Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4753**: Methods for Advanced Mathematics

Advanced GCE

Mark Scheme for June 2014

1. Annotations and abbreviations

Annotation in scoris	Meaning
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

±

Question			Answer	Marks	Guidance	
1			$\int_0^{\pi/6}(1-\sin 3x)\,dx = \left[x + \frac{1}{3}\cos 3x\right]_0^{\pi/6}$	M1	±1/3 cos 3x seen or $\int \frac{1}{3}(1-\sin u)[du]$	i.e. condone sign error
				A1	$\left[x + \frac{1}{3}\cos 3x\right]$ or $\left[\frac{1}{3}(u+\cos u)\right]$	condone '+ c'
			= π/6 – 1/3	A1cao	o.e., must be exact	isw after correct answer seen
				[3]		
2			$y = \ln(1 - \cos 2x)$, let $u = 1 - \cos 2x$	M1	$1/(1 - \cos 2x)$ soi	
			⇒ $dy/dx = dy/du \,.\, du/dx$			
			$= (1/u).\ 2\sin 2x$	M1	$d/dx\ (1 - \cos 2x) = \pm\ 2\sin 2x$	
			$= \dfrac{2\sin 2x}{1-\cos 2x}$	A1cao		
			When $x = \pi/6$, $\dfrac{dy}{dx} = \dfrac{2\sin(\pi/3)}{1-\cos(\pi/3)}$	M1	substituting π/6 or 30° into their deriv	must be in at least two places
			$= 2\sqrt{3}$	A1cao		isw after correct answer seen
				[5]		
3			$\lvert 3-2x \rvert = 4\lvert x \rvert$			
			⇒ $3 - 2x = 4x$, $x = ½$	M1A1		If 3 or more final answers offered, −1 for each incorrect additional answer
			or $3 - 2x = -4x$, $x = -1½$	M1A1	not 3/(−2)	
			or			−1 for final ans written as an inequality
			$(3 - 2x)^2 = 16x^2$	M1	squaring both sides	$(3 - 2x)^2 = 4x^2$ is M0
			⇒ $12x^2 + 12x - 9$ [= 0]	A1	correct quadratic o.e. but with single x^2 term	
			⇒ $x = ½, -1½$	A1 A1		
				[4]		

Question			Answer	Marks	Guidance	
4	(i)		$a = 2$, $b = ½$	B1B1 **[2]**		
4	(ii)		$y = 2 + \cos ½x \quad x \leftrightarrow y$ $x = 2 + \cos ½y$ $\Rightarrow \quad x - 2 = \cos ½y$ $\Rightarrow \quad \arccos(x - 2) = ½y$ $\Rightarrow \quad y = f^{-1}(x) = 2\arccos(x - 2)$ Domain $1 \le x \le 3$ Range $0 \le y \le 2\pi$	 M1 M1 A1 M1 A1 **[5]**	(may be seen later) subtracting [their] a from both sides (first) arccos(x –[their] a) = [their] $b \times y$ cao or $2\cos^{-1}(x - 2)$ domain 1 to 3, range 0 to 2π correctly specified: must be $\le$, x for domain, y or f^{-1} or $f^{-1}(x)$ for range	 need not substitute for a, b or with $x \leftrightarrow y$, need not subst for a, b may be implied by flow diagram if not stated, assume first is domain allow [1, 3], [0, 2π] not 360° (not f)
5			$dV/dr = 4\pi r^2$ $dV/dt = 10$ $dV/dt = (dV/dr)(dr/dt)$ $\Rightarrow \quad 10 = 4\pi.64.dr/dt$ $\Rightarrow \quad dr/dt = 0.0124$ cm s^{-1}	B1 B1 M1 A1 A1 **[5]**	or $12\pi r^2/3$, condone dr/dV, dV/dR a correct chain rule soi o.e. (soi) must be correct 0.012 or better or $10/256\pi$ or $5/128\pi$	 Condone use of other letters for t o.e. e.g. $dr/dt = (dr/dV)(dV/dt)$ mark final answer

