OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **4753/1**

Methods for Advanced Mathematics (C3)

Wednesday **25 MAY 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

Section A (36 marks)

1 Solve the equation $|3x + 2| = 1$. [3]

2 Given that $\arcsin x = \frac{1}{6}\pi$, find x. Find $\arccos x$ in terms of π. [3]

3 The functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are defined for the domain $x > 0$ as follows:

$$\mathrm{f}(x) = \ln x, \quad \mathrm{g}(x) = x^3.$$

Express the composite function $\mathrm{fg}(x)$ in terms of $\ln x$.

State the transformation which maps the curve $y = \mathrm{f}(x)$ onto the curve $y = \mathrm{fg}(x)$. [3]

4 The temperature T°C of a liquid at time t minutes is given by the equation

$$T = 30 + 20\mathrm{e}^{-0.05t}, \quad \text{for } t \geqslant 0.$$

Write down the initial temperature of the liquid, and find the initial rate of change of temperature.

Find the time at which the temperature is 40 °C. [6]

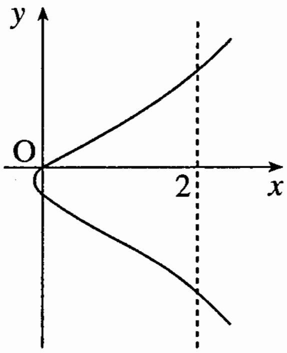
5 Using the substitution $u = 2x + 1$, show that $\int_0^1 \frac{x}{2x+1}\,\mathrm{d}x = \frac{1}{4}(2 - \ln 3)$. [6]

6 A curve has equation $y = \frac{x}{2 + 3\ln x}$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. Hence find the exact coordinates of the stationary point of the curve. [7]

7 Fig. 7 shows the curve defined implicitly by the equation

$$y^2 + y = x^3 + 2x,$$

together with the line $x = 2$.



Not to scale

Fig. 7

Find the coordinates of the points of intersection of the line and the curve.

Find $\frac{dy}{dx}$ in terms of x and y. Hence find the gradient of the curve at each of these two points. [8]

Section B (36 marks)

8 Fig. 8 shows part of the curve $y = x \sin 3x$. It crosses the x-axis at P. The point on the curve with x-coordinate $\frac{1}{6}\pi$ is Q.

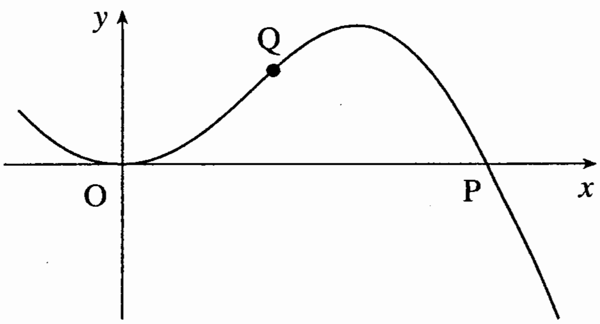


Fig. 8

(i) Find the x-coordinate of P. [3]

(ii) Show that Q lies on the line $y = x$. [1]

(iii) Differentiate $x \sin 3x$. Hence prove that the line $y = x$ touches the curve at Q. [6]

(iv) Show that the area of the region bounded by the curve and the line $y = x$ is $\frac{1}{72}(\pi^2 - 8)$. [7]

9 The function $f(x) = \ln(1 + x^2)$ has domain $-3 \leqslant x \leqslant 3$.

Fig. 9 shows the graph of $y = f(x)$.

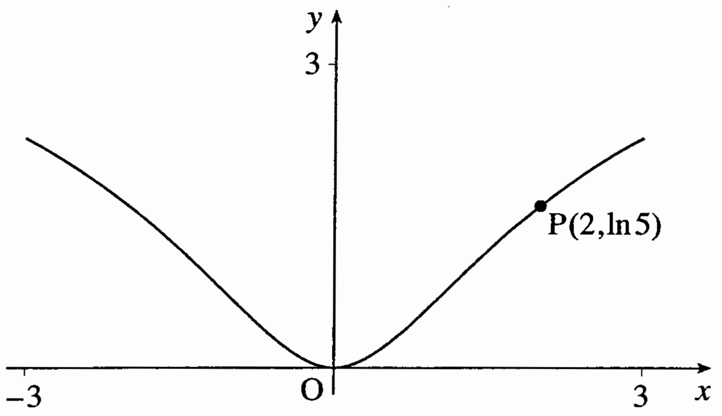


Fig. 9

(i) Show algebraically that the function is even. State how this property relates to the shape of the curve. [3]

(ii) Find the gradient of the curve at the point $P(2, \ln 5)$. [4]

(iii) Explain why the function does not have an inverse for the domain $-3 \leqslant x \leqslant 3$. [1]

The domain of $f(x)$ is now restricted to $0 \leqslant x \leqslant 3$. The inverse of $f(x)$ is the function $g(x)$.

(iv) Sketch the curves $y = f(x)$ and $y = g(x)$ on the same axes.

State the domain of the function $g(x)$.

Show that $g(x) = \sqrt{e^x - 1}$. [6]

(v) Differentiate $g(x)$. Hence verify that $g'(\ln 5) = 1\frac{1}{4}$. Explain the connection between this result and your answer to part **(ii)**. [5]