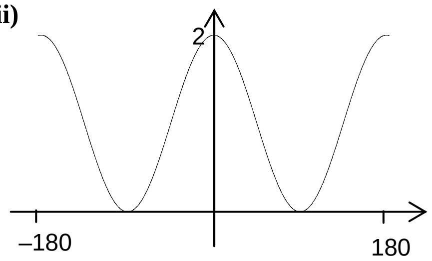
4753 (C3) Methods for Advanced Mathematics

Section A

1 $\lvert x-1\rvert < 3 \Rightarrow -3 < x-1 < 3$ $\Rightarrow \quad -2 < x < 4$	M1 A1 B1 [3]	or $x - 1 = \pm3$, or squaring $\Rightarrow$ correct quadratic $\Rightarrow$ $(x + 2)(x - 4)$ (condone factorising errors) or correct sketch showing $y = 3$ to scale $-2 <$ < 4 (penalise $\le$ once only)
2(i) $y = x\cos 2x$ $\Rightarrow \quad \frac{dy}{dx} = -2x\sin 2x + \cos 2x$	M1 B1 A1 [3]	product rule d/dx (cos2x) = –2sin2x oe cao
(ii) $\int x\cos 2x dx = \int x\frac{d}{dx}(\frac{1}{2}\sin 2x)dx$ $= \frac{1}{2}x\sin 2x - \int\frac{1}{2}\sin 2x dx$ $= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$	M1 A1 A1ft A1 [4]	parts with $u = x$, $v = ½ \sin 2x$ $+\frac{1}{4}\cos 2x$ cao – must have + c
3 *Either* $y = \frac{1}{2}\ln(x-1) \quad x \leftrightarrow y$ $\Rightarrow \quad x = \frac{1}{2}\ln(y-1)$ $\Rightarrow \quad 2x = \ln(y-1)$ $\Rightarrow \quad e^{2x} = y - 1$ $\Rightarrow \quad 1 + e^{2x} = y$ $\Rightarrow \quad g(x) = 1 + e^{2x}$	 M1 M1 E1	or $y = e^{(x-1)/2}$ attempt to invert and interchanging x with y o.e. (at any stage) $e^{\ln y-1} = y - 1$ or $\ln(e^y) = y$ used www
or $gf(x) = g(½ \ln(x-1))$ $= 1 + e^{\ln(x-1)}$ $= 1 + x - 1$ $= x$	M1 M1 E1 [3]	or fg(x) = … (correct way round) $e^{\ln(x-1)} = x - 1$ or $\ln(e^{2x}) = 2x$ www
4 $\int_0^2 \sqrt{1+4x}dx \quad$ let $u = 1+4x$, $du = 4dx$ $= \int_1^9 u^{1/2}.\frac{1}{4}du$ $= \left[\frac{1}{6}u^{3/2}\right]_1^9$ $= \frac{27}{6} - \frac{1}{6} = \frac{26}{6} = \frac{13}{3}$ or $4\frac{1}{3}$	M1 A1 B1 M1 A1cao	$u = 1 + 4x$ and du/dx = 4 or du = 4dx $\int u^{1/2}.\frac{1}{4}du$ $\int u^{1/2}du = \frac{u^{3/2}}{3/2}$ soi substituting correct limits (u or x) dep attempt to integrate
or $\frac{d}{dx}(1+4x)^{3/2} = 4.\frac{3}{2}(1+4x)^{1/2} = 6(1+4x)^{1/2}$ $\Rightarrow \quad \int_0^2 (1+4x)^{1/2}dx = \left[\frac{1}{6}(1+4x)^{3/2}\right]_0^2$ $= \frac{27}{6} - \frac{1}{6} = \frac{26}{6} = \frac{13}{3}$ or $4\frac{1}{3}$	M1 A1 A1 M1 A1cao [5]	$k(1 + 4x)^{3/2}$ $\int(1+4x)^{1/2}dx = \frac{2}{3}(1+4x)^{3/2}...$ $\times ¼$ substituting limits (dep attempt to integrate)

5(i) period 180°	B1 [1]	condone $0 \le x \le 180°$ or π
(ii) one-way stretch in x-direction scale factor ½ translation in y-direction through $\begin{pmatrix}0\\1\end{pmatrix}$	M1 A1 M1 A1 [4]	[either way round…] condone 'squeeze', 'contract' for M1 stretch used and s.f ½ condone 'move', 'shift', etc for M1 'translation' used, +1 unit $\begin{pmatrix}0\\1\end{pmatrix}$ only is M1 A0
(iii) 	M1 B1 A1 [3]	correct shape, touching x-axis at -90°, 90° correct domain (0, 2) marked or indicated (i.e. amplitude is 2)
6(i) e.g $p = 1$ and $q = -2$ $p > q$ but $1/p = 1 > 1/q = -½$	M1 E1 [2]	stating values of p, q with $p \ge 0$ and $q \le 0$ (but not $p = q = 0$) showing that $1/p > 1/q$ - if 0 used, must state that 1/0 is undefined or infinite
(ii) Both p and q positive (or negative)	B1 [1]	or $q > 0$, 'positive integers'
7(i) $\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{\frac{2}{3}x^{-1/3}}{\frac{2}{3}y^{-1/3}}$ $= -\frac{y^{1/3}}{x^{1/3}} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$ *	M1 A1 M1 E1 [4]	Implicit differentiation (must show = 0) solving for dy/dx www. Must show, or explain, one more step.
(ii) $\frac{dy}{dt} = \frac{dy}{dx}.\frac{dx}{dt}$ $= -\left(\frac{8}{1}\right)^{\frac{1}{3}}.6$ $= -12$	M1 A1 A1cao [3]	any correct form of chain rule