Question			Answer	Marks	Guidance	
6	(i)		$V = 20000e^{-0.2t}$ when $t = 1$, $V = 16374.615\ldots$ so car loses (£)3600	B1 B1 **[2]**	(soi) art 16400 condone no £, must be to nearest £100	 or B2 for correct answer
6	(ii)		When $t = 1$, $V = 13000$ $\Rightarrow \quad 13000 = 15000\ e^{-k}$ $\Rightarrow \quad -k\ [\ln e] = \ln(13000/15000)$ $\Rightarrow k = \ 0.1431\ldots = 0.143$ (3sf) *	M1 M1 A1 **[3]**	 taking lns correctly oe e.g. ln 13000 = ln15000–k [lne] cao **NB AG** must show some working if 4th d.p. not shown	If $k = 0.143$ verified ,e.g. $15000\ e^{-0.143} = 13001[.31\ldots]$, SCB1 need not have substituted for V and A e.g. $k = -\ln(13000/15000) = 0.143$
6	(iii)		$15000e^{-0.143t} = 20000e^{-0.2t}$ $\Rightarrow \quad (15000/20000) = e^{(0.143 - 0.2)t}$ $\Rightarrow \quad t = \ln 0.75 / -0.057 = 5.05$ years so after 5 years	M1* M1dep A1 **[3]**	must be correct, but could use a more accurate value for k dep * cao accept answers in the range 5 − 5.1	If M0, SCB1 for 5 – 5.1 years from correct calculations for each car, rot e.g. $t = 5$, £7358 (Brian), £7338(Kate) or (£7334 with more accurate k) o.e. e.g. ln15000–0.143t = ln 20000− 0.2t
7	(i)		False e.g. neither 25 and 27 are prime as 25 is div by 5 and 27 by 3	B1 B1 **[2]**	correct counter-example identified justified correctly	Need not explicitly say ‘false’
7	(ii)		True: one has factor of 2, the other 4, so product must have factor of 8.	B2 **[2]**	or algebraic proofs: e.g. $2n(2n+2) = 4n(n+1) = 4\times$even$\times$odd no so div by 8	B1 for stating with justification div by 4 e.g. both even, or from $4(n^2 + n)$ or $4pq$

Question			Answer	Marks	Guidance	
8	(i)		$f(-x)=\dfrac{-x}{\sqrt{2+(-x)^2}}$	M1	substituting $-x$ for x in $f(x)$	$\dfrac{-x}{\sqrt{2+-x^2}}, \dfrac{-x}{\sqrt{2+-(x^2)}}, \dfrac{-x}{\sqrt{2+(-x^2)}}$ M1A0
			$=-\dfrac{x}{\sqrt{2+x^2}}=-f(x)$	A1	1st line must be shown, must have $f(-x) = -f(x)$ oe somewhere	$\dfrac{-x}{\sqrt{2-x^2}}$ M0A0
			Rotational symmetry of order 2 about O	B1	must have 'rotate' and 'O' and 'order 2 or 180 or ½ turn'	oe e.g. reflections in both x- and y-axes
				[3]		
8	(ii)		$f'(x)=\dfrac{\sqrt{2+x^2}.1-x.\frac{1}{2}(2+x^2)^{-1/2}.2x}{(\sqrt{2+x^2})^2}$	M1	quotient or product rule used	QR: condone $u\mathrm{d}v \pm v\mathrm{d}u$, but u, v and denom must be correct
				M1	½ $u^{-1/2}$ or – ½ $v^{-3/2}$ soi	
				A1	correct expression	$x(-1/2)(2+x^2)^{-3/2}.2x+(2+x^2)^{-1/2}$.
			$=\dfrac{2+x^2-x^2}{(2+x^2)^{3/2}}=\dfrac{2}{(2+x^2)^{3/2}}$ *	A1	**NB AG**	$=(2+x^2)^{-3/2}(-x^2+2+x^2)$
			When $x = 0$, $f'(x) = 2/2^{3/2} = 1/\sqrt{2}$	B1	oe e.g. $\sqrt{2}/2$, $2^{-1/2}$, $1/2^{1/2}$, but not $2/2^{3/2}$	allow isw on these seen
				[5]		
8	(iii)		$A=\int_0^1 \dfrac{x}{\sqrt{2+x^2}}[\mathrm{d}x]$	B1	correct integral and limits	limits may be inferred from subsequent working, condone no dx
			let $u = 2 + x^2$, du = $2x$ dx		or $v = \sqrt{(2+x^2)}$, d$v = x(2+x^2)^{-1/2}$dx	
			$=\int_2^3 \dfrac{1}{2}\dfrac{1}{\sqrt{u}}\,\mathrm{d}u$	M1	$\int \dfrac{1}{2}\dfrac{1}{\sqrt{u}}[\mathrm{d}u]$ or $=\int 1[\mathrm{d}v]$ or $k(2+x^2)^{1/2}$	condone no du or dv, but not $\int \dfrac{1}{2}\dfrac{1}{\sqrt{u}}\mathrm{d}x$
			$=\left[u^{1/2}\right]_2^3$	A1	$[u^{1/2}]$ o.e. (but not $1/u^{-1/2}$) or $[v]$ or $k = 1$	
			$= \sqrt{3} - \sqrt{2}$	A1cao	must be exact	isw approximations
				[4]		

