Mark Scheme 4753
June 2005

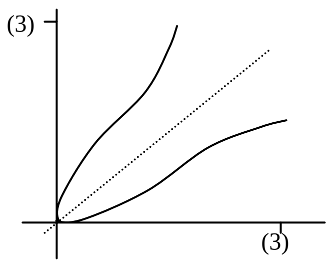
Section A

1 $3x + 2 = 1 \Rightarrow x = -1/3$ $3x + 2 = -1$ $\Rightarrow x = -1$	B1 M1 A1	$x = -1/3$ from a correct method – must be exact
or $(3x + 2)^2 = 1$ $\Rightarrow 9x^2 + 12x + 3 = 0$ $\Rightarrow 3x^2 + 4x + 1 = 0$ $\Rightarrow (3x + 1)(x + 1) = 0$ $\Rightarrow x = -1/3$ or $x = -1$	M1 B1 A1 [3]	Squaring and expanding correctly $x = -1/3$ $x = -1$
2 $x = ½$ $\cos\theta = ½$ $\Rightarrow \theta = \pi/3$	B1 M1 A1 [3]	M1A0 for 1.04… or 60°
3 $fg(x) = \ln(x^3)$ $= 3 \ln x$ Stretch s.f. 3 in y direction	M1 A1 B1 [3]	$\ln(x^3)$ $= 3 \ln x$
4 $T = 30 + 20e^0 = 50$ $dT/dt = -0.05 \times 20e^{-0.05t} = -e^{-0.05t}$ When $t = 0$, $dT/dt = -1$ When $T = 40$, $40 = 30 + 20e^{-0.05t}$ $\Rightarrow e^{-0.05t} = ½$ $\Rightarrow -0.05t = \ln ½$ $\Rightarrow t = -20 \ln ½ = 13.86..$ (mins)	B1 M1 A1cao M1 M1 A1cao [6]	50 correct derivative –1 (or 1) substituting $T = 40$ taking lns correctly or trial and improvement – one value above and one below or 13.9 or 13 mins 52 secs or better www condone secs

5 $\int_0^1 \frac{x}{2x+1}\,dx$ let $u = 2x + 1$ $\Rightarrow du = 2dx,\ x = \frac{u-1}{2}$ When $x = 0$, $u = 1$, when $x = 1$, $u = 3$ $= \int_1^3 \frac{\frac{1}{2}(u-1)}{u}\frac{1}{2}\,du = \frac{1}{4}\int_1^3 \frac{u-1}{u}\,du$ $= \frac{1}{4}\int_1^3 (1-\frac{1}{u})\,du$ $= \frac{1}{4}\left[u - \ln u\right]_1^3$ $= \frac{1}{4}[3 - \ln 3 - 1 + \ln 1]$ $= ¼ (2 – \ln 3)$	M1 A1 B1 M1 A1 E1 [6]	Substituting $\frac{x}{2x+1} = \frac{u-1}{2u}$ o.e. $\frac{1}{4}\int \frac{u-1}{u}\,du$ o.e. [condone no du] converting limits dividing through by u $\frac{1}{4}[u - \ln u]$ o.e. – ft their ¼ (only) must be some evidence of substitution
6 $y = \frac{x}{2+3\ln x}$ $\Rightarrow \frac{dy}{dx} = \frac{(2+3\ln x).1 - x.\frac{3}{x}}{(2+3\ln x)^2}$ $= \frac{2+3\ln x - 3}{(2+3\ln x)^2}$ $= \frac{3\ln x - 1}{(2+3\ln x)^2}$ When $\frac{dy}{dx} = 0$, $3\ln x – 1 = 0$ $\Rightarrow \ln x = 1/3$ $\Rightarrow x = e^{1/3}$ $\Rightarrow y = \frac{e^{1/3}}{2+1} = \frac{1}{3}e^{1/3}$	M1 B1 A1 M1 A1cao M1 A1cao [7]	Quotient rule consistent with their derivatives or product rule + chain rule on $(2+3x)^{-1}$ $\frac{d}{dx}(\ln x) = \frac{1}{x}$ soi correct expression their numerator = 0 (or equivalent step from product rule formulation) M0 if denominator = 0 is pursued $x = e^{1/3}$ substituting for their x (correctly) Must be exact: – 0.46… is M1A0
7 $y^2 + y = x^3 + 2x$ $x = 2 \Rightarrow y^2 + y = 12$ $\Rightarrow y^2 + y – 12 = 0$ $\Rightarrow (y – 3)(y + 4) = 0$ $\Rightarrow y = 3$ or $–4$. $2y\frac{dy}{dx} + \frac{dy}{dx} = 3x^2 + 2$ $\Rightarrow \frac{dy}{dx}(2y+1) = 3x^2 + 2$ $\Rightarrow \frac{dy}{dx} = \frac{3x^2+2}{2y+1}$ At (2, 3), $\frac{dy}{dx} = \frac{12+2}{6+1} = 2$ At (2, –4), $\frac{dy}{dx} = \frac{12+2}{-8+1} = -2$	M1 A1 A1 M1 A1cao M1 A1 cao A1 cao [8]	Substituting $x = 2$ $y = 3$ $y = –4$ Implicit differentiation – LHS must be correct substituting $x = 2$, $y = 3$ into their dy/dx, but must require both x and one of their y to be substituted 2 –2

Section B

8 (i) At P, $x \sin 3x = 0$ $\Rightarrow \sin 3x = 0$ $\Rightarrow 3x = \pi$ $\Rightarrow x = \pi/3$	M1 A1 A1cao [3]	$x \sin 3x = 0$ $3x = \pi$ or 180 $x = \pi/3$ or 1.05 or better
(ii) When $x = \pi/6$, $x \sin 3x = \frac{\pi}{6}\sin\frac{\pi}{2} = \frac{\pi}{6}$ $\Rightarrow$ Q($\pi/6$, $\pi/6$) lies on line $y = x$	E1 [1]	$y = \frac{\pi}{6}$ or $x \sin 3x = x \Rightarrow \sin 3x = 1$ etc. Must conclude in radians, and be exact
(iii) $y = x\sin 3x$ $\Rightarrow \frac{dy}{dx} = x.3\cos 3x + \sin 3x$ At Q, $\frac{dy}{dx} = \frac{\pi}{6}.3\cos\frac{\pi}{2} + \sin\frac{\pi}{2} = 1$ = gradient of $y = x$ So line touches curve at this point	B1 M1 A1cao M1 A1ft E1 [6]	d/dx ($\sin 3x$) = $3\cos 3x$ Product rule consistent with their derivs $3x \cos 3x + \sin 3x$ substituting $x = \pi/6$ into their derivative = 1 ft dep 1st M1 = gradient of $y = x$ (www)
(iv) Area under curve = $\int_0^{\frac{\pi}{6}} x\sin 3x dx$ Integrating by parts, $u = x$, dv/dx = $\sin 3x$ $\Rightarrow v = -\frac{1}{3}\cos 3x$ $\int_0^{\frac{\pi}{6}} x\sin 3x dx = \left[-\frac{1}{3}x\cos 3x\right]_0^{\frac{\pi}{6}} + \int_0^{\frac{\pi}{6}} \frac{1}{3}\cos 3x dx$ $= -\frac{1}{3}.\frac{\pi}{6}.\cos\frac{\pi}{2} + \frac{1}{3}.0.\cos 0 + \left[\frac{1}{9}\sin 3x\right]_0^{\frac{\pi}{6}}$ $= \frac{1}{9}$ Area under line = $\frac{1}{2}\times\frac{\pi}{6}\times\frac{\pi}{6} = \frac{\pi^2}{72}$ So area required = $\frac{\pi^2}{72} - \frac{1}{9}$ $= \frac{\pi^2 - 8}{72}$ *	 M1 A1cao A1ft M1 A1 B1 E1 [7]	 Parts with $u = x$ dv/dx = $\sin 3x \Rightarrow$ $v = -\frac{1}{3}\cos 3x$ [condone no negative] $\ldots + \left[\frac{1}{9}\sin 3x\right]_0^{\frac{\pi}{6}}$ substituting (correct) limits $\frac{1}{9}$ www $\frac{\pi^2}{72}$ www

9 (i) $f(-x) = \ln[1 + (-x)^2]$ $= \ln[1 + x^2] = f(x)$ Symmetrical about Oy	M1 E1 B1 [3]	If verifies that $f(-x) = f(x)$ using a particular point, allow SCB1 For $f(-x) = \ln(1 + x^2) = f(x)$ allow M1E0 For $f(-x) = \ln(1 +- x^2) = f(x)$ allow M1E0 or 'reflects in Oy', etc
(ii) $y = \ln(1 + x^2)$ let $u = 1 + x^2$ $dy/du = 1/u$, $du/dx = 2x$ $\frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ $= \frac{1}{u}.2x = \frac{2x}{1+x^2}$ When $x = 2$, $dy/dx = 4/5$.	M1 B1 A1 A1cao [4]	Chain rule $1/u$ soi
(iii) The function is not one to one for this domain	B1 [1]	Or many to one
(iv)  Domain for $g(x) = 0 \le x \le \ln 10$ $y = \ln(1 + x^2)$ $x \leftrightarrow y$ $x = \ln(1 + y^2)$ $\Rightarrow e^x = 1 + y^2$ $\Rightarrow e^x - 1 = y^2$ $\Rightarrow y = \sqrt{(e^x - 1)}$ so $g(x) = \sqrt{(e^x - 1)}$* *or* $g f(x) = g[\ln(1 + x^2)]$ $= \sqrt{e^{\ln(1+x^2)} - 1}$ $= (1 + x^2) - 1$ $= x$	M1 A1 B1 M1 M1 E1 M1 M1 E1 [6]	$g(x)$ is $f(x)$ reflected in $y = x$ Reasonable shape and domain, i.e. no –ve x values, inflection shown, does not cross $y = x$ line Condone y instead of x Attempt to invert function Taking exponentials $g(x) = \sqrt{(e^x - 1)}$* www forming g f(x) or f g(x) $e^{\ln(1+x^2)} = 1 + x^2$ or $\ln(1 + e^x - 1) = x$ www
(v) $g'(x) = ½ (e^x - 1)^{-1/2} . e^x$ $\Rightarrow g'(\ln 5) = ½ (e^{\ln 5} - 1)^{-1/2} . e^{\ln 5}$ $= ½ (5 - 1)^{-1/2} . 5$ $= 5/4$ Reciprocal of gradient at P as tangents are reflections in $y = x$.	B1 B1 M1 E1cao B1 [5]	$½ u^{-1/2}$ soi $\times e^x$ substituting ln 5 into g′ - must be some evidence of substitution Must have idea of reciprocal. Not 'inverse'.

Mark Scheme 4753
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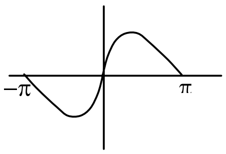
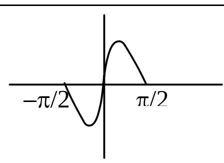
Section A

1 $y = (1 + 6x)^{1/3}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{3}(1+6x)^{-2/3}.6$ $= 2(1 + 6x)^{-2/3}$ $= 2[(1 + 6x)^{1/3}]^{-2}$ $= \frac{2}{y^2}$ *	M1 B1 A1 E1	Chain rule $\frac{1}{3}(1+6x)^{-2/3}$ or $\frac{1}{3}u^{-2/3}$ any correct expression for the derivative www
or $y^3 = 1 + 6x$ $\Rightarrow x = (y^3 - 1)/6$ $\Rightarrow dx/dy = 3y^2/6 = y^2/2$ $\Rightarrow dy/dx = 1/(dx/dy) = 2/y^2$ *	M1 A1 B1 E1	Finding x in terms of y $y^2/2$ o.e.
or $y^3 = 1 + 6x$ $\Rightarrow 3y^2\, dy/dx = 6$ $\Rightarrow dy/dx = 6/3y^2 = 2/y^2$ *	M1 A1 A1 E1 [4]	together with attempt to differentiate implicitly $3y^2\, dy/dx$ $= 6$
2 (i) When $t = 0$, $P = 5 + a = 8$ $\Rightarrow a = 3$ When $t = 1$, $5 + 3e^{-b} = 6$ $\Rightarrow e^{-b} = 1/3$ $\Rightarrow -b = \ln 1/3$ $\Rightarrow b = \ln 3 = 1.10$ (3 s.f.) **(ii)** 5 million	M1 A1 M1 M1 A1ft B1 [6]	substituting $t = 0$ into equation Forming equation using their a Taking lns on correct re-arrangement (ft their a) or $P = 5$
3 (i) $\ln (3x^2)$ **(ii)** $\ln 3x^2 = \ln(5x + 2)$ $\Rightarrow 3x^2 = 5x + 2$ $\Rightarrow 3x^2 - 5x - 2 = 0$* **(iii)** $(3x + 1)(x - 2) = 0$ $\Rightarrow x = -1/3$ or 2 $x = -1/3$ is not valid as $\ln(-1/3)$ is not defined	B1 B1 M1 E1 M1 A1cao B1ft [7]	$2\ln x = \ln x^2$ $\ln x^2 + \ln 3 = \ln 3x^2$ Anti-logging Factorising or quadratic formula ft on one positive and one negative root

4 (i) $\frac{dV}{dt} = 2$ **(ii)** tan 30 = 1/√3 = r/h ⇒ $h = \sqrt{3}\, r$ ⇒ $V = \frac{1}{3}\pi r^2 . \sqrt{3} r = \frac{\sqrt{3}}{3}\pi r^3$ * $\frac{dV}{dr} = \sqrt{3}\pi r^2$ **(iii)** When $r = 2$, dV/dr = 4√3π $\frac{dV}{dt} = \frac{dV}{dr}.\frac{dr}{dt}$ ⇒ 2 = 4√3π dr/dt ⇒ dr/dt = 1/(2√3π) or 0.092 cm s^{-1}	B1 M1 E1 B1 M1 M1 A1cao [7]	Correct relationship between r and h in any form From exact working only o.e. e.g. (3√3/3)π r^2 or $\frac{dr}{dt} = \frac{dr}{dV}.\frac{dV}{dt}$ substituting 2 for dV/dt and r = 2 into their dV/dr
5(i) $y^3 = 2xy + x^2$ ⇒ $3y^2\frac{dy}{dx} = 2x\frac{dy}{dx} + 2y + 2x$ ⇒ $(3y^2 - 2x)\frac{dy}{dx} = 2y + 2x$ ⇒ $\frac{dy}{dx} = \frac{2(x+y)}{3y^2 - 2x}$ * **(ii)** $\frac{dx}{dy} = \frac{3y^2 - 2x}{2(x+y)}$	B1 B1 M1 E1 B1cao [5]	$3y^2\frac{dy}{dx} =$ $2x\frac{dy}{dx} + 2y + 2x$ collecting dy/dx terms on one side www
6(i) $y = 1 + 2\sin x$ $y \leftrightarrow x$ ⇒ $x = 1 + 2\sin y$ ⇒ $x - 1 = 2 \sin y$ ⇒ $(x - 1)/2 = \sin y$ ⇒ $y = \arcsin(\frac{x-1}{2})$ * Domain is $-1 \le x \le 3$ **(ii)** A is (π/2, 3) B is (1, 0) C is (3, π/2)	M1 A1 E1 B1 B1cao B1cao B1ft [7]	Attempt to invert Allow π/2 = 1.57or better ft on their A

Section B

7(i) $2x - x \ln x = 0$ $\Rightarrow$ $x(2 - \ln x) = 0$ $\Rightarrow$ $(x = 0)$ or $\ln x = 2$ $\Rightarrow$ at A, $x = e^2$	M1 A1 [2]	Equating to zero
(ii) $\frac{dy}{dx} = 2 - x.\frac{1}{x} - \ln x.1$ $= 1 - \ln x$ $\frac{dy}{dx} = 0 \Rightarrow 1 - \ln x = 0$ $\Rightarrow$ $\ln x = 1$, $x = e$ When $x = e$, $y = 2e - e \ln e = e$ So B is (e, e)	M1 B1 A1 M1 A1cao B1ft [6]	Product rule for $x \ln x$ $d/dx (\ln x) = 1/x$ $1 - \ln x$ o.e. equating their derivative to zero $x = e$ $y = e$
(iii) At A, $\frac{dy}{dx} = 1 - \ln e^2 = 1 - 2$ $= -1$ At C, $\frac{dy}{dx} = 1 - \ln 1 = 1$ $1 \times -1 = -1 \Rightarrow$ tangents are perpendicular	M1 A1cao E1 [3]	Substituting x=1 or their e^2 into their derivative -1 and 1 www
(iv) Let $u = \ln x$, $dv/dx = x$ $\Rightarrow v = ½ x^2$ $\int x \ln x dx = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2.\frac{1}{x}dx$ $= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x dx$ $= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$ * $A = \int_1^e (2x - x \ln x)dx$ $= \left[x^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2\right]_1^e$ $= (e^2 - ½e^2 \ln e + ¼ e^2) - (1 - ½1^2 \ln 1 + ¼ 1^2)$ $= \frac{3}{4}e^2 - \frac{5}{4}$	M1 A1 E1 B1 B1 M1 A1 cao [7]	Parts: $u = \ln x$, $dv/dx = x \Rightarrow v = ½ x^2$ correct integral and limits $\left[x^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2\right]$ o.e. substituting limits correctly

8 (i) $f(-x) = \frac{\sin(-x)}{2-\cos(-x)}$ $= \frac{-\sin(x)}{2-\cos(x)}$ $= -f(x)$  	M1 A1 B1 [3]	substituting $-x$ for x in $f(x)$ Graph completed with rotational symmetry about O.
(ii) $f'(x) = \frac{(2-\cos x)\cos x - \sin x.\sin x}{(2-\cos x)^2}$ $= \frac{2\cos x - \cos^2 x - \sin^2 x}{(2-\cos x)^2}$ $= \frac{2\cos x - 1}{(2-\cos x)^2}$ * $f'(x) = 0$ when $2\cos x - 1 = 0$ $\Rightarrow \cos x = ½$, $x = \pi/3$ When $x = \pi/3$, $y = \frac{\sin(\pi/3)}{2-\cos(\pi/3)} = \frac{\sqrt{3}/2}{2-1/2}$ $= \frac{\sqrt{3}}{3}$ So range is $-\frac{\sqrt{3}}{3} \le y \le \frac{\sqrt{3}}{3}$	M1 A1 E1 M1 A1 M1 A1 B1ft [8]	Quotient or product rule consistent with their derivatives Correct expression numerator = 0 Substituting their $\pi/3$ into y o.e. but exact ft their $\frac{\sqrt{3}}{3}$
(iii) $\int_0^{\pi} \frac{\sin x}{2-\cos x}dx$ let $u = 2 - \cos x$ $\Rightarrow du/dx = \sin x$ When $x = 0$, $u = 1$; when $x = \pi$, $u = 3$ $= \int_1^3 \frac{1}{u}du$ $= [\ln u]_1^3$ $= \ln 3 - \ln 1 = \ln 3$	M1 B1 A1ft A1cao	$\int \frac{1}{u}du$ $u = 1$ to 3 $[\ln u]$
or $= [\ln(2-\cos x)]_0^{\pi}$ $= \ln 3 - \ln 1 = \ln 3$	M2 A1 A1 cao [4]	$[k \ln (2 - \cos x)]$ $k = 1$
(iv)  	B1ft [1]	Graph showing evidence of stretch s.f. ½ in x – direction
(v) Area is stretched with scale factor ½ So area is ½ ln 3	M1 A1ft [2]	soi ½ their ln 3

