OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4753/01

Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Monday 20 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 Solve the equation $|2x - 1| = |x|$. **[4]**

2 Given that $f(x) = 2\ln x$ and $g(x) = e^x$, find the composite function $gf(x)$, expressing your answer as simply as possible. **[3]**

3 **(i)** Differentiate $\frac{\ln x}{x^2}$, simplifying your answer. **[4]**

(ii) Using integration by parts, show that $\int \frac{\ln x}{x^2}\,dx = -\frac{1}{x}(1 + \ln x) + c$. **[4]**

4 The height h metres of a tree after t years is modelled by the equation

$$h = a - be^{-kt},$$

where a, b and k are positive constants.

(i) Given that the long-term height of the tree is 10.5 metres, and the initial height is 0.5 metres, find the values of a and b. **[3]**

(ii) Given also that the tree grows to a height of 6 metres in 8 years, find the value of k, giving your answer correct to 2 decimal places. **[3]**

5 Given that $y = x^2\sqrt{1 + 4x}$, show that $\frac{dy}{dx} = \frac{2x(5x + 1)}{\sqrt{1 + 4x}}$. **[5]**

6 A curve is defined by the equation $\sin 2x + \cos y = \sqrt{3}$.

(i) Verify that the point P $\left(\frac{1}{6}\pi, \frac{1}{6}\pi\right)$ lies on the curve. **[1]**

(ii) Find $\frac{dy}{dx}$ in terms of x and y.

Hence find the gradient of the curve at the point P. **[5]**

7 **(i)** Multiply out $(3^n + 1)(3^n - 1)$. **[1]**

(ii) Hence prove that if n is a positive integer then $3^{2n} - 1$ is divisible by 8. **[3]**

Section B (36 marks)

8

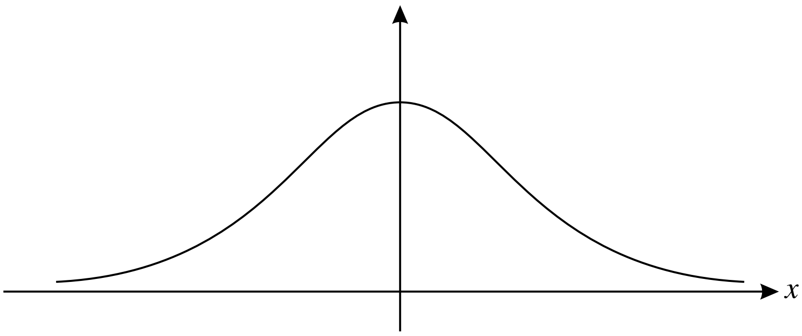

Fig. 8

Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{1}{e^x + e^{-x} + 2}$.

(i) Show algebraically that $\mathrm{f}(x)$ is an even function, and state how this property relates to the curve $y = \mathrm{f}(x)$. **[3]**

(ii) Find $\mathrm{f}'(x)$. **[3]**

(iii) Show that $\mathrm{f}(x) = \dfrac{e^x}{(e^x + 1)^2}$. **[2]**

(iv) Hence, using the substitution $u = e^x + 1$, or otherwise, find the exact area enclosed by the curve $y = \mathrm{f}(x)$, the x-axis, and the lines $x = 0$ and $x = 1$. **[5]**

(v) Show that there is only one point of intersection of the curves $y = \mathrm{f}(x)$ and $y = \frac{1}{4}e^x$, and find its coordinates. **[5]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$. The endpoints of the curve are P $(-\pi, 1)$ and Q $(\pi, 3)$, and $\mathrm{f}(x) = a + \sin bx$, where a and b are constants.

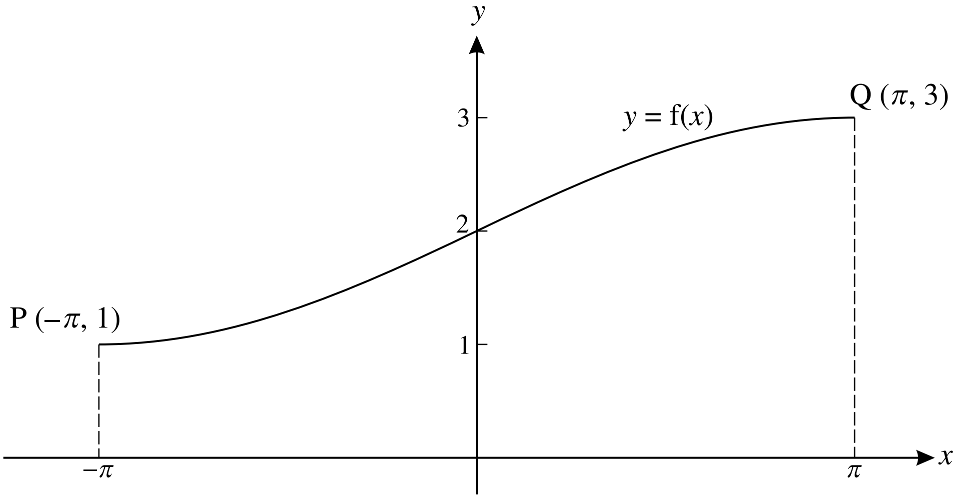

Fig. 9

(i) Using Fig. 9, show that $a = 2$ and $b = \frac{1}{2}$. **[3]**

(ii) Find the gradient of the curve $y = \mathrm{f}(x)$ at the point $(0, 2)$.

Show that there is no point on the curve at which the gradient is greater than this. **[5]**

(iii) Find $\mathrm{f}^{-1}(x)$, and state its domain and range.

Write down the gradient of $y = \mathrm{f}^{-1}(x)$ at the point $(2, 0)$. **[6]**

(iv) Find the area enclosed by the curve $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = \pi$. **[4]**