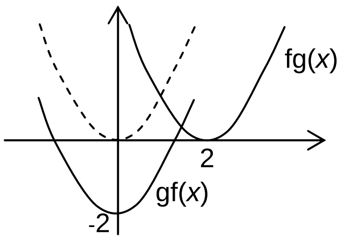
4753 (C3) Methods for Advanced Mathematics

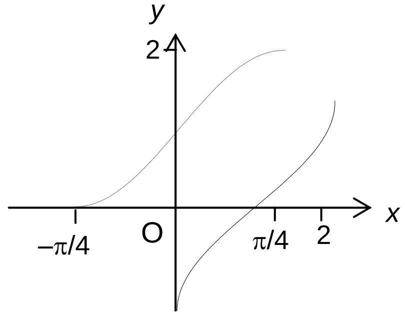
Section A

1 $y = (1+6x^2)^{1/3}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{3}(1+6x^2)^{-2/3}.12x$ $= 4x(1+6x^2)^{-2/3}$	M1 B1 A1 A1 [4]	chain rule used $\frac{1}{3}u^{-2/3}$ $\times 12x$ cao (must resolve 1/3 × 12) Mark final answer
2 (i) fg(x) = f(x – 2) = $(x-2)^2$ gf(x) = g(x^2) = $x^2 - 2$.	M1 A1 A1 [3]	forming a composite function mark final answer If fg and gf the wrong way round, M1A0A0
(ii) 	B1ft B1ft [2]	fg – must have (2, 0)labelled (or inferable from scale). Condone no y-intercept, unless wrong gf – must have (0, –2) labelled (or inferable from scale) Condone no x-intercepts, unless wrong Allow ft only if fg and gf are correct but wrong way round.
3 (i) When n = 1, 10 000 = A e^b when n = 2, 16 000 = A e^{2b} $\Rightarrow \frac{16000}{10000} = \frac{Ae^{2b}}{Ae^b} = e^b$ $\Rightarrow$ e^b = 1.6 $\Rightarrow$ b = ln 1.6 = 0.470 A = 10000/1.6 = 6250.	B1 B1 M1 E1 B1 B1 [6]	soi soi eliminating A (do not allow verification) SCB2 if initial 'B's are missing, and ratio of years = 1.6 = e^b ln 1.6 or 0.47 or better (mark final answer) cao – allow recovery from inexact b's
(ii) When n = 20, P = 6250×e$^{0.470\times20}$ = £75,550,000	M1 A1 [2]	substituting n = 20 into their equation with their A and b Allow answers from £75 000 000 to £76 000 000.
4 (i) 5 = k/100 $\Rightarrow$ k = 500*	E1 [1]	NB answer given
(ii) $\frac{dP}{dV} = -500V^{-2} = -\frac{500}{V^2}$	M1 A1 [2]	(–1)V^{-2} o.e. – allow –k/V^2
(iii) $\frac{dP}{dt} = \frac{dP}{dV}.\frac{dV}{dt}$ When V = 100, dP/dV = –500/10000 = –0.05 dV/dt = 10 $\Rightarrow$ dP/dt = –0.05 × 10 = –0.5 So P is decreasing at 0.5 Atm/s	M1 B1ft B1 A1 [4]	chain rule (any correct version) (soi) (soi) –0.5 cao

5(i) $p = 2$, $2^p - 1 = 3$, prime $p = 3$, $2^p - 1 = 7$, prime $p = 5$, $2^p - 1 = 31$, prime $p = 7$, $2^p - 1 = 127$, prime	M1 E1 [2]	Testing at least one prime testing all 4 primes (correctly) Must comment on answers being prime (allow ticks) Testing $p = 1$ is E0
(ii) $23 \times 89 = 2047 = 2^{11} - 1$ 11 is prime, 2047 is not So statement is false.	M1 E1 [2]	$2^{11} - 1$ must state or imply that 11 is prime ($p = 11$ is sufficient)
6 (i) $e^{2y} = x^2 + y$ $\Rightarrow \quad 2e^{2y}\frac{dy}{dx} = 2x + \frac{dy}{dx}$ $\Rightarrow \quad (2e^{2y} - 1)\frac{dy}{dx} = 2x$ $\Rightarrow \quad \frac{dy}{dx} = \frac{2x}{2e^{2y} - 1}$ *	M1 A1 M1 E1 [4]	Implicit differentiation – allow one slip (but with dy/dx both sides) collecting terms
(ii) Gradient is infinite when $2e^{2y} - 1 =$ 0 $\Rightarrow \quad e^{2y} = ½$ $\Rightarrow \quad 2y = \ln ½$ $\Rightarrow \quad y = ½ \ln ½ = –0.347$ (3 s.f.) $x^2 = e^{2y} - y = ½ - (–0.347)$ $= 0.8465$ $\Rightarrow \quad x = 0.920$	M1 A1 M1 A1 [4]	 must be to 3 s.f. substituting their y and solving for x cao – must be to 3 s.f., but penalise accuracy once only.