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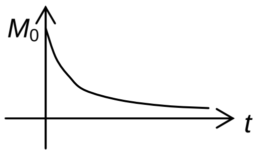
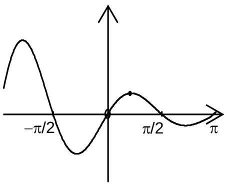
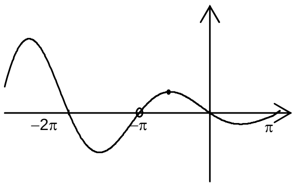
1 $\lvert 3x-2\rvert = x$ $\Rightarrow$ $3x - 2 = x \Rightarrow 2x = 2 \Rightarrow x = 1$ or $2 - 3x = x \Rightarrow 2 = 4x \Rightarrow x = ½$ *or* $(3x - 2)^2 = x^2$ $\Rightarrow 8x^2 - 12x + 4 = 0 \Rightarrow 2x^2 - 3x + 1 = 0$ $\Rightarrow (x - 1)(2x - 1) = 0$, $\Rightarrow x = 1, ½$	B1 M1 A1 M1 A1 A1 [3]	$x = 1$ solving correct quadratic
2 let $u = x$, $dv/dx = \sin 2x \Rightarrow v = -½\cos 2x$ $\Rightarrow \int_0^{\pi/6} x\sin 2x dx = \left[x.-\frac{1}{2}\cos 2x\right]_0^{\pi/6} + \int_0^{\pi/6} \frac{1}{2}\cos 2x.1.dx$ $= \frac{\pi}{6}.-\frac{1}{2}\cos\frac{\pi}{3} - 0 + \left[\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{6}}$ $= -\frac{\pi}{24} + \frac{\sqrt{3}}{8}$ $= \frac{3\sqrt{3}-\pi}{24}$ *	M1 A1 B1ft M1 B1 E1 [6]	parts with $u = x$, $dv/dx = \sin 2x$ $\ldots + \left[\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{6}}$ substituting limits $\cos \pi/3 = ½$, $\sin \pi/3 = \sqrt{3}/2$ soi www
3 (i) $x - 1 = \sin y$ $\Rightarrow$ $x = 1 + \sin y$ $\Rightarrow$ $dx/dy = \cos y$ **(ii)** When $x = 1.5$, $y = \arcsin(0.5) = \pi/6$ $\frac{dy}{dx} = \frac{1}{\cos y}$ $= \frac{1}{\cos \pi/6}$ $= 2/\sqrt{3}$	M1 A1 E1 M1 A1 M1 A1 [7]	 www condone 30° or 0.52 or better or $\frac{dy}{dx} = \frac{1}{\sqrt{1-(x-1)^2}}$ or equivalent, but must be exact
4(i) $V = \pi h^2 - \frac{1}{3}\pi h^3$ $\Rightarrow \frac{dV}{dh} = 2\pi h - \pi h^2$ **(ii)** $\frac{dV}{dt} = 0.02$ $\frac{dV}{dt} = \frac{dV}{dh}.\frac{dh}{dt}$ $\Rightarrow \frac{dh}{dt} = \frac{0.02}{dV/dh} = \frac{0.02}{2\pi h - \pi h^2}$ When $h = 0.4$, $\Rightarrow \frac{dh}{dt} = \frac{0.02}{0.8\pi - 0.16\pi} = 0.0099\,\text{m/min}$	M1 A1 B1 M1 M1dep A1cao [6]	expanding brackets (correctly) or product rule oe soi $\frac{dV}{dt} = \frac{dV}{dh}.\frac{dh}{dt}$ oe substituting $h = 0.4$ into their $\frac{dV}{dh}$ and $\frac{dV}{dt} = 0.02$ 0.01 or better or $1/32\pi$

5(i) $a^2 + b^2 = (2t)^2 + (t^2 - 1)^2$ $= 4t^2 + t^4 - 2t^2 + 1$ $= t^4 + 2t^2 + 1$ $= (t^2 + 1)^2 = c^2$	M1 M1 E1	substituting for a, b and c in terms of t Expanding brackets correctly www
(ii) $c = \sqrt{(20^2 + 21^2)} = 29$ For example: $2t = 20 \Rightarrow t = 10$ $\Rightarrow t^2 - 1 = 99$ which is not consistent with 21	B1 M1 E1 [6]	 Attempt to find t Any valid argument or E2 'none of 20, 21, 29 differ by two'.
6 (i) 	B1 B1	Correct shape Passes through $(0, M_0)$
(ii) $\frac{M}{M_0} = e^{-0.000121\times5730} = e^{-0.6933...} \approx \frac{1}{2}$	M1 E1	substituting $k = -0.00121$ and $t = 5730$ into equation (or ln eqn) showing that $M \approx \frac{1}{2} M_0$
(iii) $\frac{M}{M_0} = e^{-kT} = \frac{1}{2}$ $\Rightarrow \ln\frac{1}{2} = -kT$ $\Rightarrow \ln 2 = kT$ $\Rightarrow T = \frac{\ln 2}{k}$ *	M1 M1 E1	substituting $M/M_0 = \frac{1}{2}$ into equation (oe) taking lns correctly
(iv) $T = \frac{\ln 2}{2.88\times10^{-5}} \approx 24\,000$ years	B1 [8]	24 000 or better

Section B

7(i) $x = 1$	B1 [1]	
(ii) $\frac{dy}{dx} = \frac{(x-1)2x-(x^2+3).1}{(x-1)^2}$ $= \frac{2x^2-2x-x^2-3}{(x-1)^2}$ $= \frac{x^2-2x-3}{(x-1)^2}$ $dy/dx = 0$ when $x^2 - 2x - 3 = 0$ $\Rightarrow (x-3)(x+1) = 0$ $\Rightarrow x = 3$ or -1 When $x = 3$, $y = (9 + 3)/2 = 6$ So P is (3, 6)	M1 A1 M1 M1 A1 B1ft [6]	Quotient rule correct expression their numerator = 0 solving quadratic by any valid method $x = 3$ from correct working $y = 6$
(iii) Area $= \int_2^3 \frac{x^2+3}{x-1}dx$ $u = x - 1 \Rightarrow du/dx = 1$, $du = dx$ When $x = 2$, $u = 1$; when $x = 3$, $u = 2$ $= \int_1^2 \frac{(u+1)^2+3}{u}du$ $= \int_1^2 \frac{u^2+2u+4}{u}du$ $= \int_1^2 (u+2+\frac{4}{u})du$ * $= \left[\frac{1}{2}u^2+2u+4\ln u\right]_1^2$ $= (2 + 4 + 4\ln 2) - (½ + 2 + 4\ln 1)$ $= 3½ + 4\ln 2$	M1 B1 B1 E1 B1 M1 A1cao [7]	Correct integral and limits Limits changed, and substituting $dx = du$ substituting $\frac{(u+1)^2+3}{u}$ www $[½ u^2 + 2u + 4\ln u]$ substituting correct limits
(iv) $e^y = \frac{x^2+3}{x-1}$ $\Rightarrow e^y \frac{dy}{dx} = \frac{x^2-2x-3}{(x-1)^2}$ $\Rightarrow \frac{dy}{dx} = e^{-y}\frac{x^2-2x-3}{(x-1)^2}$ When $x = 2$, $e^y = 7 \Rightarrow$ $\Rightarrow dy/dx = \frac{1}{7}.\frac{4-4-3}{1} = -\frac{3}{7}$	M1 A1ft B1 A1cao [4]	$e^y dy/dx$ = their $f'(x)$ or $xe^y - e^y = x^2 + 3$ $\Rightarrow e^y + xe^y\frac{dy}{dx} - e^y\frac{dy}{dx} = 2x$ $\Rightarrow \frac{dy}{dx} = \frac{2x-e^y}{e^y(x-1)}$ $y = \ln 7$ or 1.95… or $e^y = 7$ or $\frac{dy}{dx} = \frac{4-7}{7(2-1)} = -\frac{3}{7}$ or –0.43 or better

8 (i) (*A*) 	B1 B1	Zeros shown every π/2. Correct shape, from −π to π
(*B*) 	M1 A1 [4]	Translated in *x*-direction π to the left
(ii) $f'(x) = -\frac{1}{5}e^{-\frac{1}{5}x}\sin x + e^{-\frac{1}{5}x}\cos x$ $f'(x) = 0$ when $-\frac{1}{5}e^{-\frac{1}{5}x}\sin x + e^{-\frac{1}{5}x}\cos x = 0$ $\Rightarrow \frac{1}{5}e^{-\frac{1}{5}x}(-\sin x + 5\cos x) = 0$ ⇒ sin *x* = 5 cos *x* $\Rightarrow \frac{\sin x}{\cos x} = 5$ ⇒ tan *x* = 5* ⇒ *x* = 1.37(34…) ⇒ *y* = 0.75 or 0.74(5…)	B1 B1 M1 E1 B1 B1 [6]	$e^{-\frac{1}{5}x}\cos x$ $\ldots -\frac{1}{5}e^{-\frac{1}{5}x}\sin x$ dividing by $e^{-\frac{1}{5}x}$ www 1.4 or better, must be in radians 0.75 or better
(iii) $f(x+\pi) = e^{-\frac{1}{5}(x+\pi)}\sin(x+\pi)$ $= e^{-\frac{1}{5}x}e^{-\frac{1}{5}\pi}\sin(x+\pi)$ $= -e^{-\frac{1}{5}x}e^{-\frac{1}{5}\pi}\sin x$ $= -e^{-\frac{1}{5}\pi}f(x)$ *	M1 A1 A1 E1	$e^{-\frac{1}{5}(x+\pi)} = e^{-\frac{1}{5}x}.e^{-\frac{1}{5}\pi}$ sin(*x* + π) = −sin *x* www
$\int_{\pi}^{2\pi} f(x)dx$ let *u* = *x* − π, d*u* = d*x* $= \int_0^{\pi} f(u+\pi)du$ $= \int_0^{\pi} -e^{-\frac{1}{5}\pi}f(u)du$ $= -e^{-\frac{1}{5}\pi}\int_0^{\pi} f(u)du$ * Area enclosed between π and 2π $= (-)e^{-\frac{1}{5}\pi} \times$ area between 0 and π.	B1 B1dep E1 B1 [8]	$\int f(u+\pi)du$ limits changed using above result or repeating work or multiplied by 0.53 or better

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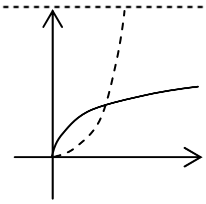
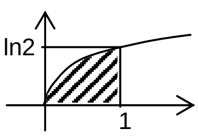
Section A

1 (i) P is (2, 1)	B1	
(ii) $\|x\| = 1\frac{1}{2}$ $\Rightarrow x = (-1\frac{1}{2})$ or $1\frac{1}{2}$ $\|x-2\|+1 = 1\frac{1}{2} \Rightarrow \|x-2\| = \frac{1}{2}$ $\Rightarrow x = (2\frac{1}{2})$ or $1\frac{1}{2}$	M1 A1 M1 E1	allow x = 1½ unsupported or $\left\|1\frac{1}{2}-2\right\|+1 = \frac{1}{2}+1 = 1\frac{1}{2}$
or by solving equation directly: $\|x-2\|+1 = \|x\|$ $\Rightarrow\ 2-x+1 = x$ $\Rightarrow\ x$ = 1½ $\Rightarrow\ y = \|x\|$ = 1½	M1 M1 A1 E1 [4]	equating from graph or listing possible cases
2 $\int_1^2 x^2 \ln x dx$ $u = \ln x$ $dv/dx = x^2 \Rightarrow v = \frac{1}{3}x^3$ $= \left[\frac{1}{3}x^3 \ln x\right]_1^2 - \int_1^2 \frac{1}{3}x^3.\frac{1}{x}dx$ $= \frac{8}{3}\ln 2 - \int_1^2 \frac{1}{3}x^2 dx$ $= \frac{8}{3}\ln 2 - \left[\frac{1}{9}x^3\right]_1^2$ $= \frac{8}{3}\ln 2 - \frac{8}{9} + \frac{1}{9}$ $= \frac{8}{3}\ln 2 - \frac{7}{9}$	M1 A1 A1 M1 A1 cao [5]	Parts with $u = \ln x$ $dv/dx = x^2 \Rightarrow v = x^3/3$ $\left[\frac{1}{9}x^3\right]$ substituting limits o.e. – not ln 1
3 (i) When $t = 0$, V = 10 000 $\Rightarrow$ 10 000 = $Ae^0 = A$ When $t = 3$, V = 6000 $\Rightarrow$ 6000 = 10 000 e^{-3k} $\Rightarrow$ $-3k$ = ln(0.6) = –0.5108… $\Rightarrow$ k = 0.17(02…)	M1 A1 M1 M1 A1 [5]	10 000 = Ae^0 A = 10 000 taking lns (correctly) on their exponential equation - not logs unless to base 10 art 0.17 or –(ln 0.6)/3 oe
(ii) 2000 = 10 000e^{-kt} $\Rightarrow$ $-kt$ = ln 0.2 $\Rightarrow$ t = –ln 0.2 / k = 9.45 (years)	 M1 A1 [2]	 taking lns on correct equation (consistent with their k) allow art 9.5, but not 9.

4 Perfect squares are 0, 1, 4, 9, 16, 25, 36, 49, 64, 81 none of which end in a 2, 3, 7 or 8. Generalisation: no perfect squares end in a 2, 3, 7 or 8.	M1 E1 B1 [3]	Listing all 1- and 2- digit squares. Condone absence of 0^2, and listing squares of 2 digit nos (i.e. $0^2 – 19^2$) For extending result to include further square numbers.
5 (i) $y = \frac{x^2}{2x+1}$ $\Rightarrow \frac{dy}{dx} = \frac{(2x+1)2x - x^2.2}{(2x+1)^2}$ $= \frac{2x^2+2x}{(2x+1)^2} = \frac{2x(x+1)}{(2x+1)^2}$ *	M1 A1 A1 E1 [4]	Use of quotient rule (or product rule) Correct numerator – condone missing bracket provided it is treated as present Correct denominator www –do not condone missing brackets
(ii) $\frac{dy}{dx} = 0$ when $2x(x + 1) = 0$ $\Rightarrow$ $x = 0$ or $−1$ $y = 0$ or $−1$	B1 B1 B1 B1 [4]	Must be from correct working: SC −1 if denominator = 0
6(i) QA = $3 – y$, PA = $6 – (3 – y) = 3 + y$ By Pythagoras, $PA^2 = OP^2 + OA^2$ $\Rightarrow (3 + y)^2 = x^2 + 3^2 = x^2 + 9$. *	B1 B1 E1 [3]	must show some working to indicate Pythagoras (e.g. $x^2 + 3^2$)
(ii) Differentiating implicitly: $2(y+3)\frac{dy}{dx} = 2x$ $\Rightarrow \frac{dy}{dx} = \frac{x}{y+3}$ *	M1 E1	Allow errors in RHS derivative (but not LHS) - notation should be correct brackets must be used
or $9 + 6y + y^2 = x^2 + 9$ $\Rightarrow 6y + y^2 = x^2$ $\Rightarrow (6+2y)\frac{dy}{dx} = 2x$ $\Rightarrow \frac{dy}{dx} = \frac{x}{y+3}$	M1 E1	Allow errors in RHS derivative (but not LHS) - notation should be correct brackets must be used
or $y = \sqrt{(x^2+9)} – 3 \Rightarrow dy/dx = ½(x^2+9)^{-1/2}.2x$ $= \frac{x}{\sqrt{x^2+9}} = \frac{x}{y+3}$	M1 E1	(cao)
(iii) $\frac{dy}{dt} = \frac{dy}{dx}.\frac{dx}{dt}$ $= \frac{4}{2+3} \times 2$ $= \frac{8}{5}$	M1 A1 A1 [3]	chain rule (soi)

Section B

7(i) When $x = -1$, $y = -1\sqrt{0} = 0$ Domain $x \ge -1$	E1 B1 [2]	 Not $y \ge -1$
(ii) $\frac{dy}{dx} = x.\frac{1}{2}(1+x)^{-1/2} + (1+x)^{1/2}$ $= \frac{1}{2}(1+x)^{-1/2}[x + 2(1+x)]$ $= \frac{2+3x}{2\sqrt{1+x}}$ *	B1 B1 M1 E1	$x.\frac{1}{2}(1+x)^{-1/2}$ … + $(1+x)^{1/2}$ taking out common factor or common denominator www
or $u = x + 1 \Rightarrow du/dx = 1$ $\Rightarrow y = (u-1)u^{1/2} = u^{3/2} - u^{1/2}$ $\Rightarrow \frac{dy}{du} = \frac{3}{2}u^{\frac{1}{2}} - \frac{1}{2}u^{-\frac{1}{2}}$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = \frac{3}{2}(x+1)^{\frac{1}{2}} - \frac{1}{2}(x+1)^{-\frac{1}{2}}$ $= \frac{1}{2}(x+1)^{-\frac{1}{2}}(3x+3-1)$ $= \frac{2+3x}{2\sqrt{1+x}}$ *	 M1 A1 M1 E1 [4]	 taking out common factor or common denominator
(iii) $dy/dx = 0$ when $3x + 2 = 0$ $\Rightarrow$ $x = -2/3$ $\Rightarrow$ $y = -\frac{2}{3}\sqrt{\frac{1}{3}}$ Range is $y \ge -\frac{2}{3}\sqrt{\frac{1}{3}}$	M1 A1cao A1 B1 ft [4]	 o.e. not $x \ge -\frac{2}{3}\sqrt{\frac{1}{3}}$ (ft their y value, even if approximate)
(iv) $\int_{-1}^{0} x\sqrt{1+x}\,dx$ let $u = 1 + x$, $du/dx = 1 \Rightarrow du = dx$ when $x = -1$, $u = 0$, when $x = 0$, $u = 1$ $= \int_0^1 (u-1)\sqrt{u}\,du$ $= \int_0^1 (u^{3/2} - u^{1/2})du$ *	 M1 B1 M1 E1	 $du = dx$ or $du/dx = 1$ or $dx/du = 1$ changing limits – allow with no working shown provided limits are present and consistent with dx and du. $(u-1)\sqrt{u}$ www – condone only final brackets missing, otherwise notation must be correct
$= \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_0^1$ $= \pm\frac{4}{15}$	B1 B1 M1 A1cao [8]	$\frac{2}{5}u^{5/2}, -\frac{2}{3}u^{3/2}$ (oe) substituting correct limits (can imply the zero limit) $\pm\frac{4}{15}$ or ± 0.27 or better, not 0.26

