C3 Jan 2006

1) $I = 3\ln x \quad I_8 = 3\ln 8 = \ln 512$

$I_2 = 3\ln 2 = \ln 8$

$I = \ln 512 - \ln 8 = \ln 64$ (4)

2) $\sec^2\theta = 1 + \tan^2\theta$

$\tan^2\theta - 4\tan\theta + 3 = 0$

$(\tan\theta - 3)(\tan\theta - 1) = 0$

$\tan\theta = 1$ or 3 (5)

$\theta = 45, 225, 71.6, 251.6$

3) a) $y' = x^2\, 6(x+1)^5 + (x+1)^6 \times 2x$

$= 6x^2(x+1)^5 + 2x(x+1)^6$

$= 2x(x+1)^5(4x+1)$ (3)

b) $y' = \dfrac{(x^2-3)2x - (x^2+3)2x}{(x^2-3)^2}$

at $x = 1 \quad y' = -3$ (3)

4) i) range $y \leq 2$ (1)

ii) $f(4) = 2 - 2 = 0$

$f(0) = 2 \quad$ so $ff(4) = 2$ (2)

iii) $0 < k \leq 2$

(2)

5) $A = \int_0^{\frac{1}{2}} (1-2x)^5 - (e^{2x-1} - 1)\, dx$

$= \dfrac{(1-2x)^6}{-12} - \dfrac{e^{2x-1}}{2} + x$

$A(\tfrac{1}{2}) = 0 - \dfrac{1}{2} + \dfrac{1}{2} = 0$

$A(0) = -\dfrac{1}{12} - \dfrac{e^{-1}}{2}$

$\text{Area} = \dfrac{1}{12} + \dfrac{e^{-1}}{2}$ (8)

6) a) $x = Ae^{Bt}$

$t = 0 \quad x = 275 \implies A = 275$

$t = 10 \quad x = 440 \implies 440 = 275e^{10B}$

$B = 0.047$

$x = 275e^{.047 \times 20} = 704$ (3)

b) i) $20 = 80e^{-0.02t}$

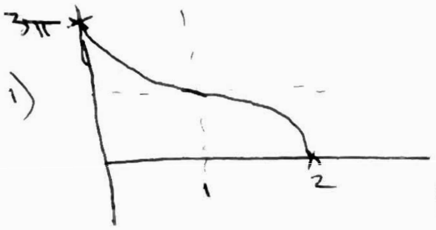
$t = \dfrac{1}{-0.02}\ln\dfrac{20}{80} = 69$ (3)

ii) $\dfrac{dy}{dt} = 80 \times -0.02e^{-0.02t}$

at $t = 30 \quad \dfrac{dy}{dt} = 0.88$ (3)

ignore −ve sign as need rate decreasing

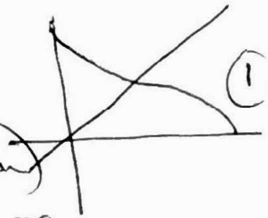
7) i)



(3)

ii) draw line $y = x$

1 root (1 intersection)



(1)

iii) $f(x) = 3\cos^{-1}(x-1) - x$

$f(1.8) = +ve \qquad f(1.9) = -ve$

change of sign so root inbetween (2)

iii) $x_1 = 2 \quad x_2 = 1.786 \quad x_3 = 1.828$

$x_4 = 1.82 \quad x_5 = 1.822$

$\alpha = 1.82$ to 2dp

$3\cos^{-1}(x-1) = x$

$\cos^{-1}(x-1) = \tfrac{1}{3}x$ (5)

$x - 1 = \cos(\tfrac{1}{3}x)$

$x = 1 + \cos(\tfrac{1}{3}x)$

$x_{n+1} = 1 + \cos(\tfrac{1}{3}x_n)$

8) i) $y' = \dfrac{-2x}{5-x^2}$ if $x=2$ $y'=-4$

$y-0=-4(x-2)$

$y=-4x+8$ tangent (5)

ii) $A = \dfrac{0.5}{3}\left(\ln 5+\ln 1+4\ln 4.75+2\ln 4 + 4\ln 2.75\right)$

$= 2.44\ u^2$ (4)

iii) Area triangle $= \frac{1}{2}\times 2\times 8$

$= 8\ u^2$

Area $R = 8-2.44 = 5.56\ u^2$ (2)

9) i) $\sin(2\theta+\theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= 2\sin\theta\cos^2\theta + (1-2\sin^2\theta)\sin\theta$

$= 2s(1-s^2)+s-2s^3$

$= 3\sin\theta - 4\sin^3\theta$ (4)

ii) max value of $\sin 3\theta$ is $y=1$

$9\sin\left(\frac{10\alpha}{3}\right) - 12\sin^3\left(\frac{10\alpha}{3}\right) = 3\sin\left(\frac{30\alpha}{3}\right)$

using part i)

so max value is $y=3$

max value of $\sin\theta$ occurs at

$\theta = 90^\circ$

so max value of $3\sin 10\alpha$ occurs

at $\alpha = 9^\circ$ (3)

iii) $\dfrac{3\sin 6\beta}{\sin 2\beta} = 4$

$3\sin 6\beta - 4\sin 2\beta = 0$

$\sin 6\beta = 3\sin 2\beta - 4\sin^3 2\beta$

from part i)

so $9\sin 2\beta - 12\sin^3 2\beta - 4\sin 2\beta = 0$

$5\sin 2\beta - 12\sin^3 2\beta = 0$

9 contd $\sin 2\beta(5-12\sin^2 2\beta)=0$

$\sin 2\beta = 0 \Rightarrow \beta = 0$ N/A

or $\sin 2\beta = \sqrt{\dfrac{5}{12}}$

$2\beta = 40.2^\circ$ or 139.8°

$\beta = 20.1^\circ$ or 69.9°

(6)