Question			Answer	Marks	Guidance	
8	(iv)	(*A*)	$y^2 = \dfrac{x^2}{2+x^2}$ $\Rightarrow$ $1/y^2 = (2 + x^2)/x^2 = 2/x^2 + 1$ *	M1 A1 **[2]**	squaring (correctly) or equivalent algebra **NB AG**	must show $\left[\sqrt{(2+x^2)}\right]^2 + 2 + x^2$ (o.e.) If argued backwards from given result without error, SCB1
8	(iv)	(*B*)	$-2y^{-3}\mathrm{d}y/\mathrm{d}x = -4x^{-3}$ $\Rightarrow$ $\mathrm{d}y/\mathrm{d}x = -4x^{-3}/-2y^{-3} = 2y^3/x^3$ * Not possible to substitute $x = 0$ and $y = 0$ into this expression	B1B1 B1 B1 **[4]**	LHS, RHS **NB AG** soi (e.g. mention of 0/0)	condone $\mathrm{d}y/\mathrm{d}x$ $-2y^{-3}$ unless pursued Condone 'can't substitute $x = 0$' o.e. (i.e. need not mention $y = 0$). Condone also 'division by 0 is infinite'
9	(i)		$xe^{-2x} = mx$ $\Rightarrow$ $e^{-2x} = m$ $\Rightarrow$ $-2x = \ln m$ $\Rightarrow$ $x = -½ \ln m$ * **or** If $x = -½ \ln m$, $y = -½ \ln m \times e^{\ln m}$ $= -½ \ln m \times m$ so P lies on $y = mx$	M1 M1 A1 M1 A1 A1 **[3]**	may be implied from 2nd line dividing by x, or subtracting $\ln x$ **NB AG** substituting correctly	o.e. e.g. $[\ln x] - 2x = \ln m + [\ln x]$ or factorising: $x(e^{-2x} - m) = 0$
9	(ii)		let $u = x$, $u' = 1$, $v = e^{-2x}$, $v' = -2e^{-2x}$ $\mathrm{d}y/\mathrm{d}x = e^{-2x} - 2xe^{-2x}$ $= e^{-2.(-\frac{1}{2}\ln m)} - 2.(-\frac{1}{2}\ln m)e^{-2.(-\frac{1}{2}\ln m)}$ $= e^{\ln m} + e^{\ln m} \ln m$ [$= m + m \ln m$]	M1* A1 M1dep A1cao **[4]**	product rule consistent with their derivs o.e. correct expression subst $x = -½ \ln m$ into their deriv dep M1* condone $e^{\ln m}$ not simplified	but not $-2(-½ \ln m)$, but mark final ans

Question			Answer	Marks	Guidance	
9	(iii)		$m + m\ln m = -m$	M1	their gradient from (ii) $= -m$	
			$\Rightarrow \quad \ln m = -2$			
			$\Rightarrow \quad m = e^{-2}$ *	A1	**NB AG**	
			or			
			$y + ½m\ln m = m(1 + \ln m)(x + ½\ln m)$ $x=-\ln m$, $y=0 \Rightarrow ½m\ln m = m(1+\ln m)(-½\ln m)$ $\Rightarrow 1 + \ln m = -1$, $\ln m = -2$, $m = e^{-2}$	B2	for fully correct methods finding x-intercept of equation of tangent and equating to $-\ln m$	
			At P, $x = 1$	B1		
			$\Rightarrow \quad y = e^{-2}$	B1	isw approximations	not $e^{-2} \times 1$
				[4]		
9	(iv)		Area under curve $= \int_0^1 xe^{-2x}dx$			
			$u = x$, $u' = 1$, $v' = e^{-2x}$, $v = -½\, e^{-2x}$	M1	parts, condone $v = k\, e^{-2x}$, provided it is used consistently in their parts formula	ignore limits until 3rd A1
			$= \left[-\frac{1}{2}xe^{-2x}\right]_0^1 + \int_0^1 \frac{1}{2}e^{-2x}dx$	A1ft	ft their v	
			$= \left[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\right]_0^1$	A1	$-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$ o.e	
			$= (-½\, e^{-2} - ¼\, e^{-2}) - (0 - ¼\, e^0)$ [$= ¼ - ¾\, e^{-2}$]	A1	correct expression	need not be simplified
			Area of triangle = ½ base × height	M1	ft their 1, e^{-2} or [$e^{-2}x^2/2$]	o.e. using isosceles triangle
			$= ½ \times 1 \times e^{-2}$	A1		M1 may be implied from 0.067…
			So area enclosed $= ¼ - 5e^{-2}/4$	A1cao	o.e. must be exact, two terms only	isw
				[7]		