8 (i) $f'(x) = 2(e^x - 1)e^x$ When $x = 0$, $f'(0) = 0$ When $x = \ln 2$, $f'(\ln 2) = 2(2 - 1)2$ $= 4$	M1 A1 B1dep M1 A1cao [5]	or $f(x) = e^{2x} - 2e^x + 1$ M1 (or $(e^x)^2 - 2e^x + 1$ plus correct deriv of $(e^x)^2$) $\Rightarrow f'(x) = 2e^{2x} - 2e^x$ A1 derivative must be correct, www $e^{\ln 2} = 2$ soi
(ii) $y = (e^x - 1)^2$ $x \leftrightarrow y$ $x = (e^y - 1)^2$ $\Rightarrow \sqrt{x} = e^y - 1$ $\Rightarrow 1 + \sqrt{x} = e^y$ $\Rightarrow y = \ln(1 + \sqrt{x})$	M1 M1 E1	reasonable attempt to invert formula taking lns similar scheme of inverting $y = \ln(1 + \sqrt{x})$
or $gf(x) = g((e^x - 1)^2)$ $= \ln(1 + e^x - 1)$ $= x$	M1 M1 E1	constructing gf or fg $\ln(e^x) = x$ or $e^{\ln(1+\sqrt{x})} = 1 + \sqrt{x}$
 Gradient at (1, ln 2) = ¼	B1 B1ft [5]	reflection in $y = x$ (must have infinite gradient at origin)
(iii) $\int (e^x - 1)^2 dx = \int (e^{2x} - 2e^x + 1)dx$ $= \frac{1}{2}e^{2x} - 2e^x + x + c$ * $\int_0^{\ln 2} (e^x - 1)^2 dx = \left[\frac{1}{2}e^{2x} - 2e^x + x\right]_0^{\ln 2}$ $= \frac{1}{2}e^{2\ln 2} - 2e^{\ln 2} + \ln 2 - (\frac{1}{2} - 2)$ $= 2 - 4 + \ln 2 - \frac{1}{2} + 2$ $= \ln 2 - \frac{1}{2}$	M1 E1 M1 M1 A1 [5]	expanding brackets (condone e^{x^2}) substituting limits $e^{\ln 2} = 2$ used must be exact
(iv)  Area = $1 \times \ln 2 - (\ln 2 - \frac{1}{2})$ $= \frac{1}{2}$	M1 B1 A1cao [3]	subtracting area in (iii) from rectangle rectangle area = $1 \times \ln 2$ must be supported

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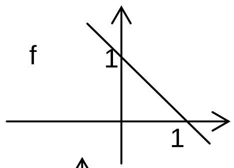
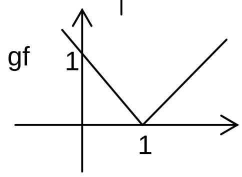
Section A

1 (i) $\frac{1}{2}(1+2x)^{-1/2} \times 2$ $= \dfrac{1}{\sqrt{1+2x}}$	M1 B1 A1 [3]	chain rule $\frac{1}{2}u^{-1/2}$ or $\frac{1}{2}(1+2x)^{-1/2}$ oe, but must resolve ½ × 2 = 1
(ii) $y = \ln(1-e^{-x})$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{1-e^{-x}}.(-e^{-x})(-1)$ $= \dfrac{e^{-x}}{1-e^{-x}}$ $= \dfrac{1}{e^{x}-1}$ *	M1 B1 A1 E1 [4]	chain rule $\dfrac{1}{1-e^{-x}}$ or $\dfrac{1}{u}$ if substituting $u = 1-e^{-x}$ $\times(-e^{-x})(-1)$ or e^{-x} www (may imply $\times e^{x}$ top and bottom)
2 $gf(x) = \|1-x\|$  	B1 B1 B1 [3]	 intercepts must be labelled line must extend either side of each axis condone no labels, but line must extend to left of y axis
3(i) Differentiating implicitly: $(4y+1)\dfrac{dy}{dx} = 18x$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{18x}{4y+1}$ When $x = 1$, $y = 2$, $\dfrac{dy}{dx} = \dfrac{18}{9} = 2$	M1 A1 M1 A1cao [4]	$(4y+1)\dfrac{dy}{dx} = \ldots$ allow $4y+1\dfrac{dy}{dx} = \ldots$ condone omitted bracket if intention implied by following line. $4y\dfrac{dy}{dx}+1$ M1 A0 substituting $x = 1$, $y = 2$ into their derivative (provided it contains x's and y's). Allow unsupported answers.
(ii) $\dfrac{dy}{dx} = 0$ when $x = 0$ $\Rightarrow 2y^2 + y = 1$ $\Rightarrow 2y^2 + y - 1 = 0$ $\Rightarrow (2y-1)(y+1) = 0$ $\Rightarrow y = \frac{1}{2}$ or $y = -1$ So coords are (0, ½) and (0, –1)	B1 M1 A1 A1 [4]	$x = 0$ from their numerator = 0 (must have a denominator) Obtaining correct quadratic and attempt to factorise or use quadratic formula $y = \dfrac{-1 \pm \sqrt{1-4\times -2}}{4}$ cao allow unsupported answers provided quadratic is shown

4(i) $T = 25 + ae^{-kt}$. When $t = 0$, $T = 100$ $\Rightarrow$ $100 = 25 + ae^0$ $\Rightarrow$ $a = 75$ When $t = 3$, $T = 80$ $\Rightarrow$ $80 = 25 + 75e^{-3k}$ $\Rightarrow$ $e^{-3k} = 55/75$ $\Rightarrow$ $-3k = \ln(55/75)$, $k = -\ln(55/75) / 3$ $= 0.1034$	M1 A1 M1 M1 A1cao [5]	substituting $t = 0$ and $T = 100$ into their equation (even if this is an incorrect version of the given equation) substituting $t = 3$ and $T = 80$ into (their) equation taking lns correctly at any stage 0.1 or better or $-\frac{1}{3}\ln(\frac{55}{75})$ o.e. if final answer
(ii) (A) $T = 25 + 75e^{-0.1034\times5}$ $= 69.72$ (B) 25°C	M1 A1 B1cao [3]	substituting $t = 5$ into their equation 69.5 to 70.5, condone inaccurate rounding due to value of k.
5 $n = 1$, $n^2 + 3n + 1 = 5$ prime $n = 2$, $n^2 + 3n + 1 = 11$ prime $n = 3$, $n^2 + 3n + 1 = 19$ prime $n = 4$, $n^2 + 3n + 1 = 29$ prime $n = 5$, $n^2 + 3n + 1 = 41$ prime $n = 6$, $n^2 + 3n + 1 = 55$ not prime so statement is false	M1 E1 [2]	One or more trials shown finding a counter-example – must state that it is not prime.
6 (i) $-\pi/2 < \arctan x < \pi/2$ $\Rightarrow$ $-\pi/4 < f(x) < \pi/4$ $\Rightarrow$ range is $-\pi/4$ to $\pi/4$	M1 A1cao [2]	$\pi/4$ or $-\pi/4$ or 45 seen not $\le$
(ii) $y = ½ \arctan x$ $x \leftrightarrow y$ $x = ½ \arctan y$ $\Rightarrow$ $2x = \arctan y$ $\Rightarrow$ $\tan 2x = y$ $\Rightarrow$ $y = \tan 2x$	M1 A1cao	tan(arctan y or x) = y or x
either $\frac{dy}{dx} = 2\sec^2 2x$	M1 A1cao	derivative of tan is $\sec^2$ used
or $y = \frac{\sin 2x}{\cos 2x} \Rightarrow \frac{dy}{dx} = \frac{2\cos^2 2x + 2\sin^2 2x}{\cos^2 2x}$ $= \frac{2}{\cos^2 2x}$	M1 A1cao	quotient rule (need not be simplified but mark final answer)
When $x = 0$, $dy/dx = 2$	B1 [5]	www
(iii) So gradient of $y = ½ \arctan x$ is ½ .	B1ft [1]	ft their ‘2’, but not 1 or 0 or ∞

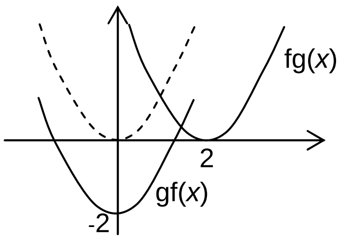
Section B

7(i) Asymptote when $1 + 2x^3 = 0$ $\Rightarrow$ $2x^3 = -1$ $\Rightarrow$ $x = -\frac{1}{\sqrt[3]{2}}$ $= -0.794$	M1 A1 A1cao [3]	 oe, condone $\pm\frac{1}{\sqrt[3]{2}}$ if positive root is rejected must be to 3 s.f.
(ii) $\frac{dy}{dx} = \frac{(1+2x^3).2x - x^2.6x^2}{(1+2x^3)^2}$ $= \frac{2x+4x^4-6x^4}{(1+2x^3)^2}$ $= \frac{2x-2x^4}{(1+2x^3)^2}$ * dy/dx = 0 when $2x(1-x^3) = 0$ $\Rightarrow$ $x = 0, y = 0$ or $x = 1$, $y = 1/3$	M1 A1 E1 M1 B1 B1 B1 B1 [8]	Quotient or product rule: (udv–vdu M0) $2x(1+2x^3)^{-1} + x^2(-1)(1+2x^3)^{-2}.6x^2$ allow one slip on derivatives correct expression – condone missing bracket if if intention implied by following line derivative = 0 x = 0 or 1 – allow unsupported answers y = 0 and 1/3 SC–1 for setting denom = 0 or extra solutions (e.g. $x = -1$)
(iii) $A = \int_0^1 \frac{x^2}{1+2x^3}\,dx$	M1	Correct integral and limits – allow $\int_1^0$
either $= \left[\frac{1}{6}\ln(1+2x^3)\right]_0^1$ $= \frac{1}{6}\ln 3$ *	M1 A1 M1 E1	$k\ln(1 + 2x^3)$ $k = 1/6$ substituting limits dep previous M1 www
or let $u = 1 + 2x^3 \Rightarrow du = 6x^2dx$ $\Rightarrow$ $A = \int_1^3 \frac{1}{6}.\frac{1}{u}\,du$ $= \left[\frac{1}{6}\ln u\right]_1^3$ $= \frac{1}{6}\ln 3$ *	 M1 A1 M1 E1 [5]	 $\frac{1}{6u}$ $\frac{1}{6}\ln u$ substituting correct limits (but must have used substitution) www

8 (i) $x\cos 2x = 0$ when $x = 0$ or $\cos 2x = 0$ $\Rightarrow$ $2x = \pi/2$ $\Rightarrow$ $x = \frac{1}{4}\pi$ $\Rightarrow$ P is $(\pi/4, 0)$	M1 M1 A1 [3]	$\cos 2x = 0$ or $x = \frac{1}{2}\cos^{-1}0$ $x = 0.785..$ or 45 is M1 M1 A0
(ii) $f(-x) = -x\cos(-2x)$ $= -x\cos 2x$ $= -f(x)$ Half turn symmetry about O.	M1 E1 B1 [3]	$-x\cos(-2x)$ $= -x\cos 2x$ Must have two of: rotational, order 2, about O, (half turn = rotational order 2)
(iii) $f'(x) = \cos 2x - 2x\sin 2x$	M1 A1 [2]	product rule
(iv) $f'(x) = 0 \Rightarrow \cos 2x = 2x\sin 2x$ $\Rightarrow$ $2x\dfrac{\sin 2x}{\cos 2x} = 1$ $\Rightarrow$ $x\tan 2x = \frac{1}{2}$ *	M1 E1 [2]	$\dfrac{\sin}{\cos} = \tan$ www
(v) $f'(0) = \cos 0 - 2.0.\sin 0 = 1$ $f''(x) = -2\sin 2x - 2\sin 2x - 4x\cos 2x$ $= -4\sin 2x - 4x\cos 2x$ $\Rightarrow$ $f''(0) = -4\sin 0 - 4.0.\cos 0 = 0$	B1ft M1 A1 E1 [4]	allow ft on (their) product rule expression product rule on $(2)x\sin 2x$ correct expression – mark final expression www
(vi) Let $u = x$, $dv/dx = \cos 2x$ $\Rightarrow v = \frac{1}{2}\sin 2x$ $\int_0^{\pi/4} x\cos 2x\,dx = \left[\frac{1}{2}x\sin 2x\right]_0^{\pi/4} - \int_0^{\pi/4}\frac{1}{2}\sin 2x\,dx$ $= \frac{\pi}{8} + \left[\frac{1}{4}\cos 2x\right]_0^{\pi/4}$ $= \frac{\pi}{8} - \frac{1}{4}$ Area of region enclosed by curve and x-axis between $x = 0$ and $x = \pi/4$	M1 A1 A1 M1 A1 B1 [6]	Integration by parts with $u = x$, $dv/dx = \cos 2x$ $\left[\frac{1}{4}\cos 2x\right]$ - sign consistent with their previous line substituting limits – dep using parts www or graph showing correct area – condone P for $\pi/4$.

4753 (C3) Methods for Advanced Mathematics

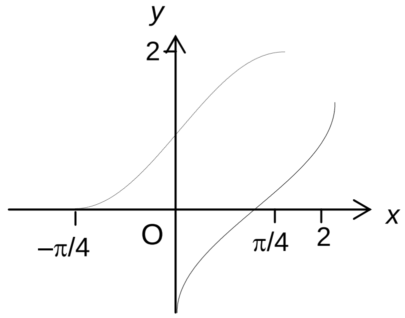
Section A

1 $y=(1+6x^2)^{1/3}$ $\Rightarrow \dfrac{dy}{dx}=\dfrac{1}{3}(1+6x^2)^{-2/3}.12x$ $=4x(1+6x^2)^{-2/3}$	M1 B1 A1 A1 [4]	chain rule used $\frac{1}{3}u^{-2/3}$ $\times 12x$ cao (must resolve 1/3 × 12) Mark final answer
2 (i) fg(x) = f(x – 2) = $(x-2)^2$ gf(x) = g(x^2) = $x^2 - 2$.	M1 A1 A1 [3]	forming a composite function mark final answer If fg and gf the wrong way round, M1A0A0
(ii) 	B1ft B1ft [2]	fg – must have (2, 0)labelled (or inferable from scale). Condone no y-intercept, unless wrong gf – must have (0, –2) labelled (or inferable from scale) Condone no x-intercepts, unless wrong Allow ft only if fg and gf are correct but wrong way round.
3 (i) When n = 1, 10 000 = A e^b when n = 2, 16 000 = A e^{2b} $\Rightarrow \dfrac{16000}{10000}=\dfrac{Ae^{2b}}{Ae^b}=e^b$ $\Rightarrow$ e^b = 1.6 $\Rightarrow$ b = ln 1.6 = 0.470 A = 10000/1.6 = 6250.	B1 B1 M1 E1 B1 B1 [6]	soi soi eliminating A (do not allow verification) SCB2 if initial 'B's are missing, and ratio of years = 1.6 = e^b ln 1.6 or 0.47 or better (mark final answer) cao – allow recovery from inexact b's
(ii) When n = 20, P = 6250×$e^{0.470\times 20}$ = £75,550,000	M1 A1 [2]	substituting n = 20 into their equation with their A and b Allow answers from £75 000 000 to £76 000 000.
4 (i) 5 = k/100 $\Rightarrow$ k = 500*	E1 [1]	NB answer given
(ii) $\dfrac{dP}{dV}=-500V^{-2}=-\dfrac{500}{V^2}$	M1 A1 [2]	$(-1)V^{-2}$ o.e. – allow $-k/V^2$
(iii) $\dfrac{dP}{dt}=\dfrac{dP}{dV}.\dfrac{dV}{dt}$ When V = 100, dP/dV = –500/10000 = –0.05 dV/dt = 10 $\Rightarrow$ dP/dt = –0.05 × 10 = –0.5 So P is decreasing at 0.5 Atm/s	M1 B1ft B1 A1 [4]	chain rule (any correct version) (soi) (soi) –0.5 cao

5(i) $p = 2$, $2^p - 1 = 3$, prime $p = 3$, $2^p - 1 = 7$, prime $p = 5$, $2^p - 1 = 31$, prime $p = 7$, $2^p - 1 = 127$, prime	M1 E1 [2]	Testing at least one prime testing all 4 primes (correctly) Must comment on answers being prime (allow ticks) Testing $p = 1$ is E0
(ii) $23\times89 = 2047 = 2^{11} - 1$ 11 is prime, 2047 is not So statement is false.	M1 E1 [2]	$2^{11} - 1$ must state or imply that 11 is prime ($p = 11$ is sufficient)
6 (i) $e^{2y} = x^2 + y$ $\Rightarrow 2e^{2y}\frac{dy}{dx} = 2x + \frac{dy}{dx}$ $\Rightarrow (2e^{2y} - 1)\frac{dy}{dx} = 2x$ $\Rightarrow \frac{dy}{dx} = \frac{2x}{2e^{2y} - 1}$ *	M1 A1 M1 E1 [4]	Implicit differentiation – allow one slip (but with dy/dx both sides) collecting terms
(ii) Gradient is infinite when $2e^{2y} - 1 = 0$ $\Rightarrow e^{2y} = ½$ $\Rightarrow 2y = \ln ½$ $\Rightarrow y = ½ \ln ½ = -0.347$ (3 s.f.) $x^2 = e^{2y} - y = ½ - (-0.347)$ $= 0.8465$ $\Rightarrow x = 0.920$	M1 A1 M1 A1 [4]	 must be to 3 s.f. substituting their y and solving for x cao – must be to 3 s.f., but penalise accuracy once only.