8(i) When $x = 1$ $y = 1^2 - (\ln 1)/8 = 1$ Gradient of PR = $(1 + 7/8)/1 = 1\frac{7}{8}$	B1 M1 A1 [3]	 1.9 or better
(ii) $\frac{dy}{dx} = 2x - \frac{1}{8x}$ When $x = 1$, $dy/dx = 2 - 1/8 = 1\frac{7}{8}$ Same as gradient of PR, so PR touches curve	B1 B1dep E1 [3]	cao 1.9 or better dep 1st B1 dep gradients exact
(iii) Turning points when $dy/dx = 0$ $\Rightarrow 2x - \frac{1}{8x} = 0$ $\Rightarrow 2x = \frac{1}{8x}$ $\Rightarrow x^2 = 1/16$ $\Rightarrow x = ¼ \ (x > 0)$ When $x = ¼$, $y = \frac{1}{16} - \frac{1}{8}\ln\frac{1}{4} = \frac{1}{16} + \frac{1}{8}\ln 4$ So TP is $(\frac{1}{4}, \frac{1}{16} + \frac{1}{8}\ln 4)$	M1 M1 A1 M1 A1cao [5]	setting their derivative to zero multiplying through by x allow verification substituting for x in y o.e. but must be exact, not $1/4^2$. Mark final answer.
(iv) $\frac{d}{dx}(x\ln x - x) = x.\frac{1}{x} + 1.\ln x - 1 = \ln x$	M1 A1	product rule $\ln x$
Area = $\int_1^2 (x^2 - \frac{1}{8}\ln x)dx$ $= \left[\frac{1}{3}x^3 - \frac{1}{8}(x\ln x - x)\right]_1^2$ $= (\frac{8}{3} - \frac{1}{4}\ln 2 + \frac{1}{4}) - (\frac{1}{3} - \frac{1}{8}\ln 1 + \frac{1}{8})$ $= \frac{7}{3} + \frac{1}{8} - \frac{1}{4}\ln 2$ $= \frac{59}{24} - \frac{1}{4}\ln 2$ *	M1 M1 A1 M1 E1 [7]	correct integral and limits (soi) – condone no dx $\int \ln x dx = x\ln x - x$ used (or derived using integration by parts) $\frac{1}{3}x^3 - \frac{1}{8}(x\ln x - x)$ – bracket required substituting correct limits must show at least one step

9(i) Asymptotes when (√)($2x - x^2$) = 0 ⇒ $x(2 - x) = 0$ ⇒ $x = 0$ or 2 so $a = 2$ Domain is $0 < x < 2$	M1 A1 B1ft [3]	 or by verification $x > 0$ and $x < 2$, not ≤
(ii) $y = (2x - x^2)^{-1/2}$ let $u = 2x - x^2$, $y = u^{-1/2}$ ⇒ $dy/du = -½ u^{-3/2}$, $du/dx = 2 - 2x$ ⇒ $\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}=-\frac{1}{2}(2x-x^2)^{-3/2}.(2-2x)$ $=\frac{x-1}{(2x-x^2)^{3/2}}$ *	M1 B1 A1 E1	chain rule (or within correct quotient rule) $-½ u^{-3/2}$ or $-\frac{1}{2}(2x-x^2)^{-3/2}$ or $\frac{1}{2}(2x-x^2)^{-1/2}$ in quotient rule × $(2 - 2x)$ www – penalise missing brackets here
$dy/dx = 0$ when $x - 1 = 0$ ⇒ $x = 1$, $y = 1/\sqrt{(2-1)} = 1$ Range is $y \geq 1$	M1 A1 B1 B1ft [8]	extraneous solutions M0
(iii) (*A*) $g(-x)=\frac{1}{\sqrt{1-(-x)^2}}=\frac{1}{\sqrt{1-x^2}}=g(x)$	M1 E1	Expression for g(–*x*) – must have g(–*x*) = g(*x*) seen
(*B*) $g(x-1)=\frac{1}{\sqrt{1-(x-1)^2}}$ $=\frac{1}{\sqrt{1-x^2+2x-1}}=\frac{1}{\sqrt{2x-x^2}}=f(x)$	M1 E1	 must expand bracket
(*C*) f(*x*) is g(*x*) translated 1 unit to the right. But g(*x*) is symmetrical about O*y* So f(*x*) is symmetrical about *x* = 1.	M1 M1 A1	 dep both M1s
or $f(1 - x) = g(-x)$, $f(1 + x) = g(x)$ ⇒ $f(1 + x) = f(1 - x)$ ⇒ f(*x*) is symmetrical about *x* = 1.	M1 E1 A1 [7]	or $f(1-x)=\frac{1}{\sqrt{2-2x-(1-x)^2}}=\frac{1}{\sqrt{2-2x-1+2x-x^2}}=\frac{1}{\sqrt{1-x^2}}$ $f(1+x)=\frac{1}{\sqrt{2+2x-(1+x)^2}}=\frac{1}{\sqrt{2+2x-1-2x-x^2}}=\frac{1}{\sqrt{1-x^2}}$