Section B

7(i) $y = 2x\ln(1+x)$ $\Rightarrow \frac{dy}{dx} = \frac{2x}{1+x} + 2\ln(1+x)$ When $x = 0$, $dy/dx = 0 + 2\ln 1 = 0$ $\Rightarrow$ origin is a stationary point.	M1 B1 A1 E1 [4]	product rule $d/dx(\ln(1+x)) = 1/(1+x)$ soi www (i.e. from correct derivative)
(ii) $\frac{d^2y}{dx^2} = \frac{(1+x).2 - 2x.1}{(1+x)^2} + \frac{2}{1+x}$ $= \frac{2}{(1+x)^2} + \frac{2}{1+x}$ When $x = 0$, $d^2y/dx^2 = 2 + 2 = 4 > 0$ $\Rightarrow$ (0, 0) is a min point	M1 A1ft A1 M1 E1 [5]	Quotient or product rule on their $2x/(1+x)$ correctly applied to their $2x/(1+x)$ o.e., e.g. $\frac{4+2x}{(1+x)^2}$ cao substituting $x = 0$ into their d^2y/dx^2 www – dep previous A1
(iii) Let $u = 1 + x \Rightarrow du = dx$ $\Rightarrow \int \frac{x^2}{1+x}dx = \int \frac{(u-1)^2}{u}du$ $= \int \frac{(u^2 - 2u + 1)}{u}du$ $= \int (u - 2 + \frac{1}{u})du$ * $\Rightarrow \int_0^1 \frac{x^2}{1+x}dx = \int_1^2 (u - 2 + \frac{1}{u})du$ $= \left[\frac{1}{2}u^2 - 2u + \ln u\right]_1^2$ $= 2 - 4 + \ln 2 - (\frac{1}{2} - 2 + \ln 1)$ $= \ln 2 - \frac{1}{2}$	 M1 E1 B1 B1 M1 A1 [6]	 $\frac{(u-1)^2}{u}$ www (but condone du omitted except in final answer) changing limits (or substituting back for x and using 0 and 1) $\left[\frac{1}{2}u^2 - 2u + \ln u\right]$ substituting limits (consistent with u or x) cao
(iv) $A = \int_0^1 2x\ln(1+x)dx$ Parts: $u = \ln(1+x)$, $du/dx = 1/(1+x)$ $dv/dx = 2x \Rightarrow v = x^2$ $= \left[x^2\ln(1+x)\right]_0^1 - \int_0^1 \frac{x^2}{1+x}dx$ $= \ln 2 - \ln 2 + \frac{1}{2} = \frac{1}{2}$	 M1 A1 M1 A1 [4]	 soi substituting their $\ln 2 - \frac{1}{2}$ for $\int_0^1 \frac{x^2}{1+x}dx$ cao

8 (i) Stretch in x-direction s.f. ½ translation in y-direction 1 unit up	M1 A1 M1 A1 [4]	(in either order) – allow 'contraction' dep 'stretch' allow 'move', 'shift', etc – direction can be inferred from $\begin{pmatrix}0\\1\end{pmatrix}$ or $\begin{pmatrix}0\\1\end{pmatrix}$ dep 'translation'. $\begin{pmatrix}0\\1\end{pmatrix}$ alone is M1 A0
(ii) $A=\int_{-\pi/4}^{\pi/4}(1+\sin 2x)dx$ $=\left[x-\frac{1}{2}\cos 2x\right]_{-\pi/4}^{\pi/4}$ $= \pi/4 - ½ \cos \pi/2 + \pi/4 + ½ \cos (-\pi/2)$ $= \pi/2$	M1 B1 M1 A1 [4]	correct integral and limits. Condone dx missing; limits may be implied from subsequent working. substituting their limits (if zero lower limit used, must show evidence of substitution) or 1.57 or better – cao (www)
(iii) $y = 1 + \sin 2x$ ⇒ $dy/dx = 2\cos 2x$ When $x = 0$, $dy/dx = 2$ So gradient at (0, 1) on f(x) is 2 ⇒ gradient at (1, 0) on $f^{-1}(x) = ½$	M1 A1 A1ft B1ft [4]	differentiating – allow 1 error (but not $x + 2\cos 2x$) If 1, then must show evidence of using reciprocal, e.g. 1/1
(iv) Domain is $0 \le x \le 2$. 	B1 M1 A1 [3]	Allow 0 to 2, but not $0 < x < 2$ or y instead of x clear attempt to reflect in $y = x$ correct domain indicated (0 to 2), and reasonable shape
(v) $y = 1 + \sin 2x$ $x \leftrightarrow y$ $x = 1 + \sin 2y$ ⇒ $\sin 2y = x - 1$ ⇒ $2y = \arcsin(x - 1)$ ⇒ $y = ½ \arcsin(x - 1)$	M1 A1 [2]	or $\sin 2x = y - 1$ cao