Section B

7(i) $y = 2x\ln(1+x)$ $\Rightarrow \frac{dy}{dx} = \frac{2x}{1+x} + 2\ln(1+x)$ When $x = 0$, $dy/dx = 0 + 2\ln 1 = 0$ $\Rightarrow$ origin is a stationary point.	M1 B1 A1 E1 [4]	product rule $d/dx(\ln(1+x)) = 1/(1+x)$ soi www (i.e. from correct derivative)
(ii) $\frac{d^2y}{dx^2} = \frac{(1+x).2 - 2x.1}{(1+x)^2} + \frac{2}{1+x}$ $= \frac{2}{(1+x)^2} + \frac{2}{1+x}$ When $x = 0$, $d^2y/dx^2 = 2 + 2 = 4 > 0$ $\Rightarrow$ (0, 0) is a min point	M1 A1ft A1 M1 E1 [5]	Quotient or product rule on their $2x/(1+x)$ correctly applied to their $2x/(1+x)$ o.e., e.g. $\frac{4+2x}{(1+x)^2}$ cao substituting $x = 0$ into their d^2y/dx^2 www – dep previous A1
(iii) Let $u = 1 + x \Rightarrow du = dx$ $\Rightarrow \int \frac{x^2}{1+x}dx = \int \frac{(u-1)^2}{u}du$ $= \int \frac{(u^2-2u+1)}{u}du$ $= \int (u - 2 + \frac{1}{u})du$ * $\Rightarrow \int_0^1 \frac{x^2}{1+x}dx = \int_1^2 (u-2+\frac{1}{u})du$ $= \left[\frac{1}{2}u^2 - 2u + \ln u\right]_1^2$ $= 2 - 4 + \ln 2 - (½ - 2 + \ln 1)$ $= \ln 2 - ½$	 M1 E1 B1 B1 M1 A1 [6]	 $\frac{(u-1)^2}{u}$ www (but condone du omitted except in final answer) changing limits (or substituting back for x and using 0 and 1) $\left[\frac{1}{2}u^2 - 2u + \ln u\right]$ substituting limits (consistent with u or x) cao
(iv) $A = \int_0^1 2x\ln(1+x)dx$ Parts: $u = \ln(1+x)$, $du/dx = 1/(1+x)$ $dv/dx = 2x \Rightarrow v = x^2$ $= \left[x^2\ln(1+x)\right]_0^1 - \int_0^1 \frac{x^2}{1+x}dx$ $= \ln 2 - \ln 2 + ½ = ½$	 M1 A1 M1 A1 [4]	 soi substituting their $\ln 2 - ½$ for $\int_0^1 \frac{x^2}{1+x}dx$ cao

8 (i) Stretch in x-direction s.f. ½ translation in y-direction 1 unit up	M1 A1 M1 A1 [4]	(in either order) – allow 'contraction' dep 'stretch' allow 'move', 'shift', etc – direction can be inferred from $\begin{pmatrix}0\\1\end{pmatrix}$ or $\begin{pmatrix}0\\1\end{pmatrix}$ dep 'translation'. $\begin{pmatrix}0\\1\end{pmatrix}$ alone is M1 A0
(ii) $A=\int_{-\pi/4}^{\pi/4}(1+\sin 2x)dx$ $=\left[x-\frac{1}{2}\cos 2x\right]_{-\pi/4}^{\pi/4}$ $= \pi/4 - ½ \cos \pi/2 + \pi/4 + ½ \cos (-\pi/2)$ $= \pi/2$	M1 B1 M1 A1 [4]	correct integral and limits. Condone dx missing; limits may be implied from subsequent working. substituting their limits (if zero lower limit used, must show evidence of substitution) or 1.57 or better – cao (www)
(iii) $y = 1 + \sin 2x$ $\Rightarrow$ $dy/dx = 2\cos 2x$ When $x = 0$, $dy/dx = 2$ So gradient at (0, 1) on f(x) is 2 $\Rightarrow$ gradient at (1, 0) on $f^{-1}(x) = ½$	M1 A1 A1ft B1ft [4]	differentiating – allow 1 error (but not $x + 2\cos 2x$) If 1, then must show evidence of using reciprocal, e.g. 1/1
(iv) Domain is $0 \le x \le 2$. 	B1 M1 A1 [3]	Allow 0 to 2, but not $0 < x < 2$ or y instead of x clear attempt to reflect in $y = x$ correct domain indicated (0 to 2), and reasonable shape
(v) $y = 1 + \sin 2x \quad x \leftrightarrow y$ $x = 1 + \sin 2y$ $\Rightarrow$ $\sin 2y = x - 1$ $\Rightarrow$ $2y = \arcsin(x - 1)$ $\Rightarrow$ $y = ½ \arcsin(x - 1)$	M1 A1 [2]	or $\sin 2x = y - 1$ cao

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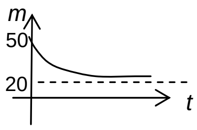
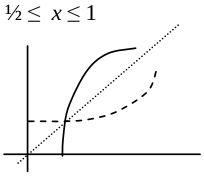
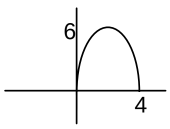
Section A

1 $\lvert 2x-1\rvert \le 3$ $\Rightarrow$ $-3 \le 2x - 1 \le 3$ $\Rightarrow$ $-2 \le 2x \le 4$ $\Rightarrow$ $-1 \le x \le 2$ *or* $(2x-1)^2 \le 9$ $\Rightarrow$ $4x^2 - 4x - 8 \le 0$ $\Rightarrow$ $(4)(x+1)(x-2) \le 0$ $\Rightarrow$ $-1 \le x \le 2$	M1 A1 M1 A1 M1 A1 A1 A1 [4]	$2x - 1 \le 3$ (or =) $x \le 2$ $2x - 1 \ge -3$ (or =) $x \ge -1$ squaring and forming quadratic = 0 (or $\le$) factorising or solving to get $x = -1, 2$ $x \ge -1$ $x \le 2$ (www)
2 Let $u = x$, $dv/dx = e^{3x} \Rightarrow v = e^{3x}/3$ $\Rightarrow$ $\int x e^{3x}\,dx = \frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}.1.dx$ $= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c$	M1 A1 A1 B1 [4]	parts with $u = x$, $dv/dx = e^{3x} \Rightarrow v$ $= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}$ $+c$
3 (i) $f(-x) = f(x)$ Symmetrical about Oy.	B1 B1 [2]	
(ii) (A) even (B) neither (C) odd	B1 B1 B1 [3]	
4 Let $u = x^2 + 2 \Rightarrow du = 2x\,dx$ $\int_1^4 \frac{x}{x^2+2}dx = \int_3^{18} \frac{1/2}{u}du$ $= \frac{1}{2}[\ln u]_3^{18}$ $= ½ (\ln 18 - \ln 3)$ $= ½ \ln(18/3)$ $= ½ \ln 6$*	M1 A1 M1 E1 [4]	$\int \frac{1/2}{u}du$ or $k \ln(x^2 + 1)$ $½ \ln u$ or $½ \ln(x^2 + 2)$ substituting correct limits (u or x) must show working for ln 6
5 $y = x^2 \ln x$ $\Rightarrow$ $\frac{dy}{dx} = x^2.\frac{1}{x} + 2x\ln x$ $= x + 2x\ln x$ $dy/dx = 0$ when $x + 2x\ln x = 0$ $\Rightarrow$ $x(1 + 2\ln x) = 0$ $\Rightarrow$ $\ln x = -½$ $\Rightarrow$ $x = e^{-½} = 1/\sqrt{e}$ *	M1 B1 A1 M1 M1 E1 [6]	product rule $d/dx (\ln x) = 1/x$ soi oe their deriv = 0 or attempt to verify $\ln x = -½ \Rightarrow x = e^{-½}$ or $\ln(1/\sqrt{e}) = -½$

6(i) Initial mass = $20 + 30\,e^0 = 50$ grams Long term mass = 20 grams	M1A1 B1 [3]	
(ii) $30 = 20 + 30\,e^{-0.1t}$ $\Rightarrow$ $e^{-0.1t} = 1/3$ $\Rightarrow$ $-0.1t = \ln(1/3) = -1.0986\ldots$ $\Rightarrow$ $t = 11.0$ mins	M1 M1 A1 [3]	 anti-logging correctly 11, 11.0, 10.99, 10.986 (not more than 3 d.p)
(iii) 	B1 B1 [2]	correct shape through (0, 50) – ignore negative values of t $\to 20$ as $t \to \infty$
7 $x^2 + xy + y^2 = 12$ $\Rightarrow$ $2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$ $\Rightarrow$ $(x+2y)\frac{dy}{dx} = -2x - y$ $\Rightarrow$ $\frac{dy}{dx} = -\frac{2x+y}{(x+2y)}$	M1 B1 A1 M1 A1 [5]	Implicit differentiation $x\frac{dy}{dx} + y$ correct equation collecting terms in dy/dx and factorising oe cao

Section B

8(i) $y = 1/(1+\cos\pi/3) = 2/3$.	B1 [1]	or 0.67 or better
(ii) $f'(x) = -1(1+\cos x)^{-2}.-\sin x$ $= \dfrac{\sin x}{(1+\cos x)^2}$ When $x = \pi/3$, $f'(x) = \dfrac{\sin(\pi/3)}{(1+\cos(\pi/3))^2}$ $= \dfrac{\sqrt{3}/2}{(1\frac{1}{2})^2} = \dfrac{\sqrt{3}}{2} \times \dfrac{4}{9} = \dfrac{2\sqrt{3}}{9}$	M1 B1 A1 M1 A1 [5]	chain rule or quotient rule d/dx (cos x) = –sin x soi correct expression substituting $x = \pi/3$ oe or 0.38 or better. (0.385, 0.3849)
(iii) deriv $= \dfrac{(1+\cos x)\cos x - \sin x.(-\sin x)}{(1+\cos x)^2}$ $= \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$ $= \dfrac{\cos x + 1}{(1+\cos x)^2}$ $= \dfrac{1}{1+\cos x}$ * Area $= \int_0^{\pi/3} \dfrac{1}{1+\cos x}\,dx$ $= \left[\dfrac{\sin x}{1+\cos x}\right]_0^{\pi/3}$ $= \dfrac{\sin \pi/3}{1+\cos\pi/3}(-0)$ $= \dfrac{\sqrt{3}}{2} \times \dfrac{2}{3} = \dfrac{\sqrt{3}}{3}$	M1 A1 M1dep E1 B1 M1 A1 cao [7]	Quotient or product rule – condone uv' – u'v for M1 correct expression $\cos^2 x + \sin^2 x = 1$ used dep M1 www substituting limits or $1/\sqrt{3}$ - must be exact
(iv) $y = 1/(1 + \cos x)$ $x \leftrightarrow y$ $x = 1/(1 + \cos y)$ $\Rightarrow$ $1 + \cos y = 1/x$ $\Rightarrow$ $\cos y = 1/x - 1$ $\Rightarrow$ $y = \arccos(1/x - 1)$ * Domain is $\frac{1}{2} \le x \le 1$ 	M1 A1 E1 B1 B1 [5]	attempt to invert equation www reasonable reflection in $y = x$

9 (i) $y=\sqrt{4-x^2}$ $\Rightarrow$ $y^2 = 4 - x^2$ $\Rightarrow$ $x^2 + y^2 = 4$ which is equation of a circle centre O radius 2 Square root does not give negative values, so this is only a semi-circle.	M1 A1 B1 [3]	squaring $x^2 + y^2 = 4$ + comment (correct) oe, e.g. f is a function and therefore single valued
(ii) (*A*) Grad of OP = *b*/*a* $\Rightarrow$ grad of tangent = $-\dfrac{a}{b}$ (*B*) $f'(x)=\dfrac{1}{2}(4-x^2)^{-1/2}.(-2x)$ $= -\dfrac{x}{\sqrt{4-x^2}}$ $\Rightarrow$ $f'(a)=-\dfrac{a}{\sqrt{4-a^2}}$ (*C*) $b = \sqrt{(4 - a^2)}$ so $f'(a)=-\dfrac{a}{b}$ as before	M1 A1 M1 A1 B1 E1 [6]	 chain rule or implicit differentiation oe substituting *a* into their f'(*x*)
(iii) Translation through $\begin{pmatrix}2\\0\end{pmatrix}$ followed by stretch scale factor 3 in *y*-direction 	M1 A1 M1 A1 M1 A1 [6]	Translation in *x*-direction through $\begin{pmatrix}2\\0\end{pmatrix}$ or 2 to right ('shift', 'move' M1 A0) $\begin{pmatrix}2\\0\end{pmatrix}$ alone is SC1 stretch in *y*-direction (condone *y* 'axis') (scale) factor 3 elliptical (or circular) shape through (0, 0) and (4, 0) and (2, 6) (soi) –1 if whole ellipse shown
(iv) $y = 3f(x - 2)$ $= 3\sqrt{(4 - (x - 2)^2)}$ $= 3\sqrt{(4 - x^2 + 4x - 4)}$ $= 3\sqrt{(4x - x^2)}$ $\Rightarrow$ $y^2 = 9(4x - x^2)$ $\Rightarrow$ $9x^2 + y^2 = 36x$ *	M1 A1 E1 [3]	or substituting $3\sqrt{(4 - (x - 2)^2)}$ oe for *y* in $9x^2 + y^2$ $4x - x^2$ www

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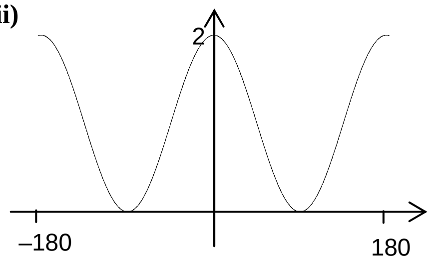
Section A

1 $\|x-1\|<3 \Rightarrow -3<x-1<3$ $\Rightarrow \quad -2<x<4$	M1 A1 B1 [3]	or $x-1=\pm 3$, or squaring $\Rightarrow$ correct quadratic $\Rightarrow$ $(x+2)(x-4)$ (condone factorising errors) or correct sketch showing $y = 3$ to scale $-2<$ <4 (penalise $\le$ once only)
2(i) $y = x\cos 2x$ $\Rightarrow \quad \frac{dy}{dx} = -2x\sin 2x + \cos 2x$	M1 B1 A1 [3]	product rule d/dx ($\cos 2x$) = $-2\sin 2x$ oe cao
(ii) $\int x\cos 2x\,dx = \int x\frac{d}{dx}(\frac{1}{2}\sin 2x)dx$ $= \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x\,dx$ $= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$	M1 A1 A1ft A1 [4]	parts with $u = x$, $v = ½ \sin 2x$ $+\frac{1}{4}\cos 2x$ cao – must have + c
3 *Either* $y = \frac{1}{2}\ln(x-1) \quad x \leftrightarrow y$ $\Rightarrow \quad x = \frac{1}{2}\ln(y-1)$ $\Rightarrow \quad 2x = \ln(y-1)$ $\Rightarrow \quad e^{2x} = y-1$ $\Rightarrow \quad 1+e^{2x} = y$ $\Rightarrow \quad g(x) = 1+e^{2x}$	 M1 M1 E1	or $y = e^{(x-1)/2}$ attempt to invert and interchanging x with y o.e. (at any stage) $e^{\ln y-1} = y-1$ or $\ln(e^y) = y$ used www
or $gf(x) = g(½ \ln(x-1))$ $= 1 + e^{\ln(x-1)}$ $= 1 + x - 1$ $= x$	M1 M1 E1 [3]	or $fg(x) = \ldots$ (correct way round) $e^{\ln(x-1)} = x-1$ or $\ln(e^{2x}) = 2x$ www
4 $\int_0^2 \sqrt{1+4x}\,dx \quad$ let $u = 1+4x$, $du = 4dx$ $= \int_1^9 u^{1/2}.\frac{1}{4}du$ $= \left[\frac{1}{6}u^{3/2}\right]_1^9$ $= \frac{27}{6} - \frac{1}{6} = \frac{26}{6} = \frac{13}{3}$ or $4\frac{1}{3}$	M1 A1 B1 M1 A1cao	$u = 1+4x$ and du/dx = 4 or du = 4dx $\int u^{1/2}.\frac{1}{4}du$ $\int u^{1/2}du = \frac{u^{3/2}}{3/2}$ soi substituting correct limits (u or x) dep attempt to integrate
or $\frac{d}{dx}(1+4x)^{3/2} = 4.\frac{3}{2}(1+4x)^{1/2} = 6(1+4x)^{1/2}$ $\Rightarrow \quad \int_0^2 (1+4x)^{1/2}dx = \left[\frac{1}{6}(1+4x)^{3/2}\right]_0^2$ $= \frac{27}{6} - \frac{1}{6} = \frac{26}{6} = \frac{13}{3}$ or $4\frac{1}{3}$	M1 A1 A1 M1 A1cao [5]	$k(1+4x)^{3/2}$ $\int (1+4x)^{1/2}dx = \frac{2}{3}(1+4x)^{3/2}\ldots$ $\times ¼$ substituting limits (dep attempt to integrate)

5(i) period 180°	B1 [1]	condone $0 \le x \le 180°$ or π
(ii) one-way stretch in x-direction scale factor ½ translation in y-direction through $\begin{pmatrix}0\\1\end{pmatrix}$	 M1 A1 M1 A1 [4]	[either way round…] condone 'squeeze', 'contract' for M1 stretch used and s.f ½ condone 'move', 'shift', etc for M1 'translation' used, +1 unit $\begin{pmatrix}0\\1\end{pmatrix}$ only is M1 A0
(iii) 	M1 B1 A1 [3]	correct shape, touching x-axis at -90°, 90° correct domain (0, 2) marked or indicated (i.e. amplitude is 2)
6(i) e.g $p = 1$ and $q = -2$ $p > q$ but $1/p = 1 > 1/q = -½$	M1 E1 [2]	stating values of p, q with $p \ge 0$ and $q \le 0$ (but not $p = q = 0$) showing that $1/p > 1/q$ - if 0 used, must state that 1/0 is undefined or infinite
(ii) Both p and q positive (or negative)	B1 [1]	or $q > 0$, 'positive integers'
7(i) $\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{\frac{2}{3}x^{-1/3}}{\frac{2}{3}y^{-1/3}}$ $= -\frac{y^{1/3}}{x^{1/3}} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$ *	M1 A1 M1 E1 [4]	Implicit differentiation (must show = 0) solving for dy/dx www. Must show, or explain, one more step.
(ii) $\frac{dy}{dt} = \frac{dy}{dx}.\frac{dx}{dt}$ $= -\left(\frac{8}{1}\right)^{\frac{1}{3}}.6$ $= -12$	M1 A1 A1cao [3]	any correct form of chain rule

8(i) When $x = 1$ $y = 1^2 - (\ln 1)/8 = 1$ Gradient of PR = $(1 + 7/8)/1 = 1\frac{7}{8}$	B1 M1 A1 [3]	 1.9 or better
(ii) $\frac{dy}{dx} = 2x - \frac{1}{8x}$ When $x = 1$, $dy/dx = 2 - 1/8 = 1\frac{7}{8}$ Same as gradient of PR, so PR touches curve	B1 B1dep E1 [3]	cao 1.9 or better dep 1st B1 dep gradients exact
(iii) Turning points when $dy/dx = 0$ $\Rightarrow 2x - \frac{1}{8x} = 0$ $\Rightarrow 2x = \frac{1}{8x}$ $\Rightarrow x^2 = 1/16$ $\Rightarrow x = ¼\ (x > 0)$ When $x = ¼$, $y = \frac{1}{16} - \frac{1}{8}\ln\frac{1}{4} = \frac{1}{16} + \frac{1}{8}\ln 4$ So TP is $(\frac{1}{4}, \frac{1}{16} + \frac{1}{8}\ln 4)$	M1 M1 A1 M1 A1cao [5]	setting their derivative to zero multiplying through by x allow verification substituting for x in y o.e. but must be exact, not $1/4^2$. Mark final answer.
(iv) $\frac{d}{dx}(x\ln x - x) = x.\frac{1}{x} + 1.\ln x - 1 = \ln x$	M1 A1	product rule $\ln x$
Area = $\int_1^2 (x^2 - \frac{1}{8}\ln x)dx$ $= \left[\frac{1}{3}x^3 - \frac{1}{8}(x\ln x - x)\right]_1^2$ $= (\frac{8}{3} - \frac{1}{4}\ln 2 + \frac{1}{4}) - (\frac{1}{3} - \frac{1}{8}\ln 1 + \frac{1}{8})$ $= \frac{7}{3} + \frac{1}{8} - \frac{1}{4}\ln 2$ $= \frac{59}{24} - \frac{1}{4}\ln 2$ *	M1 M1 A1 M1 E1 [7]	correct integral and limits (soi) – condone no dx $\int \ln x dx = x\ln x - x$ used (or derived using integration by parts) $\frac{1}{3}x^3 - \frac{1}{8}(x\ln x - x)$ – bracket required substituting correct limits must show at least one step

