

x intercept $\frac{1}{2}x - a = 1$ $x = 2+2a$

so if $|\ln(\frac{1}{2}x-a)| = -\ln(\frac{1}{2}x-a)$ then

x lies between $2a$ and $2a+2$

$2a < x \leq 2a+2$

8) i) $y = x^8 e^{-x^2} = uv$ (3)

$y' = e^{-x^2} \times 8x^7 - 2x^9 e^{-x^2} = 0$ TP

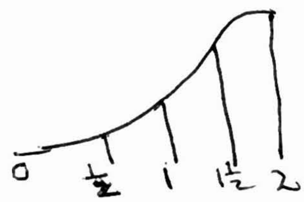
$e^{-x^2} \times 2x^7(4 - x^2) = 0$ (2)

$x = 0$ or $x^2 = 4 \Rightarrow x = \pm 2$

so Q x coord is $x = +2$

[11]

ii)



$h = \frac{1}{2}$

use table

$A = \frac{\frac{1}{2}}{3}\left(0 + 4.68880 + 4(0.00304 + 2.70127) + 2(0.36788)\right)$ (3)

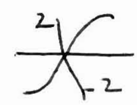
$A = 2.707$ to 3dp (1)

iii) Area B ≐ area rectangle − 2 × Area A

$\doteq 4 \times 2^8 e^{-4} - 2 \times 2.707$

$\doteq 13.3$ (2)

9) range f $-2 \le y \le +2$



range g $y \le 4$

(2)

ii) $f(0.5) = 2\sin 0.5$ (rads mode)

$= 0.95885$

$g(0.95885) = 2.16$ (2)

$g(0.5) = 3.5$ (1)

$f(3.5)$ does not exist as $-\frac{\pi}{2} < x < \frac{\pi}{2}$ (1)

iv) $f^{-1}(x) = \sin^{-1}\frac{x}{2}$

$f^{-1}g(x) = \sin^{-1}\left(\frac{4 - 2x^2}{2}\right)$

$= \sin^{-1}(2 - x^2)$

For the $\sin^{-1}(x)$ fn to exist x lies between ± 1

so $2 - x^2 < +1$ (1) (2)

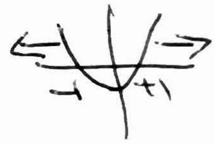
and $2 - x^2 > -1$ (2)

from (1) $2 - x^2 < +1$

$-x^2 + 1 < 0$

$x^2 - 1 > 0$

$(x+1)(x-1) > 0$



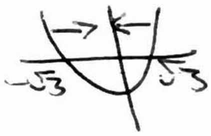
$x > 1$ + $x < -1$

from (2) $2 - x^2 > -1$

$-x^2 + 3 > 0$

$x^2 - 3 < 0$

$(x + \sqrt{3})(x - \sqrt{3}) < 0$



$x < \sqrt{3}$ & $x > -\sqrt{3}$ (2)

combining the regions

$-\sqrt{3}$ -1 0 1 $\sqrt{3}$

So if $f^{-1}g(x)$ is not defined then x lies outside the regions above

$x < -\sqrt{3}$ $\quad -1 < x < 1 \quad$ $x > \sqrt{3}$

(2)

[12]