RECOGNISING ACHIEVEMENT

ADVANCED GCE **4753/01**

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

MONDAY 2 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (36 marks)

1 Solve the inequality $|2x - 1| \leqslant 3$. [4]

2 Find $\int x\mathrm{e}^{3x}\,\mathrm{d}x$. [4]

3 **(i)** State the algebraic condition for the function $\mathrm{f}(x)$ to be an even function.

What geometrical property does the graph of an even function have? [2]

(ii) State whether the following functions are odd, even or neither.

(A) $\mathrm{f}(x) = x^2 - 3$

(B) $\mathrm{g}(x) = \sin x + \cos x$

(C) $\mathrm{h}(x) = \dfrac{1}{x + x^3}$ [3]

4 Show that $\displaystyle\int_1^4 \frac{x}{x^2 + 2}\,\mathrm{d}x = \tfrac{1}{2}\ln 6$. [4]

5 Show that the curve $y = x^2 \ln x$ has a stationary point when $x = \dfrac{1}{\sqrt{\mathrm{e}}}$. [6]

6 In a chemical reaction, the mass m grams of a chemical after t minutes is modelled by the equation

$$m = 20 + 30\mathrm{e}^{-0.1t}.$$

(i) Find the initial mass of the chemical.

What is the mass of chemical in the long term? [3]

(ii) Find the time when the mass is 30 grams. [3]

(iii) Sketch the graph of m against t. [2]

7 Given that $x^2 + xy + y^2 = 12$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [5]

Section B (36 marks)

8 Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{1}{1 + \cos x}$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$.

P is the point on the curve with x-coordinate $\frac{1}{3}\pi$.

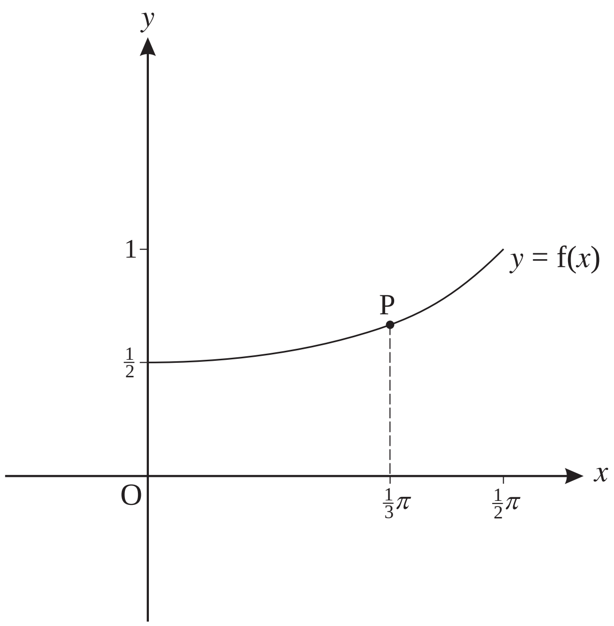

Fig. 8

(i) Find the y-coordinate of P. [1]

(ii) Find $\mathrm{f}'(x)$. Hence find the gradient of the curve at the point P. [5]

(iii) Show that the derivative of $\dfrac{\sin x}{1 + \cos x}$ is $\dfrac{1}{1 + \cos x}$. Hence find the exact area of the region enclosed by the curve $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = \frac{1}{3}\pi$. [7]

(iv) Show that $\mathrm{f}^{-1}(x) = \arccos\left(\dfrac{1}{x} - 1\right)$. State the domain of this inverse function, and add a sketch of $y = \mathrm{f}^{-1}(x)$ to a copy of Fig. 8. [5]

[Question 9 is printed overleaf.]

9 The function $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = \sqrt{4 - x^2}$ for $-2 \leqslant x \leqslant 2$.

(i) Show that the curve $y = \sqrt{4 - x^2}$ is a semicircle of radius 2, and explain why it is not the whole of this circle. [3]

Fig. 9 shows a point P (a, b) on the semicircle. The tangent at P is shown.

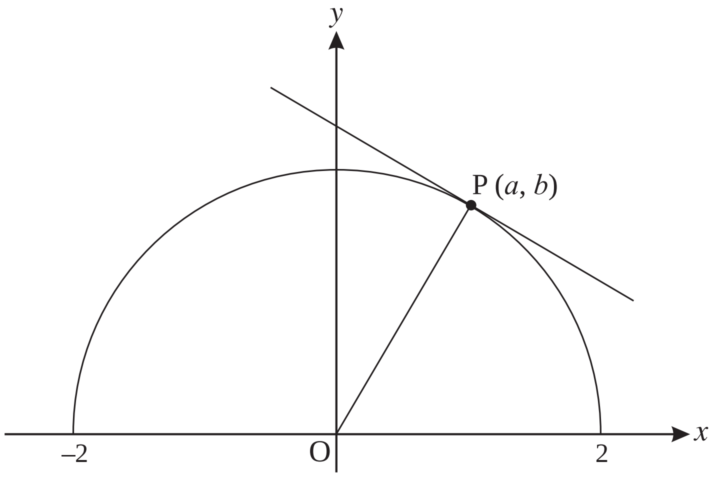

Fig. 9

(ii) (A) Use the gradient of OP to find the gradient of the tangent at P in terms of a and b.

(B) Differentiate $\sqrt{4 - x^2}$ and deduce the value of $\mathrm{f}'(a)$.

(C) Show that your answers to parts (A) and (B) are equivalent. [6]

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = 3\mathrm{f}(x - 2)$, for $0 \leqslant x \leqslant 4$.

(iii) Describe a sequence of two transformations that would map the curve $y = \mathrm{f}(x)$ onto the curve $y = \mathrm{g}(x)$.

Hence sketch the curve $y = \mathrm{g}(x)$. [6]

(iv) Show that if $y = \mathrm{g}(x)$ then $9x^2 + y^2 = 36x$. [3]