9(i) Asymptotes when (√)$(2x - x^2) = 0$ ⇒ $x(2 - x) = 0$ ⇒ $x = 0$ or 2 so $a = 2$ Domain is $0 < x < 2$	M1 A1 B1ft [3]	 or by verification $x > 0$ and $x < 2$, not ≤
(ii) $y = (2x - x^2)^{-1/2}$ let $u = 2x - x^2$, $y = u^{-1/2}$ ⇒ $dy/du = -½\, u^{-3/2}$, $du/dx = 2 - 2x$ ⇒ $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = -\frac{1}{2}(2x - x^2)^{-3/2}.(2 - 2x)$ $= \frac{x - 1}{(2x - x^2)^{3/2}}$ *	M1 B1 A1 E1	chain rule (or within correct quotient rule) $-½\, u^{-3/2}$ or $-\frac{1}{2}(2x - x^2)^{-3/2}$ or $\frac{1}{2}(2x - x^2)^{-1/2}$ in quotient rule × $(2 - 2x)$ www – penalise missing brackets here
$dy/dx = 0$ when $x - 1 = 0$ ⇒ $x = 1$, $y = 1/\sqrt{(2 - 1)} = 1$ Range is $y \geq 1$	M1 A1 B1 B1ft [8]	extraneous solutions M0
(iii) (*A*) $g(-x) = \frac{1}{\sqrt{1-(-x)^2}} = \frac{1}{\sqrt{1-x^2}} = g(x)$	M1 E1	Expression for $g(-x)$ – must have $g(-x) = g(x)$ seen
(*B*) $g(x-1) = \frac{1}{\sqrt{1-(x-1)^2}}$ $= \frac{1}{\sqrt{1-x^2+2x-1}} = \frac{1}{\sqrt{2x-x^2}} = f(x)$	M1 E1	 must expand bracket
(*C*) $f(x)$ is $g(x)$ translated 1 unit to the right. But $g(x)$ is symmetrical about Oy So $f(x)$ is symmetrical about $x = 1$.	M1 M1 A1	 dep both M1s
or $f(1 - x) = g(-x)$, $f(1 + x) = g(x)$ ⇒ $f(1 + x) = f(1 - x)$ ⇒ $f(x)$ is symmetrical about $x = 1$.	M1 E1 A1 [7]	or $f(1-x) = \frac{1}{\sqrt{2-2x-(1-x)^2}} = \frac{1}{\sqrt{2-2x-1+2x-x^2}} = \frac{1}{\sqrt{1-x^2}}$ $f(1+x) = \frac{1}{\sqrt{2+2x-(1+x)^2}} = \frac{1}{\sqrt{2+2x-1-2x-x^2}} = \frac{1}{\sqrt{1-x^2}}$

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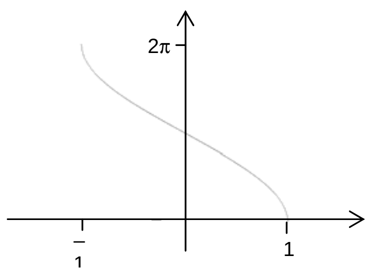
Section A

1 $\int_0^{\pi/6} \sin 3x \, dx = \left[-\frac{1}{3}\cos 3x\right]_0^{\frac{\pi}{6}}$ $= -\frac{1}{3}\cos\frac{\pi}{2} + \frac{1}{3}\cos 0$ $= \frac{1}{3}$	B1 M1 A1cao [3]	$\left[-\frac{1}{3}\cos 3x\right]$ or $\left[-\frac{1}{3}\cos u\right]$ substituting correct limits in ±*k* **cos** … 0.33 or better.
2(i) $100 = Ae^0 = A \Rightarrow A = 100$ $50 = 100\,e^{-1500k}$ $\Rightarrow e^{-1500k} = 0.5$ $\Rightarrow -1500k = \ln 0.5$ $\Rightarrow k = -\ln 0.5 / 1500 = 4.62 \times 10^{-4}$	M1A1 M1 M1 A1 [5]	 $50 = A\,e^{-1500k}$ ft their '*A*' if used taking lns correctly 0.00046 or better
(ii) $1 = 100e^{-kt}$ $\Rightarrow -kt = \ln 0.01$ $\Rightarrow t = -\ln 0.01 / k$ $= 9966$ years	M1 M1 A1 [3]	ft their *A* and *k* taking lns correctly art 9970
3 	M1 B1 A1 [3]	Can use degrees or radians reasonable shape (condone extra range) passes through (–1, 2π), (0, π) and (1, 0) good sketches – look for curve reasonably vertical at (–1, 2π) and (1, 0), negative gradient at (0, π). Domain and range must be clearly marked and correct.
4 $g(x) = 2\|x-1\|$ $\Rightarrow b = 2\|0-1\| = 2$ or (0, 2) $2\|x-1\| = 0$ $\Rightarrow x = 1$, so $a = 1$ or (1, 0)	 B1 M1 A1 [3]	Allow unsupported answers. www $\|x\| = 1$ is A0 www

5(i)	$e^{2y} = 1 + \sin x$ $\Rightarrow$ $2e^{2y}dy/dx = \cos x$ $\Rightarrow$ $dy/dx = \dfrac{\cos x}{2e^{2y}}$	M1 B1 A1 [3]	Their $2e^{2y} \times dy/dx$ $2e^{2y}$ o.e. cao
(ii)	$2y = \ln(1 + \sin x)$ $\Rightarrow$ $y = ½ \ln(1 + \sin x)$ $\Rightarrow$ $dy/dx = \dfrac{1}{2}\dfrac{\cos x}{1+\sin x}$ $= \dfrac{\cos x}{2e^{2y}}$ as before	 B1 M1 B1 E1 [4]	 chain rule (can be within 'correct' quotient rule with $dv/dx = 0$) $1/u$ or $1/(1 + \sin x)$ soi www
6	$f\,f(x) = \dfrac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}$ $= \dfrac{x+1+x-1}{x+1-x+1}$ $= 2x/2 = x$* $f^{-1}(x) = f(x)$ Symmetrical about $y = x$.	M1 M1 E1 B1 B1 [5]	correct expression without subsidiary denominators e.g. $= \dfrac{x+1+x-1}{x-1} \times \dfrac{x-1}{x+1-x+1}$ stated, or shown by inverting
7(i)	(*A*) $(x-y)(x^2+xy+y^2)$ $= x^3 + x^2y + xy^2 - yx^2 - xy^2 - y^3$ $= x^3 - y^3$ * (*B*) $(x + ½ y)^2 + ¾ y^2$ $= x^2 + xy + ¼ y^2 + ¾ y^2$ $= x^2 + xy + y^2$	M1 E1 M1 E1 [4]	expanding - allow tabulation www $(x + ½ y)^2 = x^2 + ½ xy + ½ xy + ¼ y^2$ o.e. cao www
(ii)	$x^3 - y^3 = (x-y)[(x + ½ y)^2 + ¾ y^2]$ $(x + ½ y)^2 + ¾ y^2 > 0$ [as squares ≥ 0] $\Rightarrow$ if $x - y > 0$ then $x^3 - y^3 > 0$ $\Rightarrow$ if $x > y$ then $x^3 > y^3$ *	M1 M1 E1 [3]	substituting results of (i)

8(i) A: $1 + \ln x = 0$ $\Rightarrow \ln x = -1$ so A is $(e^{-1}, 0)$ $\Rightarrow x = e^{-1}$ B: $x = 0, y = e^{0-1} = e^{-1}$ so B is $(0, e^{-1})$ C: $f(1) = e^{1-1} = e^0 = 1$ $g(1) = 1 + \ln 1 = 1$	M1 A1 B1 E1 E1 [5]	 SC1 if obtained using symmetry condone use of symmetry Penalise A = e^{-1}, B = e^{-1}, or co-ords wrong way round, but condone labelling errors.
(ii) *Either* by invertion: e.g. $y = e^{x-1}$ $x \leftrightarrow y$ $x = e^{y-1}$ $\Rightarrow \ln x = y - 1$ $\Rightarrow 1 + \ln x = y$	 M1 E1	 taking lns or exps
or by composing e.g. $f\,g(x) = f(1 + \ln x)$ $= e^{1 + \ln x - 1}$ $= e^{\ln x} = x$	 M1 E1 [2]	 $e^{1 + \ln x - 1}$ or $1 + \ln(e^{x-1})$
(iii) $\int_0^1 e^{x-1}dx = \left[e^{x-1}\right]_0^1$ $= e^0 - e^{-1}$ $= 1 - e^{-1}$	M1 M1 A1cao [3]	$\left[e^{x-1}\right]$ o.e or $u = x - 1 \Rightarrow \left[e^u\right]$ substituting correct limits for x or u o.e. not e^0, must be exact.
(iv) $\int \ln x dx = \int \ln x \frac{d}{dx}(x)dx$ $= x\ln x - \int x.\frac{1}{x}dx$ $= x \ln x - x + c$ $\Rightarrow \int_{e^{-1}}^1 g(x)dx = \int_{e^{-1}}^1 (1 + \ln x)dx$ $= [x + x\ln x - x]_{e^{-1}}^1$ $= [x\ln x)]_{e^{-1}}^1$ $= 1\ln 1 - e^{-1}\ln(e^{-1})$ $= e^{-1}$ *	M1 A1 A1cao B1ft DM1 E1 [6]	parts: $u = \ln x$, $du/dx = 1/x$, $v = x$, $dv/dx = 1$ condone no 'c' ft their '$x \ln x - x$' (provided 'algebraic') substituting limits dep B1 www
(v) Area $= \int_0^1 f(x)\,d\,x - \int_{e^{-1}}^1 g(x)\,d\,x$ $= (1 - e^{-1}) - e^{-1}$ $= 1 - 2/e$	M1 A1cao	Must have correct limits 0.264 or better.
or Area OCB = area under curve – triangle $= 1 - e^{-1} - ½ \times 1 \times 1$ $= ½ - e^{-1}$ *or* Area OAC = triangle – area under curve $= ½ \times 1 \times 1 - e^{-1}$ $= ½ - e^{-1}$ Total area $= 2(½ - e^{-1}) = 1 - 2/e$	 M1 A1cao [2]	 OCA or OCB = $½ - e^{-1}$ 0.264 or better

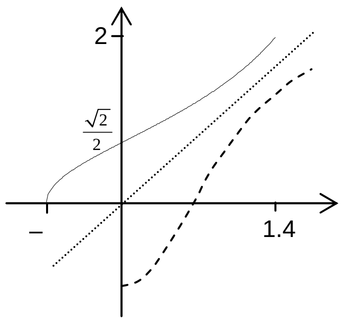
9(i) $a = 1/3$	B1 [1]	or 0.33 or better
(ii) $\frac{dy}{dx} = \frac{(3x-1)2x - x^2.3}{(3x-1)^2}$ $= \frac{6x^2 - 2x - 3x^2}{(3x-1)^2}$ $= \frac{3x^2 - 2x}{(3x-1)^2}$ $= \frac{x(3x-2)}{(3x-1)^2}$ *	M1 A1 E1 [3]	quotient rule www – must show both steps; penalise missing brackets.
(iii) $dy/dx = 0$ when $x(3x-2) = 0$ $\Rightarrow$ $x = 0$ or $x = 2/3$, so at P, $x = 2/3$ when $x = \frac{2}{3}$, $y = \frac{(2/3)^2}{3\times(2/3)-1} = \frac{4}{9}$ when $x = 0.6$, $dy/dx = -0.1875$ when $x = 0.8$, $dy/dx = 0.1633$ Gradient increasing $\Rightarrow$ minimum	M1 A1 M1 A1cao B1 B1 E1 [7]	if denom = 0 also then M0 o.e e.g. $0.\dot{6}$, but must be exact o.e e.g. $0.\dot{4}$, but must be exact –3/16, or –0.19 or better 8/49 or 0.16 or better o.e. e.g. 'from negative to positive'. Allow ft on their gradients, provided –ve and +ve respectively. Accept table with indications of signs of gradient.
(iv) $\int \frac{x^2}{3x-1}dx$ $u = 3x-1 \Rightarrow du = 3dx$ $= \int \frac{\frac{(u+1)^2}{9}}{u}\frac{1}{3}du$ $= \frac{1}{27}\int \frac{(u+1)^2}{u}du = \frac{1}{27}\int \frac{u^2+2u+1}{u}du$ $= \frac{1}{27}\int (u+2+\frac{1}{u})du$ * Area = $\int_{2/3}^{1} \frac{x^2}{3x-1}dx$ When $x = 2/3$, $u = 1$, when $x = 1$, $u = 2$ $= \frac{1}{27}\int_1^2 (u+2+1/u)du$ $= \frac{1}{27}\left[\frac{1}{2}u^2 + 2u + \ln u\right]_1^2$ $= \frac{1}{27}[(2+4+\ln 2) - (\frac{1}{2}+2+\ln 1)]$ $= \frac{1}{27}(3\frac{1}{2}+\ln 2)$ $[= \frac{7+2\ln 2}{54}]$	B1 M1 M1 E1 B1 M1 A1cao [7]	$\frac{\frac{(u+1)^2}{9}}{u}$ o.e. × 1/3 (du) expanding Condone missing du's $\left[\frac{1}{2}u^2 + 2u + \ln u\right]$ substituting correct limits, dep integration o.e., but must evaluate ln 1 = 0 and collect terms.

4753 (C3) Methods for Advanced Mathematics

1 $e^{2x} - 5e^{x} = 0$ $\Rightarrow$ $e^{x}(e^{x} - 5) = 0$ $\Rightarrow$ $e^{x} = 5$ $\Rightarrow$ $x = \ln 5$ or 1.6094	M1 M1 A1 A1 [4]	factoring out e^{x} or dividing $e^{2x} = 5e^{x}$ by e^{x} $e^{2x} / e^{x} = e^{x}$ ln 5 or 1.61 or better, mark final answer –1 for additional solutions, e.g. $x = 0$
or $\ln(e^{2x}) = \ln(5e^{x})$ $\Rightarrow$ $2x = \ln 5 + x$ $\Rightarrow$ $x = \ln 5$ or 1.6094	M1 A1 A1 A1 [4]	taking lns on $e^{2x} = 5e^{x}$ $2x$, $\ln 5 + x$ ln 5 or 1.61 or better, mark final answer –1 for additional solutions, e.g. $x = 0$
2 (i) When $t = 0$, $T = 100$ $\Rightarrow$ $100 = 20 + b$ $\Rightarrow$ $b = 80$ When $t = 5$, $T = 60$ $\Rightarrow$ $60 = 20 + 80e^{-5k}$ $\Rightarrow$ $e^{-5k} = ½$ $\Rightarrow$ $k = \ln 2 / 5 = 0.139$	M1 A1 M1 A1 [4]	substituting $t = 0$, $T = 100$ cao substituting $t = 5$, $T = 60$ 1/5 ln 2 or 0.14 or better
(ii) $50 = 20 + 80e^{-kt}$ $\Rightarrow$ $e^{-kt} = 3/8$ $\Rightarrow$ $t = \ln(8/3) / k = 7.075$ mins	M1 A1 [2]	Re-arranging and taking lns correctly – ft their b and k answers in range 7 to 7.1
3(i) $\frac{dy}{dx} = \frac{1}{3}(1+3x^2)^{-2/3}.6x$ $= 2x(1+3x^2)^{-2/3}$	M1 B1 A1 [3]	chain rule $1/3\, u^{-2/3}$ or $\frac{1}{3}(1+3x^2)^{-2/3}$ o.e but must be '2' (not 6/3) mark final answer
(ii) $3y^2 \frac{dy}{dx} = 6x$ $\Rightarrow$ $dy/dx = 6x/3y^2$ $= \frac{2x}{(1+3x^2)^{2/3}} = 2x(1+3x^2)^{-2/3}$	M1 A1 A1 E1 [4]	$3y^2 \frac{dy}{dx}$ $= 6x$ if deriving $2x(1+3x^2)^{-2/3}$, needs a step of working

4(i) $\int_0^1 \frac{2x}{x^2+1}\,dx = \left[\ln(x^2+1)\right]_0^1$ $= \ln 2$	M2 A1 [3]	$[\ln(x^2 + 1)]$ cao (must be exact)
or let $u = x^2 + 1$, $du = 2x\,dx$ $\Rightarrow$ $\int_0^1 \frac{2x}{x^2+1}\,dx = \int_1^2 \frac{1}{u}\,du$ $= \left[\ln u\right]_1^2$ $= \ln 2$	M1 A1 A1 [3]	$\int \frac{1}{u}\,du$ or $\left[\ln(1+x^2)\right]_0^1$ with correct limits cao (must be exact)
(ii) $\int_0^1 \frac{2x}{x+1}\,dx = \int_0^1 \frac{2x+2-2}{x+1}\,dx = \int_0^1 (2-\frac{2}{x+1})\,dx$ $= \left[2x - 2\ln(x+1)\right]_0^1$ $= 2 - 2\ln 2$	M1 A1, A1 A1 A1 [5]	dividing by $(x + 1)$ 2, $-2/(x+ 1)$
or $\int_0^1 \frac{2x}{x+1}\,dx$ let $u = x + 1$, $\Rightarrow du = dx$ $= \int_1^2 \frac{2(u-1)}{u}\,du$ $= \int_1^2 (2-\frac{2}{u})\,du$ $= \left[2u - 2\ln u\right]_1^2$ $= 4 - 2\ln 2 - (2 - 2\ln 1)$ $= 2 - 2\ln 2$	M1 B1 M1 A1 A1 [5]	substituting $u = x + 1$ and $du = dx$ (or $du/dx = 1$) *and* correct limits used for u or x $2(u - 1)/u$ dividing through by u $2u - 2\ln u$ allow ft on $(u - 1)/u$ (i.e. with 2 omitted) o.e. cao (must be exact)
5 (i) $a = 0$, $b = 3$, $c = 2$	B(2,1,0)	or $a = 0$, $b = -3$, $c = -2$
(ii) $a = 1$, $b = -1$, $c = 1$ *or* $a = 1$, $b = 1$, $c = -1$	B(2,1,0) [4]	
6 $f(-x) = -f(x)$, $g(-x) = g(x)$ $g\,f(-x) = g\,[-f\,(x)]$ $= g\,f\,(x)$ $\Rightarrow$ g f is even	B1B1 M1 E1 [4]	condone f and g interchanged forming $gf(-x)$ or $gf(x)$ and using $f(-x) = -f(x)$ www
7 Let $\arcsin x = \theta$ $\Rightarrow$ $x = \sin\theta$ $\theta = \arccos y \Rightarrow y = \cos\theta$ $\sin^2\theta + \cos^2\theta = 1$ $\Rightarrow$ $x^2 + y^2 = 1$	 M1 M1 E1 [3]	

8(i) At P, $x\cos 3x = 0$ $\Rightarrow$ $\cos 3x = 0$ $\Rightarrow$ $3x = \pi/2, 3\pi/2$ $\Rightarrow$ $x = \pi/6, \pi/2$ So P is $(\pi/6, 0)$ and Q is $(\pi/2, 0)$	M1 M1 A1 A1 [4]	or verification $3x = \pi/2, (3\pi/2\ldots)$ dep both Ms condone degrees here
(ii) $\frac{dy}{dx} = -3x\sin 3x + \cos 3x$ At P, $\frac{dy}{dx} = -\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2} = -\frac{\pi}{2}$ At TPs $\frac{dy}{dx} = -3x\sin 3x + \cos 3x = 0$ $\Rightarrow$ $\cos 3x = 3x\sin 3x$ $\Rightarrow$ $1 = 3x\sin 3x / \cos 3x = 3x\tan 3x$ $\Rightarrow$ $x\tan 3x = 1/3$ *	M1 B1 A1 M1 A1cao M1 E1 [7]	Product rule $d/dx(\cos 3x) = -3\sin 3x$ cao (so for $dy/dx = -3x\sin 3x$ allow B1) mark final answer substituting their $-\pi/6$ (must be rads) $-\pi/2$ $dy/dx = 0$ and $\sin 3x / \cos 3x = \tan 3x$ used www
(iii) $A = \int_0^{\pi/6} x\cos 3x\,dx$ Parts with $u = x$, $dv/dx = \cos 3x$ $du/dx = 1$, $v = 1/3 \sin 3x$ $\Rightarrow$ $A = \left[\frac{1}{3}x\sin 3x\right]_0^{\frac{\pi}{6}} - \int_0^{\pi/6}\frac{1}{3}\sin 3x\,dx$ $= \left[\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x\right]_0^{\frac{\pi}{6}}$ $= \frac{\pi}{18} - \frac{1}{9}$	B1 M1 A1 A1 M1dep A1 cao [6]	Correct integral and limits (soi) – ft their P, but must be in radians can be without limits dep previous A1. substituting correct limits, dep 1st M1: ft their P provided in radians o.e. but must be exact

