OCR
RECOGNISING ACHIEVEMENT

Friday 20 January 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Differentiate $x^2 \tan 2x$. **[3]**

2 The functions f(x) and g(x) are defined as follows.

$$f(x) = \ln x, \quad x > 0$$
$$g(x) = 1 + x^2, \quad x \in \mathbb{R}$$

Write down the functions fg(x) and gf(x), and state whether these functions are odd, even or neither. **[4]**

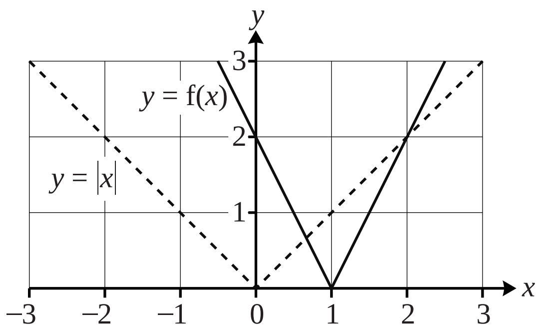
3 Show that $\int_0^{\frac{\pi}{2}} x \cos \frac{1}{2}x \, dx = \frac{\sqrt{2}}{2}\pi + 2\sqrt{2} - 4$. **[5]**

4 Prove or disprove the following statement:

'No cube of an integer has 2 as its units digit.' **[2]**

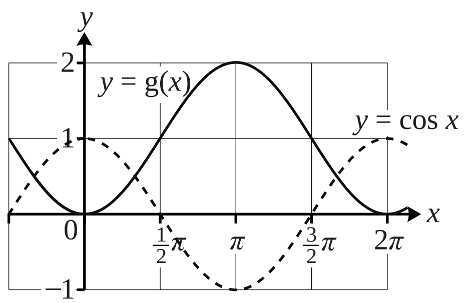
5 Each of the graphs of $y = f(x)$ and $y = g(x)$ below is obtained using a sequence of two transformations applied to the corresponding dashed graph. In each case, state suitable transformations, and hence find expressions for f(x) and g(x).

(i)



[3]

(ii)



[3]

6 Oil is leaking into the sea from a pipeline, creating a circular oil slick. The radius r metres of the oil slick t hours after the start of the leak is modelled by the equation

$$r = 20(1 - e^{-0.2t}).$$

(i) Find the radius of the slick when $t = 2$, and the rate at which the radius is increasing at this time. **[4]**

(ii) Find the rate at which the area of the slick is increasing when $t = 2$. **[4]**

7 Fig. 7 shows the curve $x^3 + y^3 = 3xy$. The point P is a turning point of the curve.

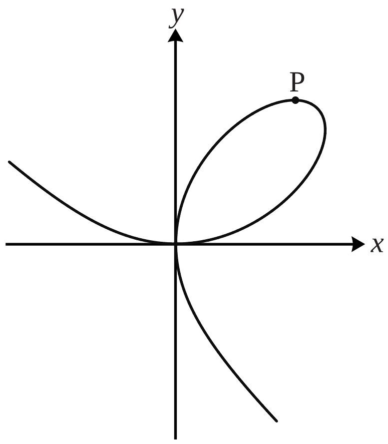


Fig. 7

(i) Show that $\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$. **[4]**

(ii) Hence find the exact x-coordinate of P. **[4]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = \frac{x}{\sqrt{x-2}}$, together with the lines $y = x$ and $x = 11$. The curve meets these lines at P and Q respectively. R is the point (11, 11).

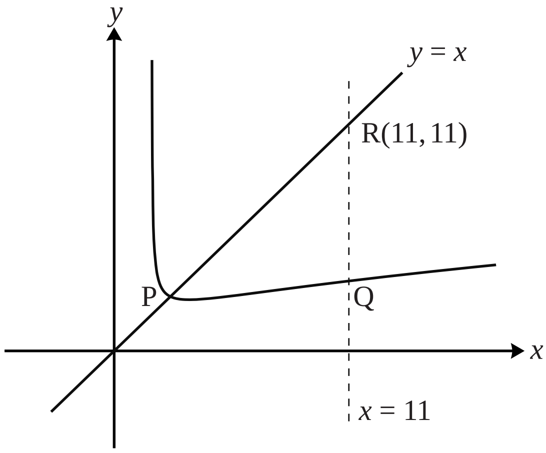


Fig. 8

(i) Verify that the x-coordinate of P is 3. **[2]**

(ii) Show that, for the curve, $\frac{dy}{dx} = \frac{x-4}{2(x-2)^{\frac{3}{2}}}$.

Hence find the gradient of the curve at P. Use the result to show that the curve is **not** symmetrical about $y = x$. **[7]**

(iii) Using the substitution $u = x - 2$, show that $\int_3^{11} \frac{x}{\sqrt{x-2}}\, dx = 25\frac{1}{3}$.

Hence find the area of the region PQR bounded by the curve and the lines $y = x$ and $x = 11$. **[9]**

9 Fig. 9 shows the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$. The function $y = \mathrm{f}(x)$ is given by

$$\mathrm{f}(x) = \ln\left(\frac{2x}{1+x}\right), \quad x > 0.$$

The curve $y = \mathrm{f}(x)$ crosses the x-axis at P, and the line $x = 2$ at Q.

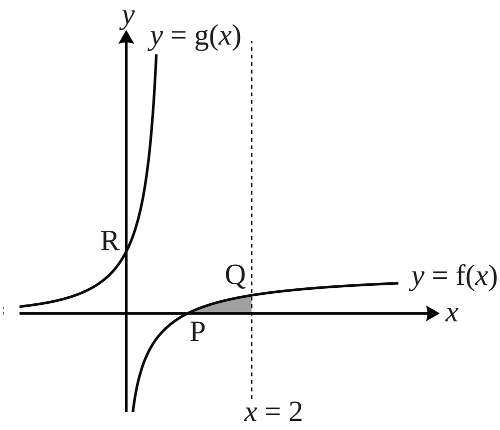


Fig. 9

(i) Verify that the x-coordinate of P is 1.

Find the exact y-coordinate of Q. **[2]**

(ii) Find the gradient of the curve at P. [Hint: use $\ln\frac{a}{b} = \ln a - \ln b$.] **[4]**

The function $\mathrm{g}(x)$ is given by

$$\mathrm{g}(x) = \frac{\mathrm{e}^x}{2 - \mathrm{e}^x}, \quad x < \ln 2.$$

The curve $y = \mathrm{g}(x)$ crosses the y-axis at the point R.

(iii) Show that $\mathrm{g}(x)$ is the inverse function of $\mathrm{f}(x)$.

Write down the gradient of $y = \mathrm{g}(x)$ at R. **[5]**

(iv) Show, using the substitution $u = 2 - \mathrm{e}^x$ or otherwise, that $\int_0^{\ln\frac{4}{3}} \mathrm{g}(x)\,\mathrm{d}x = \ln\frac{3}{2}$.

Using this result, show that the exact area of the shaded region shown in Fig. 9 is $\ln\frac{32}{27}$.
[Hint: consider its reflection in $y = x$.] **[7]**

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