OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4753/1

Methods for Advanced Mathematics (C3)

Wednesday **18 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 6 printed pages and 2 blank pages.

Section A (36 marks)

1 Given that $y = (1 + 6x)^{\frac{1}{3}}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{y^2}$. [4]

2 A population is P million at time t years. P is modelled by the equation

$$P = 5 + a\mathrm{e}^{-bt},$$

where a and b are constants.

The population is initially 8 million, and declines to 6 million after 1 year.

(i) Use this information to calculate the values of a and b, giving b correct to 3 significant figures. [5]

(ii) What is the long-term population predicted by the model? [1]

3 **(i)** Express $2\ln x + \ln 3$ as a single logarithm. [2]

(ii) Hence, given that x satisfies the equation

$$2\ln x + \ln 3 = \ln(5x + 2),$$

show that x is a root of the quadratic equation $3x^2 - 5x - 2 = 0$. [2]

(iii) Solve this quadratic equation, explaining why only one root is a valid solution of

$$2\ln x + \ln 3 = \ln(5x + 2).$$ [3]

4 Fig. 4 shows a cone. The angle between the axis and the slant edge is 30°. Water is poured into the cone at a constant rate of 2 cm^3 per second. At time t seconds, the radius of the water surface is r cm and the volume of water in the cone is $V\,\text{cm}^3$.

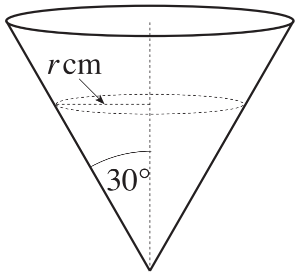


Fig. 4

(i) Write down the value of $\dfrac{\mathrm{d}V}{\mathrm{d}t}$. [1]

(ii) Show that $V = \dfrac{\sqrt{3}}{3}\pi r^3$, and find $\dfrac{\mathrm{d}V}{\mathrm{d}r}$. [3]

[You may assume that the volume of a cone of height h and radius r is $\frac{1}{3}\pi r^2 h$.]

(iii) Use the results of parts **(i)** and **(ii)** to find the value of $\dfrac{\mathrm{d}r}{\mathrm{d}t}$ when $r = 2$. [3]

5 A curve is defined implicitly by the equation

$$y^3 = 2xy + x^2.$$

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2(x+y)}{3y^2 - 2x}$. [4]

(ii) Hence write down $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x and y. [1]

6 The function $f(x)$ is defined by $f(x) = 1 + 2\sin x$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Show that $f^{-1}(x) = \arcsin\left(\dfrac{x-1}{2}\right)$ and state the domain of this function. [4]

Fig. 6 shows a sketch of the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

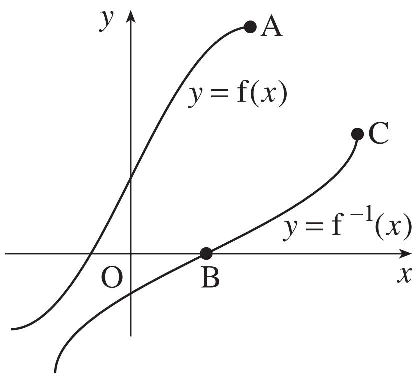


Fig. 6

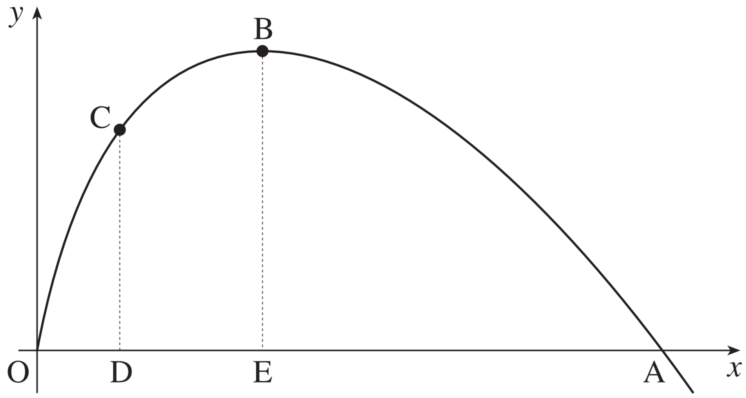
(ii) Write down the coordinates of the points A, B and C. [3]

Section B (36 marks)

7 Fig. 7 shows the curve

$$y = 2x - x \ln x, \text{ where } x > 0.$$

The curve crosses the x-axis at A, and has a turning point at B. The point C on the curve has x-coordinate 1. Lines CD and BE are drawn parallel to the y-axis.



Not to scale

Fig. 7

(i) Find the x-coordinate of A, giving your answer in terms of e. [2]

(ii) Find the exact coordinates of B. [6]

(iii) Show that the tangents at A and C are perpendicular to each other. [3]

(iv) Using integration by parts, show that

$$\int x \ln x \, dx = \tfrac{1}{2}x^2 \ln x - \tfrac{1}{4}x^2 + c.$$

Hence find the exact area of the region enclosed by the curve, the x-axis and the lines CD and BE. [7]

[Question 8 is printed overleaf.]

8 The function $f(x) = \dfrac{\sin x}{2 - \cos x}$ has domain $-\pi \leqslant x \leqslant \pi$.

Fig. 8 shows the graph of $y = f(x)$ for $0 \leqslant x \leqslant \pi$.

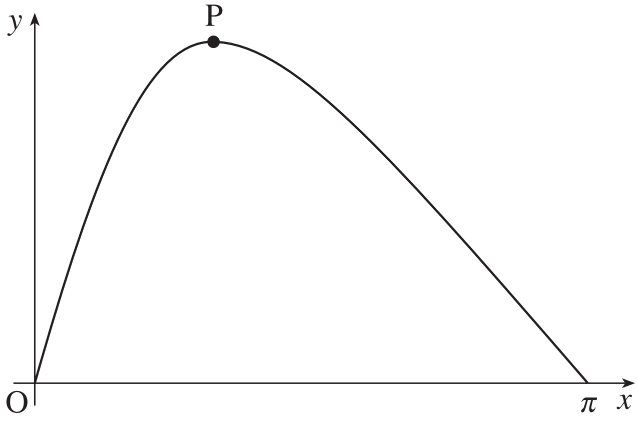


Fig. 8

(i) Find $f(-x)$ in terms of $f(x)$. Hence sketch the graph of $y = f(x)$ for the complete domain $-\pi \leqslant x \leqslant \pi$. [3]

(ii) Show that $f'(x) = \dfrac{2\cos x - 1}{(2 - \cos x)^2}$. Hence find the exact coordinates of the turning point P.

State the range of the function $f(x)$, giving your answer exactly. [8]

(iii) Using the substitution $u = 2 - \cos x$ or otherwise, find the exact value of $\displaystyle\int_0^{\pi} \frac{\sin x}{2 - \cos x}\,dx$. [4]

(iv) Sketch the graph of $y = f(2x)$. [1]

(v) Using your answers to parts **(iii)** and **(iv)**, write down the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{\sin 2x}{2 - \cos 2x}\,dx$. [2]