9(i) $f'(x)=\frac{(x^2+1)4x-(2x^2-1)2x}{(x^2+1)^2}$ $=\frac{4x^3+4x-4x^3+2x}{(x^2+1)^2}=\frac{6x}{(x^2+1)^2}$ * When $x > 0$, $6x > 0$ and $(x^2+1)^2 > 0$ $\Rightarrow f'(x) > 0$	M1 A1 E1 M1 E1 [5]	Quotient or product rule correct expression www attempt to show or solve f'(x) > 0 numerator > 0 and denominator > 0 or, if solving, $6x > 0 \Rightarrow x > 0$
(ii) $f(2)=\frac{8-1}{4+1}=1\frac{2}{5}$ Range is $-1 \le y \le 1\frac{2}{5}$	B1 B1 [2]	 must be ≤, y or f(x)
(iii) $f'(x)$ max when $f''(x) = 0$ $\Rightarrow 6 - 18x^2 = 0$ $\Rightarrow x^2 = 1/3$, $x = 1/\sqrt{3}$ $\Rightarrow f'(x)=\frac{6/\sqrt{3}}{(1\frac{1}{3})^2}=\frac{6}{\sqrt{3}}.\frac{9}{16}=\frac{9\sqrt{3}}{8}=1.95$	M1 A1 M1 A1 [4]	 (±)1/√3 oe (0.577 or better) substituting 1/√3 into f'(x) 9√3/8 o.e. or 1.95 or better (1.948557..)
(iv) Domain is $-1 < x < 1\frac{2}{5}$ Range is $0 \le y \le 2$ 	B1 B1 M1 A1 cao [4]	ft their 1.4 but not $x \ge -1$ or $0 \le g(x) \le 2$ (not f) Reasonable reflection in $y = x$ from (–1, 0) to (1.4, 2), through (0, √2/2) allow omission of one of –1, 1.4, 2, √2/2
(v) $y=\frac{2x^2-1}{x^2+1}$ $x \leftrightarrow y$ $x=\frac{2y^2-1}{y^2+1}$ $\Rightarrow xy^2 + x = 2y^2 - 1$ $\Rightarrow x + 1 = 2y^2 - xy^2 = y^2(2 - x)$ $\Rightarrow y^2=\frac{x+1}{2-x}$ $\Rightarrow y=\sqrt{\frac{x+1}{2-x}}$ *	 M1 M1 M1 E1 [4]	(could start from g) Attempt to invert clearing fractions collecting terms in y^2 and factorising www

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

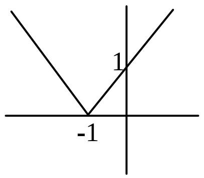
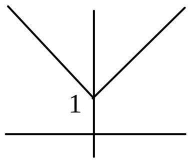
Advanced GCE **4753**

Methods for Advanced Mathematics (C3)

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

Section A

1 $\int_0^{\pi/6} \cos 3x \, \mathrm{d}x = \left[\frac{1}{3}\sin 3x\right]_0^{\pi/6}$ $= \frac{1}{3}\sin\frac{\pi}{2} - 0$ $= 1/3$	M1 B1 A1cao [3]	$k \sin 3x$, $k > 0$, $k \neq 3$ $k = (\pm)1/3$ 0.33 or better	or M1 for $u = 3x \Rightarrow \int \frac{1}{3}\cos u \, \mathrm{d}u$ condone 90° in limit or M1 for $\left[\frac{1}{3}\sin u\right]$ so: $\sin 3x$: M1B0, $-\sin 3x$: M0B0, $\pm 3\sin 3x$: M0B0, $-1/3 \sin 3x$: M0B1
2 $\mathrm{fg}(x) = \lvert x+1 \rvert$ $\mathrm{g\,f}(x) = \lvert x \rvert + 1$   	B1 B1 B1 B1 [4]	soi from correctly-shaped graphs (i.e. without intercepts) graph of $\lvert x+1 \rvert$ only graph of $\lvert x \rvert + 1$	but must indicate which is which bod gf if negative x values are missing 'V' shape with (–1, 0) and (0, 1) labelled 'V' shape with (0, 1) labelled (0, 1)
3(i) $y = (1+3x^2)^{1/2}$ $\Rightarrow dy/dx = \frac{1}{2}(1+3x^2)^{-1/2}.6x$ $= 3x(1 + 3x^2)^{-1/2}$	M1 B1 A1 [3]	chain rule ½ $u^{-1/2}$ o.e., but must be '3'	 can isw here
(ii) $y = x(1+3x^2)^{1/2}$ $\Rightarrow dy/dx = x.\frac{3x}{\sqrt{1+3x^2}} + 1.(1+3x^2)^{1/2}$ $= \frac{3x^2+1+3x^2}{\sqrt{1+3x^2}}$ $= \frac{1+6x^2}{\sqrt{1+3x^2}}$ *	M1 A1ft M1 E1 [4]	product rule ft their dy/dx from (i) common denominator or factoring $(1+3x^2)^{-1/2}$ www	 must show this step for M1 E1

4 $p = 100/x = 100\,x^{-1}$ $\Rightarrow$ $dp/dx = -100x^{-2} = -100/x^2$ $dp/dt = dp/dx \times dx/dt$ $dx/dt = 10$ When $x = 50$, $dp/dx = (-100/50^2)$ $\Rightarrow dp/dt = 10 \times -0.04 = -0.4$	M1 A1 M1 B1 M1dep A1cao [6]	attempt to differentiate $-100x^{-2}$ o.e. o.e. soi soi substituting $x = 50$ into their dp/dx dep 2nd M1 o.e. e.g. decreasing at 0.4	condone poor notation if chain rule correct *or* $x = 50 + 10\,t$ B1 $\Rightarrow P = 100/x = 100/(50 + 10\,t)$ $\Rightarrow dP/dt = -100(50 + 10\,t)^{-2} \times 10 = -1000/(50 + 10t)^{-2}$ M1 A1 When $t = 0$, $dP/dt = -1000/50^2 = -0.4$ A1
5 $y^3 = xy - x^2$ $\Rightarrow$ $3y^2\,dy/dx = x\,dy/dx + y - 2x$ $\Rightarrow$ $3y^2\,dy/dx - x\,dy/dx = y - 2x$ $\Rightarrow$ $(3y^2 - x)\,dy/dx = y - 2x$ $\Rightarrow$ $dy/dx = (y - 2x)/(3y^2 - x)$ * TP when $dy/dx = 0 \Rightarrow y - 2x = 0$ $\Rightarrow$ $y = 2x$ $\Rightarrow$ $(2x)^3 = x.2x - x^2$ $\Rightarrow$ $8x^3 = x^2$ $\Rightarrow$ $x = 1/8$ *(or 0)	B1 B1 M1 E1 M1 M1 E1 [7]	$3y^2dy/dx$ $x\,dy/dx + y - 2x$ collecting terms in dy/dx only or $x = 1/8$ and $dy/dx = 0 \Rightarrow y = ¼$ or $(1/4)^3 = (1/8)(1/4) - (1/8)^2$ or verifying e.g. $1/64 = 1/64$	must show '$x\,dy/dx + y$' on one side or $x = 1/8 \Rightarrow y^3 = (1/8)y - 1/64$ M1 verifying that $y = ¼$ is a solution (must show evidence*) M1 $\Rightarrow dy/dx = (¼ - 2(1/8))/(\ldots) = 0$ E1 *just stating that $y = ¼$ is M1 M0 E0
6 $f(x) = 1 + 2\sin 3x = y$ $x \leftrightarrow y$ $x = 1 + 2\sin 3y$ $\Rightarrow$ $\sin 3y = (x - 1)/2$ $\Rightarrow$ $3y = \arcsin[(x - 1)/2]$ $\Rightarrow y = \frac{1}{3}\arcsin\left[\frac{x-1}{2}\right]$ so $f^{-1}(x) = \frac{1}{3}\arcsin\left[\frac{x-1}{2}\right]$ Range of f is -1 to 3 $\Rightarrow$ $-1 \le x \le 3$	M1 A1 A1 A1 M1 A1 [6]	attempt to invert must be $y = \ldots$ or $f^{-1}(x) = \ldots$ or $-1 \le (x - 1)/2 \le 1$ must be 'x', not y or $f(x)$	at least one step attempted, or reasonable attempt at flow chart inversion (or any other variable provided same used on each side) condone <'s for M1 allow unsupported correct answers; -1 to 3 is M1 A0
7 (*A*) True , (*B*) True , (C) False Counterexample, e.g. $\sqrt{2} + (-\sqrt{2}) = 0$	B2,1,0 B1 [3]		

8(i) When $x = 1$, $y = 3\ln 1 + 1 - 1^2$ $= 0$	E1 [1]		
(ii) $\frac{dy}{dx} = \frac{3}{x} + 1 - 2x$ $\Rightarrow$ At R, $\frac{dy}{dx} = 0 = \frac{3}{x} + 1 - 2x$ $\Rightarrow$ $3 + x - 2x^2 = 0$ $\Rightarrow$ $(3 - 2x)(1 + x) = 0$ $\Rightarrow$ $x = 1.5$, (or -1) $\Rightarrow$ $y = 3\ln 1.5 + 1.5 - 1.5^2$ $= 0.466$ (3 s.f.) $\frac{d^2y}{dx^2} = -\frac{3}{x^2} - 2$ When $x = 1.5$, d^2y/dx^2 $(= -10/3) < 0 \Rightarrow$ max	M1 A1cao M1 M1 A1 M1 A1cao B1ft E1 [9]	d/dx (ln x) = 1/x re-arranging into a quadratic = 0 factorising or formula or completing square substituting their x ft their dy/dx on equivalent work www – don't need to calculate 10/3	 SC1 for $x = 1.5$ unsupported, SC3 if verified but condone rounding errors on 0.466
(iii) Let $u = \ln x$, $du/dx = 1/x$ $dv/dx = 1$, $v = x$ $\Rightarrow$ $\int \ln x dx = x\ln x - \int x.\frac{1}{x}dx$ $= x\ln x - \int 1.dx$ $= x\ln x - x + c$ $\Rightarrow$ $A = \int_1^{2.05}(3\ln x + x - x^2)dx$ $= \left[3x\ln x - 3x + \frac{1}{2}x^2 - \frac{1}{3}x^3\right]_1^{2.05}$ $= -2.5057 + 2.833..$ $= 0.33$ (2 s.f.)	 M1 A1 A1 B1 B1ft M1dep A1 cao [7]	 parts condone no c correct integral and limits (soi) $\left[3 \times \text{their}' x\ln x - x' + \frac{1}{2}x^2 - \frac{1}{3}x^3\right]$ substituting correct limits dep 1st B1	 allow correct result to be quoted (SC3)

9(i) (0, ½)	B1 [1]	allow y = ½ , but not (x =) ½ or (½ ,0) nor P = 1/2	
(ii) $\frac{dy}{dx}=\frac{(1+e^{2x})2e^{2x}-e^{2x}.2e^{2x}}{(1+e^{2x})^2}$ $=\frac{2e^{2x}}{(1+e^{2x})^2}$ When x = 0, dy/dx = $2e^0/(1+e^0)^2$ = ½	M1 A1 A1 B1ft [4]	Quotient or product rule correct expression – condone missing bracket cao – mark final answer follow through their derivative	product rule: $\frac{dy}{dx}=e^{2x}.2e^{2x}(-1)(1+e^{2x})^{-2}+2e^{2x}(1+e^{2x})^{-1}$ $-\frac{2e^{2x}}{(1+e^{2x})^2}$ from $(u\text{d}v - v\text{d}u)/v^2$ SC1
(iii) $A=\int_0^1 \frac{e^{2x}}{1+e^{2x}}dx$ $=\left[\frac{1}{2}\ln(1+e^{2x})\right]_0^1$ *or* let u = 1 + e^{2x}, du/dx = 2 e^{2x} $\Rightarrow$ $A=\int_2^{1+e^2}\frac{1/2}{u}du = \left[\frac{1}{2}\ln u\right]_2^{1+e^2}$ $=\frac{1}{2}\ln(1+e^2)-\frac{1}{2}\ln 2$ $=\frac{1}{2}\ln\left[\frac{1+e^2}{2}\right]$*	B1 M1 A1 M1 A1 M1 E1 [5]	correct integral and limits (soi) $k\ln(1 + e^{2x})$ k = ½ or $v = e^{2x}$, dv/dx = $2e^{2x}$ o.e. [½ ln u] or [½ ln (v + 1)] substituting correct limits www	condone no dx allow missing dx's or incompatible limits, but penalise missing brackets
(iv) $g(-x)=\frac{1}{2}\left[\frac{e^{-x}-e^{x}}{e^{-x}+e^{x}}\right]=-\frac{1}{2}\left[\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}\right]=-g(x)$ Rotational symmetry of order 2 about O	M1 E1 B1 [3]	substituting –x for x in g(x) completion www – taking out –ve must be clear must have 'rotational' 'about O', 'order 2' (oe)	not g(–x) ≠ g(x). Condone use of f for g.
(v)(A) $g(x)+\frac{1}{2}=\frac{1}{2}.\frac{e^x-e^{-x}}{e^x+e^{-x}}+\frac{1}{2}=\frac{1}{2}.(\frac{e^x-e^{-x}+e^x+e^{-x}}{e^x+e^{-x}})$ $=\frac{1}{2}.(\frac{2e^x}{e^x+e^{-x}})$ $=\frac{e^x.e^x}{e^x(e^x+e^{-x})}=\frac{e^{2x}}{e^{2x}+1}=f(x)$ (B) Translation $\begin{pmatrix}0\\1/2\end{pmatrix}$ (C) Rotational symmetry [of order 2]about P	M1 A1 E1 M1 A1 B1 [6]	combining fractions (correctly) translation in y direction up ½ unit dep 'translation' used o.e. condone omission of 180°/order 2	allow 'shift', 'move' in correct direction for M1. $\begin{pmatrix}0\\1/2\end{pmatrix}$ alone is SC1.

OCR
RECOGNISING ACHIEVEMENT

GCE

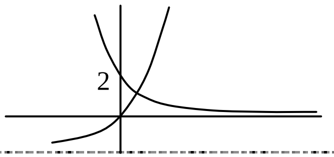
Mathematics (MEI)

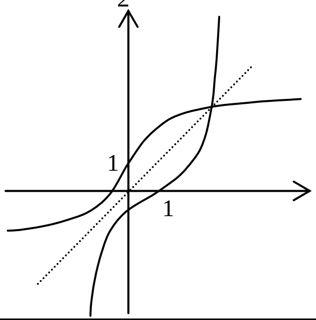
Advanced GCE

Unit **4753:** Methods for Advanced Mathematics

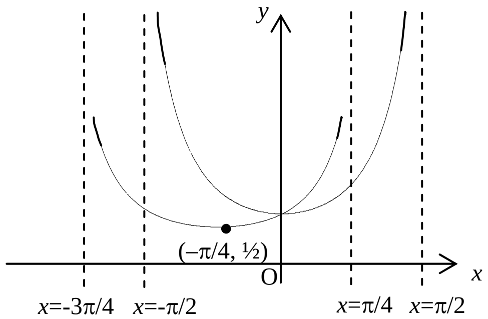
Mark Scheme for January 2011

Oxford Cambridge and RSA Examinations

1 $y=\sqrt[3]{1+x^2}=(1+x^2)^{1/3}$ $\Rightarrow \frac{dy}{dx}=\frac{1}{3}(1+x^2)^{-\frac{2}{3}}.2x$ $=\frac{2}{3}x(1+x^2)^{-\frac{2}{3}}$	M1 M1 B1 A1 [4]	$(1+x^2)^{1/3}$ chain rule $(1/3)\ u^{-2/3}$ (soi) cao, mark final answer	Do not allow MR for square root their $dy/du \times du/dx$ (available for wrong indices) no ft on ½ index oe e.g. $\frac{2x(1+x^2)^{-\frac{2}{3}}}{3}$, $\frac{2x}{3\sqrt[3]{(1+x^2)^2}}$, etc but must combine 2 with 1/3.
2 $\lvert 2x+1\rvert \geq 4$ $\Rightarrow$ $2x+1\geq 4 \Rightarrow x\geq 1½$ or $2x+1\leq -4 \Rightarrow x\leq -2½$	 M1 A1 M1 A1 [4]	 allow M1 for 1½ seen allow M1 for –2½ seen	Same scheme for other methods, e.g. squaring, graphing Penalise both > and < once only. –1 if both correct but final ans expressed incorrectly, e.g $-2½ \geq x \geq 1½$ or $1½ \leq x \leq -2½$ (or even $-2½ \leq x \leq 1½$ from previously correct work) e.g. SC3
3 $A=\pi r^2$ $\Rightarrow$ $dA/dr = 2\pi r$ When $r=2$, $dA/dr=4\pi$, $dA/dt=1$ $\frac{dA}{dt}=\frac{dA}{dr}.\frac{dr}{dt}$ $\Rightarrow$ $1=4\pi.dr/dt$ $\Rightarrow$ $dr/dt=1/4\pi=0.0796$ (mm/s)	 M1A1 A1 M1 A1 [5]	 $2\pi r$ soi (at any stage) chain rule (o.e) cao: 0.08 or better condone truncation	 M1A0 if incorrect notation, e.g. dy/dx, dr/dA, if seen. $2r$ is M1A0 must be dA/dr (soi) and dA/dt any correct form stated with relevant variables , e.g. $\frac{dr}{dt}=\frac{dr}{dA}.\frac{dA}{dt}$, $\frac{dr}{dt}=\frac{dr}{dA}/\frac{dt}{dA}$, etc. allow $1/4\pi$ but mark final answer
4 $\sin\theta = BC/AC$, $\cos\theta = AB/AC$ $AB^2+BC^2=AC^2$ $\Rightarrow$ $(AB/AC)^2+(BC/AC)^2=1$ $\Rightarrow$ $\cos^2\theta+\sin^2\theta=1$ Valid for (0° <) θ < 90°	M1 A1 B1 [3]	or a/b, c/b condone taking AC = 1 Must use Pythagoras allow ≤, or 'between 0 and 90' or < 90 allow < π/2 or 'acute'	allow o/h, a/h etc if clearly marked on triangle. but must be stated arguing backwards unless ⇔ used A0
5(i) 	B1 B1 B1 [3]	shape of $y=e^x-1$ and through O shape of $y=2e^{-x}$ through (0, 2) (not (2,0))	for first and second B1s graphs must include negative x values condone no asymptote $y=-1$ shown asymptotic to x-axis (shouldn't cross)
(ii) $e^x-1=2e^{-x}$ $\Rightarrow$ $e^{2x}-e^x=2$ $\Rightarrow$ $(e^x)^2-e^x-2=0$ $\Rightarrow$ $(e^x-2)(e^x+1)=0$ $\Rightarrow$ $e^x=2$ (or –1) $\Rightarrow$ $x=\ln 2$ $\Rightarrow$ $y=1$	M1 M1 B1 B1 B1cao [5]	equating re-arranging into a quadratic in $e^x=0$ stated www www www	 allow one error but must have $e^{2x}=(e^x)^2$ (soi) award even if not from quadratic method (i.e. by 'fitting') provided www allow for unsupported answers, provided www need not have used a quadratic, provided www

6 $(x+y)^2 = 4x$ $\Rightarrow$ $2(x+y)(1+\frac{dy}{dx}) = 4$ $\Rightarrow$ $1+\frac{dy}{dx} = \frac{4}{2(x+y)} = \frac{2}{x+y}$ $\Rightarrow$ $\frac{dy}{dx} = \frac{2}{x+y} - 1$ *	M1 A1 A1	Implicit differentiation of LHS correct expression = 4 www (**AG**)	Award no marks for solving for y and attempting to differentiate allow one error but must include dy/dx ignore superfluous dy/dx = … for M1, and for both A1s if not pursued condone missing brackets A0 if missing brackets in earlier working
or $x^2 + 2xy + y^2 = 4x$ $\Rightarrow$ $2x + 2x\frac{dy}{dx} + 2y + 2y\frac{dy}{dx} = 4$ $\Rightarrow$ $\frac{dy}{dx}(2x+2y) = 4 - 2x - 2y$ $\Rightarrow$ $\frac{dy}{dx} = \frac{4}{2x+2y} - 1 = \frac{2}{x+y} - 1$ *	M1dep A1 A1	Implicit differentiation of LHS dep correct expansion correct expression = 4 (oe after re-arrangement) www (**AG**)	allow 1 error provided 2xdy/dx and 2ydy/dx are correct, but must expand $(x + y)^2$ correctly for M1 (so $x^2 + y^2 = 4x$ is M0) ignore superfluous dy/dx = … for M1, and for both A1s if not pursued A0 if missing brackets in earlier working
When $x = 1$, $y = 1$, $\frac{dy}{dx} = \frac{2}{1+1} - 1 = 0$ *	B1 [4]	(**AG**) oe (e.g. from $x + y = 2$)	or e.g $2/(x + y) - 1 = 0 \Rightarrow x + y = 2, \Rightarrow 4 = 4x, \Rightarrow x = 1, y = 1$ (oe)
7 (i) bounds $-\pi + 1$, $\pi + 1$ $\Rightarrow$ $-\pi + 1 < f(x) < \pi + 1$	B1B1 B1cao [3]	or … < y < … or $(-\pi + 1, \pi + 1)$	not … < x < …, not 'between …'
(ii) $y = 2\arctan x + 1$ $x \leftrightarrow y$ $x = 2\arctan y + 1$ $\Rightarrow$ $\frac{x-1}{2} = \arctan y$ $\Rightarrow$ $y = \tan(\frac{x-1}{2}) \Rightarrow f^{-1}(x) = \tan(\frac{x-1}{2})$ 	M1 A1 A1 B1 B1 [5]	attempt to invert formula or $\frac{y-1}{2} = \arctan x$ reasonable reflection in $y = x$ (1, 0) intercept indicated.	one step is enough, i.e. $y - 1 = 2\arctan x$ or $x - 1 = 2\arctan y$ need not have interchanged x and y at this stage allow $y = …$ curves must cross on $y = x$ line if present (or close enough to imply intention) curves shouldn't touch or cross in the third quadrant

8(i) $\int_0^1 \frac{x^3}{1+x}\,dx$ let $u = 1+x,\ du = dx$ when $x = 0$, $u = 1$, when $x = 1$, $u = 2$ $= \int_1^2 \frac{(u-1)^3}{u}\,du$ $= \int_1^2 \frac{(u^3 - 3u^2 + 3u - 1)}{u}\,du$ $= \int_1^2 (u^2 - 3u + 3 - \frac{1}{u})\,du$ * $\int_0^1 \frac{x^3}{1+x}\,dx = \left[\frac{1}{3}u^3 - \frac{3}{2}u^2 + 3u - \ln u\right]_1^2$ $= (\frac{8}{3} - 6 + 6 - \ln 2) - (\frac{1}{3} - \frac{3}{2} + 3 - \ln 1)$ $= \frac{5}{6} - \ln 2$	B1 B1 M1 A1dep B1 M1 A1cao [7]	$a = 1$, $b = 2$ $(u-1)^3/u$ expanding (correctly) dep du = dx (o.e.) **AG** $\left[\frac{1}{3}u^3 - \frac{3}{2}u^2 + 3u - \ln u\right]$ substituting correct limits dep integrated must be exact – must be 5/6	seen anywhere, e.g. in new limits e.g. du/dx = 1, condone missing dx's and du's, allow du = 1 upper – lower; may be implied from 0.140… must have evaluated ln 1 = 0
(ii) $y = x^2 \ln(1 + x)$ $\Rightarrow \frac{dy}{dx} = x^2 . \frac{1}{1+x} + 2x.\ln(1+x)$ $= \frac{x^2}{1+x} + 2x\ln(1+x)$ When $x = 0$, dy/dx = 0 + 0.ln 1 = 0 (⇒ Origin is a stationary point)	M1 B1 A1 M1 A1cao [5]	Product rule d/dx (ln(1 + x)) = 1/(1 + x) cao (oe) mark final ans substituting x = 0 into **correct** deriv www	 or d/dx (ln u) = 1/u where u = 1 + x ln1+x is A0 when x = 0, dy/dx = 0 with no evidence of substituting M1A0 but condone missing bracket in ln(1+x)
(iii) $A = \int_0^1 x^2 \ln(1+x)\,dx$ let $u = \ln(1+x),\ dv/dx = x^2$ $\frac{du}{dx} = \frac{1}{1+x},\ v = \frac{1}{3}x^3$ $\Rightarrow A = \left[\frac{1}{3}x^3 \ln(1+x)\right]_0^1 - \int_0^1 \frac{1}{3}\frac{x^3}{1+x}\,dx$ $= \frac{1}{3}\ln 2 - (\frac{5}{18} - \frac{1}{3}\ln 2)$ $= \frac{1}{3}\ln 2 - \frac{5}{18} + \frac{1}{3}\ln 2$ $= \frac{2}{3}\ln 2 - \frac{5}{18}$	B1 M1 A1 B1 B1ft A1 [6]	Correct integral and limits parts correct $= \frac{1}{3}\ln 2 - ...$ … – 1/3 (result from part (i)) cao	condone no dx, limits (and integral) can be implied by subsequent work u, du/dx, dv/dx and v all correct (oe) condone missing brackets condone missing bracket, can re-work from scratch oe e.g. $= \frac{12\ln 2 - 5}{18}, \frac{1}{3}\ln 4 - \frac{5}{18}$, etc but must have evaluated ln 1 =0 Must combine the two ln terms

9(i) $\frac{d}{dx}(\frac{\sin x}{\cos x}) = \frac{\cos x.\cos x - \sin x.(-\sin x)}{\cos^2 x}$ $= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$ *	M1 A1 A1 [3]	Quotient (or product) rule (**AG**)	product rule: $\frac{1}{\cos x}.\cos x + \sin x(-\frac{1}{\cos^2 x})(-\sin x)$ but must show evidence of using chain rule on 1/cos *x* (or d/d*x* (sec *x*) = sec *x* tan *x* used)
(ii) Area = $\int_0^{\pi/4} \frac{1}{\cos^2 x} dx$ $= [\tan x]_0^{\pi/4}$ = tan(π/4) – tan 0 = 1	B1 M1 A1 [3]	correct integral and limits (soi) $[\tan x]$ or $\left[\frac{\sin x}{\cos x}\right]$	condone no d*x*; limits can be implied from subsequent work unsupported scores M0
(iii) f(0) = 1/cos²(0) = 1 g(*x*) = 1/2cos²(*x* + π/4) g(0) = 1/2cos²(π/4) = 1 (⇒ f and g meet at (0, 1))	B1 M1 A1 [3]	must show evidence	or f(π/4) = 1/cos²(π/4) = 2 so g(0) = ½ f(π/4) = 1
(iv) Translation in *x*-direction through –π/4 Stretch in *y*-direction scale factor ½  	M1 A1 M1 A1 B1ft B1ft B1 B1dep [8]	must be in *x*-direction, or $\begin{pmatrix}-\pi/4\\0\end{pmatrix}$ must be in *y*-direction asymptotes correct min point (–π/4, ½) curves intersect on *y*-axis correct curve, dep B3, with asymptote lines indicated and correct, and TP in correct position	'shift' or 'move' for 'translation' M1 A0; $\begin{pmatrix}-\pi/4\\0\end{pmatrix}$ alone SC1 'contract' or 'compress' or 'squeeze' for 'stretch' M1A0; 'enlarge' M0 stated or on graph; condone no *x* = …, ft π/4 to right only (viz. –π/4, 3π/4) stated or on graph; ft π/4 to right only (viz. (π/4, ½)) '*y*-values halved', or '*x*-values reduced by π/4, are M0 (not geometric transformations), but for M1 condone mention of *x*- and *y*- values provided transformation words are used.
(v) Same as area in (ii), but stretched by s.f. ½ . So area = ½ .	B1ft [1]	½ area in (ii)	or $\int_{-\pi/4}^{0} g(x)\,dx = \frac{1}{2}\int_{-\pi/4}^{0} \frac{1}{\cos^2(x+\pi/4)}dx = \frac{1}{2}[\tan(x+\pi/4)]_{-\pi/4}^{0}$ = ½ allow unsupported

GCE

Mathematics (MEI)

Advanced GCE

Unit **4753:** Methods for Advanced Mathematics

Mark Scheme for June 2011

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Marking instructions for GCE Mathematics (MEI): Pure strand

1. You are advised to work through the paper yourself first. Ensure you familiarise yourself with the mark scheme before you tackle the practice scripts.

2. You will be required to mark ten practice scripts. This will help you to understand the mark scheme and will not be used to assess the quality of your marking. Mark the scripts yourself first, using the annotations. Turn on the comments box and make sure you understand the comments. You must also look at the definitive marks to check your marking. If you are unsure why the marks for the practice scripts have been awarded in the way they have, please contact your Team Leader.

3. When you are confident with the mark scheme, mark the ten standardisation scripts. Your Team Leader will give you feedback on these scripts and approve you for marking. (If your marking is not of an acceptable standard your Team Leader will give you advice and you will be required to do further work. You will only be approved for marking if your Team Leader is confident that you will be able to mark candidate scripts to an acceptable standard.)

4. Mark strictly to the mark scheme. If in doubt, consult your Team Leader using the messaging system within *scoris*, by email or by telephone. Your Team Leader will be monitoring your marking and giving you feedback throughout the marking period.

   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

   Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

5. The following types of marks are available.

   M
   A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

   A
   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

   B
   Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

6. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

7. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

   Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

8. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

9. **Rules for crossed out and/or replaced work**

   If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.

   If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

   If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

   If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

   NB Follow these maths-specific instructions rather than those in the assessor handbook.

10. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

    Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

11. Annotations should be used whenever appropriate during your marking.

    The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

    For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

12. For answers scoring no marks, you must either award NR (no response) or 0, as follows:

    Award NR (no response) if:
    - Nothing is written at all in the answer space
    - There is a comment which does not in any way relate to the question being asked ("can't do", "don't know", etc.)
    - There is any sort of mark that is not an attempt at the question (a dash, a question mark, etc.)

    The hash key [#] on your keyboard will enter NR.

    Award 0 if:
    - There is an attempt that earns no credit. This could, for example, include the candidate copying all or some of the question, or any working that does not earn any marks, whether crossed out or not.

13. The following abbreviations may be used in this mark scheme.

M1	method mark (M2, etc, is also used)
A1	accuracy mark
B1	independent mark
E1	mark for explaining
U1	mark for correct units
G1	mark for a correct feature on a graph
M1 dep*	method mark dependent on a previous mark, indicated by *
cao	correct answer only
ft	follow through
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
sc	special case
soi	seen or implied
www	without wrong working

14. Annotating scripts. The following annotations are available:

✓and ✗
BOD Benefit of doubt
FT Follow through
ISW Ignore subsequent working (after correct answer obtained)
M0, M1 Method mark awarded 0, 1
A0, A1 Accuracy mark awarded 0, 1
B0, B1 Independent mark awarded 0,1
SC Special case
^ Omission sign
MR Misread
Highlighting is also available to highlight any particular points on a script.

15. The comments box will be used by the Principal Examiner to explain his or her marking of the practice scripts for your information. Please refer to these comments when checking your practice scripts.

    Please do not type in the comments box yourself. Any questions or comments you have for your Team Leader should be communicated by the *scoris* messaging system, e-mail or by telephone.

16. Write a brief report on the performance of the candidates. Your Team Leader will tell you when this is required. The Assistant Examiner's Report Form (AERF) can be found on the Cambridge Assessment Support Portal. This should contain notes on particular strengths displayed, as well as common errors or weaknesses. Constructive criticisms of the question paper/mark scheme are also appreciated.

17. Link Additional Objects with work relating to a question to those questions (a chain link appears by the relevant question number) – see scoris assessor Quick Reference Guide page 19-20 for instructions as to how to do this – this guide is on the Cambridge Assessment Support Portal and new users may like to download it with a shortcut on your desktop so you can open it easily! For AOs containing just formulae or rough working not attributed to a question, tick at the top to indicate seen but not linked. When you submit the script, *scoris* asks you to confirm that you have looked at all the additional objects. Please ensure that you have checked all Additional Objects thoroughly.

18. The schedule of dates for the marking of this paper is displayed under 'OCR Subject Specific Details' on the Cambridge Assessment Support Portal. It is vitally important that you meet these requirements. If you experience problems that mean you may not be able to meet the deadline then you must contact your Team Leader without delay.

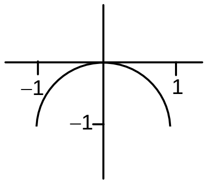
1 $\|2x-1\|=\|x\|$ $\Rightarrow$ $2x - 1 = x$, $x = 1$ or $-(2x-1) = x$, $x = 1/3$	M1A1 M1A1 [4]	www www, or $2x - 1 = -x$ must be exact for A1 (e.g. not 0.33, but allow $0.\dot{3}$) condone doing both equalities in one line e.g. $-x = 2x - 1 = x$, etc	allow unsupported answers or from graph or squaring $\Rightarrow 3x^2 - 4x + 1 = 0$ M1 $\Rightarrow (3x-1)(x-1) = 0$ M1 factorising, formula or comp. square $\Rightarrow x = 1, 1/3$ A1 A1 allow M1 for sign errors in factorisation –1 if more than two solutions offered, but isw inequalities
2 $gf(x) = e^{2\ln x}$ $= e^{\ln x^2}$ $= x^2$	M1 M1 A1 [3]	Forming $gf(x)$ (soi)	Doing fg: $2\ln(e^x) = 2x$ SC1 Allow x^2 (but not $2x$) unsupported
3(i) $\frac{dy}{dx} = \frac{x^2.\frac{1}{x} - \ln x.2x}{x^4}$ $= \frac{x - 2x\ln x}{x^4}$ $= \frac{1-2\ln x}{x^3}$	M1 B1 A1 A1 [4]	quotient rule with $u = \ln x$ and $v = x^2$ $d/dx(\ln x) = 1/x$ soi correct expression (o.e.) o.e. cao, mark final answer, but must have divided top and bottom by x	Consistent with their derivatives. $u\mathrm{d}v \pm v\mathrm{d}u$ in the quotient rule is M0 Condone $\ln x.2x = \ln 2x^2$ for this A1 (provided $\ln x.2x$ is shown) e.g. $\frac{1}{x^3} - \frac{2\ln x}{x^3}$, $x^{-3} - 2x^{-3}\ln x$
or $\frac{dy}{dx} = -2x^{-3}\ln x + x^{-2}(\frac{1}{x})$ $= -2x^{-3}\ln x + x^{-3}$	M1 B1 A1 A1 [4]	product rule with $u = x^{-2}$ and $v = \ln x$ $d/dx(\ln x) = 1/x$ soi correct expression o.e. cao, mark final answer, must simplify the $x^{-2}.(1/x)$ term.	or vice-versa
(ii) $\int \frac{\ln x}{x^2}dx$ let $u = \ln x$, $du/dx = 1/x$ $dv/dx = 1/x^2$, $v = -x^{-1}$ $= -\frac{1}{x}\ln x + \int \frac{1}{x}.\frac{1}{x}dx$ $= -\frac{1}{x}\ln x + \int \frac{1}{x^2}dx$ $= -\frac{1}{x}\ln x - \frac{1}{x} + c$ $= -\frac{1}{x}(\ln x + 1) + c$ *	M1 A1 A1 A1 [4]	Integration by parts with $u = \ln x$, $du/dx = 1/x$, $dv/dx = 1/x^2$, $v = -x^{-1}$ must be correct, condone + c condone missing c NB **AG** must have c shown in final answer	Must be correct at this stage . Need to see $1/x^2$

4(i) $h = a - be^{-kt} \Rightarrow a = 10.5$ (their)$a - be^0 = 0.5$ $\Rightarrow b = 10$	B1 M1 A1cao [3]	a need not be substituted	
(ii) $h = 10.5 - 10e^{-kt}$ When $t = 8$, $h = 10.5 - 10e^{-8k} = 6$ $\Rightarrow$ $10e^{-8k} = 4.5$ $\Rightarrow$ $-8k = \ln 0.45$ $\Rightarrow$ $k = \ln 0.45/(-8) = 0.09981\ldots = 0.10$	M1 M1 A1 [3]	ft their a and b (even if made up) taking lns correctly on a correct re-arrangement - ft a, b if not eased cao (www) but allow 0.1	allow M1 for $a - be^{-8k} = 6$ allow a and b unsubstituted allow their 0.45 (or 4.5) to be negative
5 $y = x^2(1 + 4x)^{1/2}$ $\Rightarrow$ $\frac{dy}{dx} = x^2.\frac{1}{2}(1+4x)^{-1/2}.4 + 2x(1+4x)^{1/2}$ $= 2x(1+4x)^{-1/2}(x+1+4x)$ $= \frac{2x(5x+1)}{\sqrt{1+4x}}$ *	M1 B1 A1 M1 A1 [5]	product rule with $u = x^2$, $v = \sqrt{(1 + 4x)}$ ½ $(\ldots)^{-1/2}$ soi correct expression factorising or combining fractions NB **AG**	consistent with their derivatives; condone wrong index in v used for M1 only (need not factor out the $2x$) must have evidence of $x + 1 + 4x$ oe or $2x(5x + 1)(1 + 4x)^{-½}$ or $2x(5x + 1)/(1 + 4x)^{½}$
6(i) $\sin(\pi/3) + \cos(\pi/6) = \sqrt{3}/2 + \sqrt{3}/2 = \sqrt{3}$	B1 [1]	must be exact, must show working	Not just $\sin(\pi/3) + \cos(\pi/6) = \sqrt{3}$, if substituting for y and solving for x (or vv) must evaluate $\sin \pi/3$ e.g. not $\arccos(\sqrt{3} - \sin \pi/3)$
(ii) $2\cos 2x - \sin y\frac{dy}{dx} = 0$ $\Rightarrow$ $2\cos 2x = \sin y\frac{dy}{dx}$ $\Rightarrow$ $\frac{dy}{dx} = \frac{2\cos 2x}{\sin y}$ When $x = \pi/6$, $y = \pi/6$ $\Rightarrow$ $\frac{dy}{dx} = \frac{2\cos\pi/3}{\sin\pi/6} = 2$	M1 A1 A1cao M1dep A1 [5]	Implicit differentiation correct expression substituting dep 1st M1 www	allow one error, but must have (±) $\sin y\, dy/dx$. Ignore $dy/dx = \ldots$ unless pursued. $2\cos 2x\, dx - \sin y\, dy = 0$ is M1A1 (could differentiate wrt y, get dx/dy, etc.) $\frac{-2\cos 2x}{-\sin y}$ is A0 or 30°
7 (i) $(3^n + 1)(3^n - 1) = (3^n)^2 - 1$ or $3^{2n} - 1$	B1 [1]	mark final answer	or $9^n - 1$; penalise 3^{n^2} if it looks like 3 to the power n^2.
(ii) 3^n is odd $\Rightarrow 3^n + 1$ and $3^n - 1$ both even As consecutive even nos, one must be divisible by 4, so product is divisible by 8.	M1 M1 A1 [3]	3^n is odd $\Rightarrow 3^n + 1$ and $3^n - 1$ both even completion	Induction: If true for n, $3^{2n} - 1 = 8k$, so $3^{2n} = 1 + 8k$, M1 $3^{2(n+1)} - 1 = 9\times(8k + 1) - 1 = 72k + 8 = 8(9k + 1)$ so div by 8. A1 When $n = 1$, $3^2 - 1 = 8$ div by 8, true A1(or similar with 9^n)

8(i) $f(-x)=\frac{1}{e^{-x}+e^{-(-x)}+2}$ $= f(x)$, [$\Rightarrow$ f is even *] Symmetrical about O*y*	M1 A1 B1 [3]	substituting $-x$ for x in $f(x)$ condone 'reflection in *y*-axis'	Can imply that $e^{-(-x)} = e^x$ from $f(-x)=\frac{1}{e^{-x}+e^x+2}$ Must mention axis
(ii) $f'(x)=-(e^x+e^{-x}+2)^{-2}(e^x-e^{-x})$ or $= \frac{(e^x+e^{-x}+2).0-(e^x-e^{-x})}{(e^x+e^{-x}+2)^2}$ $=\frac{(e^{-x}-e^x)}{(e^x+e^{-x}+2)^2}$	B1 M1 A1 [3]	d/d*x* (e^x) = e^x and d/d*x*(e^{-x}) = $-e^{-x}$ soi chain or quotient rule condone missing bracket on top if correct thereafter o.e. mark final answer	. If differentiating $\frac{e^x}{(e^x+1)^2}$ withhold A1 (unless result in (iii) proved here) e.g. $\frac{1}{(e^x+e^{-x}+2)^2}\times(e^{-x}-e^x)$
(iii) $f(x)=\frac{e^x}{e^{2x}+1+2e^x}$ $=\frac{e^x}{(e^x+1)^2}$ *	M1 A1 [2]	× top and bottom by e^x (correctly) condone e^{x^2} for M1 but not A1 **NB AG**	or $\frac{e^x}{(e^x+1)^2}=\frac{e^x}{e^{2x}+2e^x+1}$ M1, $=\frac{1}{e^x+e^{-x}+2}$ A1 condone no $e^{2x} = (e^x)^2$,for both M1 and A1
(iv) $A=\int_0^1 \frac{e^x}{(e^x+1)^2}dx$ let $u = e^x + 1$, $du = e^x\, dx$ when $x = 0$, $u = 2$; when $x = 1$, $u = e + 1$ $\Rightarrow A=\int_2^{1+e}\frac{1}{u^2}du$ $=\left[-\frac{1}{u}\right]_2^{1+e}$ $=-\frac{1}{1+e}+\frac{1}{2}=\frac{1}{2}-\frac{1}{1+e}$	B1 M1 A1 M1 A1cao [5]	correct integral and limits $\int \frac{1}{u^2}(du)$ $\left[-\frac{1}{u}\right]$ substituting correct limits (dep 1st M1 and integration) o.e. mark final answer. Must be exact Don't allow e^1.	condone no d*x*, must use $f(x) = \frac{e^x}{(e^x+1)^2}$. Limits may be implied by subsequent work. If 0.231.. unsupported, allow 1st B1 only or by inspection $\left[\frac{k}{e^x+1}\right]$ M1 $\left[-\frac{1}{e^x+1}\right]$ A1 upper–lower; 2 and 1+e (or 3.7..)for *u*, or 0 and 1 for *x* if substituted back (correctly) e.g. $\frac{e-1}{2(1+e)}$. Can isw 0.231, which may be used as evidence of M1. Can isw numerical ans (e.g. 0.231) but not algebraic errors
(v) Curves intersect when $f(x)=\frac{1}{4}e^x$ $\Rightarrow$ $(e^x + 1)^2 = 4$ $\Rightarrow$ $e^x = 1$ or -3 so as $e^x > 0$, only one solution $e^x = 1 \Rightarrow x = 0$ when $x = 0$, $y = ¼$	M1 M1 A1 B1 B1 [5]	soi or equivalent quadratic – must be correct getting $e^x = 1$ and discounting other soln $x = 0$ www (for this value) $y = ¼$ www (for the x value)	$\frac{e^x}{(e^x+1)^2}$ *or* $\frac{1}{e^x+e^{-x}+2}=\frac{1}{4}e^x$ With e^{2x} or $(e^x)^2$, condone e^{x^2}, e^0 www e.g. $e^x = -1$ [or $e^x + 1 = -2$] not possible www unless verified Do not allow unsupported. A sketch is not sufficient

9(i) When $x = 0$, $f(x) = a = 2$* When $x = \pi$, $f(\pi) = 2 + \sin b\pi = 3$ $\Rightarrow$ $\sin b\pi = 1$ $\Rightarrow$ $b\pi = ½\pi$, so $b = ½$ * *or* $1 = a + \sin(-\pi b)$ $(= a - \sin \pi b)$ $3 = a + \sin(\pi b)$ $\Rightarrow$ $2 = 2 \sin \pi b$, $\sin \pi b = 1$, $\pi b = \pi/2$, $b = ½$ $\Rightarrow$ $3 = a + 1$ or $1 = a - 1 \Rightarrow a = 2$ (oe for b)	B1 M1 A1 [3]	NB **AG** 'a is the y-intercept' not enough but allow verification (2+sin 0 = 2) or when $x = -\pi$, $f(-\pi) = 2 + \sin(-b\pi) = 1$ $\Rightarrow \sin(-b\pi) = -1$ condone using degrees $\Rightarrow -b\pi = -½\pi$, $b = ½$ **NB AG** M1 for both points substituted A1 solving for b or a A1 substituting to get a (or b)	or equiv transformation arguments : e.g. 'curve is shifted up 2 so $a = 2$'. e.g. period of sine curve is 4π, or stretched by sf. 2 in x-direction (not squeezed or squashed by ½) $\Rightarrow b = ½$ If verified allow M1A0 If $y = 2 + \sin ½x$ verified at two points, SC2 A sequence of sketches starting from $y = \sin x$ showing clearly the translation and the stretch (in either order) can earn full marks
(ii) $f'(x) = ½ \cos ½x$ $\Rightarrow$ $f'(0) = ½$ Maximum value of $\cos ½x$ is 1 $\Rightarrow$ max value of gradient is ½	M1 A1 A1 M1 A1 [5]	$\pm k \cos ½x$ cao www or $f''(x) = -¼ \sin ½x$ $f''(x) = 0 \Rightarrow x = 0$, so max val of $f'(x)$ is ½	
(iii) $y = 2 + \sin ½x$ $x \leftrightarrow y$ $x = 2 + \sin ½y$ $\Rightarrow$ $x - 2 = \sin ½y$ $\Rightarrow$ $\arcsin(x - 2) = ½y$ $\Rightarrow$ $y = f^{-1}(x) = 2\arcsin(x - 2)$ Domain $1 \le x \le 3$ Range $-\pi \le y \le \pi$ Gradient at (2, 0) is 2	M1 A1 A1 B1 B1 B1ft [6]	Attempt to invert formula or $\arcsin(y - 2) = ½x$ must be $y = \ldots$ or $f^{-1}(x) = \ldots$ or [1, 3] or $[-\pi, \pi]$ or $-\pi \le f^{-1}(x) \le \pi$ ft their answer in (ii) (except ±1) 1/their ½	viz solve for x in terms of y or vice-versa – one step enough condone use of a and b in inverse function, e.g. $[\arcsin(x - a)]/b$ or $\sin^{-1}(y - 2)$ condone no bracket for 1st A1 only or $2\sin^{-1}(x - 2)$, condone $f'(x)$, must have bracket in final ans but not $1 \le y \le 3$ but not $-\pi \le x \le \pi$. Penalise <'s (or '1 to 3','$-\pi$ to π') once only or by differentiating $\arcsin(x - 2)$ or implicitly
(iv) $A = \int_0^{\pi} (2 + \sin \frac{1}{2}x)\,dx$ $= \left[2x - 2\cos\frac{1}{2}x\right]_0^{\pi}$ $= 2\pi - (-2)$ $= 2\pi + 2$ $(= 8.2831\ldots)$	M1 M1 A1 A1cao [4]	correct integral and limits $\left[2x - k\cos\frac{1}{2}x\right]$ where k is positive $k = 2$ answers rounding to 8.3	soi from subsequent work, condone no dx but not 180 Unsupported correct answers score 1st M1 only.

Question			Answer	Marks	Guidance	
1			$\int_1^2 \frac{1}{\sqrt{3x-2}}\,dx = \left[\frac{2}{3}(3x-2)^{1/2}\right]_1^2$	M1 A2	$[k\,(3x-2)^{1/2}]$ $k = 2/3$	
			$= \frac{2}{3}.2 - \frac{2}{3}.1$ $= 2/3$*	M1dep A1	substituting limits dep 1st M1 NB **AG**	
			OR $u = 3x-2 \Rightarrow du/dx = 3$ $\Rightarrow \int_1^2 \frac{1}{\sqrt{3x-2}}\,dx = \int_1^4 \frac{1}{\sqrt{u}}.\frac{1}{3}\,du$	M1 A1	$\int \frac{1}{\sqrt{u}}$ $\times\ 1/3\ (du)$	or $w^2 = 3x-2 \Rightarrow \int \frac{1}{w}$ $\times\ 2/3\ w\ (dw)$
			$= \left[\frac{2}{3}u^{1/2}\right]_1^4 = \frac{2}{3}.2 - \frac{2}{3}.1$	A1	$\left[\frac{2}{3}u^{1/2}\right]$ o.e.	$\left[\frac{2}{3}w\right]$
			$= 2/3$*	M1dep A1 **[5]**	substituting correct limits dep 1st M1 NB **AG**	upper – lower, 1 to 4 for u or 1 to 2 for w or substituting back (correctly) for x and using 1 to 2
2			$\lvert 2x+1 \rvert > 4$ $\Rightarrow\ 2x+1 > 4,\ x > 3/2$ or $2x+1 < -4,$ $x < -2\tfrac{1}{2}$	B1 M1 A1 **[3]**	$x > 3/2$ mark final ans; if from $\lvert 2x \rvert > 3$ B0 o.e. , e.g. $-(2x+1) > 4$ (or $2x+1 = -4$) if $\lvert 2x+1 \rvert < -4$, M0 $x < -2\tfrac{1}{2}$ mark final ans allow 3 for correct unsupported answers	by squaring: $4x^2 + 4x - 15 >$ (or =) 0 M1 $x > 3/2$ A1 $x < -2\tfrac{1}{2}$ A1 Penalise $\geq$ and $\leq$ once only $3/2 < x < -2\tfrac{1}{2}$ SC2 (mark final ans)
3			$e^{2y} = 5 - e^{-x}$ $\Rightarrow\ 2e^{2y}\frac{dy}{dx} = e^{-x}$ $\Rightarrow\ \frac{dy}{dx} = \frac{e^{-x}}{2e^{2y}}$	B1 B1	$2e^{2y}\frac{dy}{dx} = \ldots$ $= e^{-x}$	or $y = \ln\sqrt{(5 - e^{-x})}$ o.e (e.g. $\tfrac{1}{2}\ln(5-e^{-x})$) B1 $\Rightarrow dy/dx = e^{-x}/[2(5-e^{-x})]$ o.e. B1 (but must be correct)
			At (0, ln2) $\frac{dy}{dx} = \frac{e^0}{2e^{2\ln 2}}$	M1dep	substituting $x = 0$, $y = \ln 2$ into their dy/dx dep 1st B1 – allow one slip	or substituting $x = 0$ into their correct dy/dx
			$= \frac{1}{8}$	A1cao **[4]**		

Question			Answer	Marks	Guidance	
4	(i)		$1 - 9a^2 = 0$ $\Rightarrow a^2 = 1/9 \Rightarrow a = 1/3$	M1 A1 **[2]**	or $1 - 9x^2 = 0$ or 0.33 or better $\sqrt{(1/9)}$ is A0	$\sqrt{(1 - 9a^2)} = 1 - 3a$ is M0 not $a = \pm 1/3$ nor $x = 1/3$
4	(ii)		Range $0 \le y \le 1$	B1 **[1]**	or $0 \le f(x) \le 1$ or $0 \le f \le 1$, not $0 \le x \le 1$ $0 \le y \le \sqrt{1}$ is B0	allow also [0,1], or 0 to 1 inclusive, but not 0 to 1 or (0,1)
4	(iii)			M1 M1 A1 **[3]**	curve goes from $x = -3a$ to $x = 3a$ (or −1 to 1) vertex at origin curve, 'centre' (0,−1), from (−1, −1) to (1, −1) (y-coords of −1 can be inferred from vertex at O and correct scaling)	must have evidence of using s.f. 3 allow also if s.f.3 is stated and stretch is reasonably to scale allow from $(-3a, -1)$ to $(3a, -1)$ provided $a = 1/3$ or $x = [\pm]\ 1/3$ in (i) A0 for badly inconsistent scale(s)
5	(i)		When $t = 0$, $P = 7 - 2 = 5$, so 5 (million) In the long term $e^{-kt} \to 0$ So long-term population is 7 (million)	B1 M1 A1 **[3]**	allow substituting a large number for t (for both marks)	allow 7 unsupported
5	(ii)		$P = 7 - 2e^{-kt}$ When $t = 1$, $P = 5.5$ $\Rightarrow\ 5.5 = 7 - 2e^{-k}$ $\Rightarrow\ e^{-k} = (7 - 5.5)/2 = 0.75$ $\Rightarrow\ -k = \ln((7 - 5.5)/2)$ $\Rightarrow\ k = 0.288$ (3 s.f.)	M1 M1 A1 **[3]**	re-arranging and anti-logging – allow 1 slip (e.g. arith of 7 – 5.5, or k for $-k$) or $\ln 2 - k = \ln 1.5$ o.e. 0.3 or better allow $\ln(4/3)$ or $-\ln(3/4)$ if final ans	but penalise negative lns, e.g. $\ln(-1.5) = \ln(-2) - k$ rounding from a correct value of $k = 0.2876820725\ldots$, penalise truncation, and incorrect work with negatives

Question			Answer	Marks	Guidance	
6	(i)		$y = 2 \text{ arc sin } ½ = 2 \times \pi/6$ $= \pi/3$	M1 A1 **[2]**	$y = 2 \arcsin ½$ must be in terms of π − can isw approximate answers	1.047… implies M1
6	(ii)		$y = 2 \arcsin x$ $\quad x \leftrightarrow y$ $\Rightarrow x = 2 \arcsin y$ $\Rightarrow x/2 = \arcsin y$ $\Rightarrow y = \sin(x/2)$ [so $g(x) = \sin(x/2)$] $\Rightarrow dy/dx = ½ \cos(½ x)$ At Q, $x = \pi/3$ $\Rightarrow dy/dx = ½ \cos \pi/6 = ½ \sqrt{3}/2 = \sqrt{3}/4$ $\Rightarrow$ gradient at P $= 4/\sqrt{3}$	M1 A1 A1cao M1 A1 B1 ft **[6]**	or $y/2 = \arcsin x$ but must interchange x and y at some stage substituting their $\pi/3$ into their derivative must be exact, with their $\cos(\pi/6)$ evaluated o.e. e.g. $4\sqrt{3}/3$ but must be exact ft their $\sqrt{3}/4$ unless 1	or $f'(x) = 2/\sqrt{(1-x^2)}$ $f'(½) = 2/\sqrt{¾} = 4/\sqrt{3}$ cao
7	(i)		$s(-x) = f(-x) + g(-x)$ $= -f(x) + -g(x)$ $= -(f(x) + g(x))$ $= -s(x)$ (so s is odd)	M1 A1 **[2]**	must have $s(-x) = \ldots$	
7	(ii)		$p(-x) = f(-x)g(-x)$ $=(-f(x)) \times (-g(x))$ $= f(x)g(x) = p(x)$ so p is even	M1 A1 **[2]**	must have $p(-x) = \ldots$ Allow SC1 for showing that $p(-x) = p(x)$ using two specific odd functions, but in this case they must still show that p is even	e.g. $f(x) = x$, $g(x) = x^3$, $p(x) = x^4$ $p(-x) = (-x)^4 = x^4 = p(x)$, so p even condone f and g being the same function

Question			Answer	Marks	Guidance	
8	(i)		$\frac{dy}{dx} = \sin 2x + 2x\cos 2x$	M1 A1	d/dx(sin 2x) = 2cos 2x soi cao, mark final answer	can be inferred from dy/dx = 2x cos 2x e.g. dy/dx = tan 2x + 2x is A0
			dy/dx = 0 when sin 2x + 2x cos 2x = 0 $\Rightarrow \frac{\sin 2x + 2x\cos 2x}{\cos 2x} = 0$	M1	equating their derivative to zero, **provided it has two terms**	
			$\Rightarrow$ tan 2x + 2x = 0 *	A1 **[4]**	must show evidence of division by cos 2x	
8	(ii)		At P, x sin 2x = 0 $\Rightarrow$ sin 2x = 0, 2x = (0), $\pi \Rightarrow x = \pi/2$	M1 A1	$x = \pi/2$	Finding $x = \pi/2$ using the given line equation is M0
			At P, dy/dx = sin π + 2(π/2) cos π = $-\pi$	B1 ft	ft their $\pi/2$ and their derivative	
			Eqn of tangent: $y - 0 = -\pi(x - \pi/2)$ $\Rightarrow y = -\pi x + \pi^2/2$	M1	substituting 0, their $\pi/2$ and their $-\pi$ into $y - y_1 = m(x - x_1)$	or their $-\pi$ into $y = mx+c$, and then evaluating c: $y = (-\pi)x + c$, $0 = (-\pi)(\pi/2) + c$ M1 $\Rightarrow c = \pi^2/2$ $\Rightarrow y = -\pi x + \pi^2/2 \Rightarrow 2\pi x + 2y = \pi^2$ *A1
			$\Rightarrow 2\pi x + 2y = \pi^2$ *	A1	**NB AG**	
			When $x = 0$, $y = \pi^2/2$, so Q is $(0, \pi^2/2)$	M1A1 **[7]**	can isw inexact answers from $\pi^2/2$	
8	(iii)		Area = triangle OPQ – area under curve	M1	soi (or area under PQ – area under curve	area under line may be expressed in integral form
			Triangle OPQ = ½ × π/2 × π^2/2 [= π^3/8]	B1cao	allow art 3.9	or using integral: $\int_0^{\pi/2}(\frac{1}{2}\pi^2 - \pi x)\,dx = \left[\frac{1}{2}\pi^2 x - \frac{1}{2}\pi x^2\right]_0^{\pi/2} = \frac{\pi^3}{4} - \frac{\pi^3}{8} [= \frac{\pi^3}{8}]$
			Parts: $u = x$, dv/dx = sin 2x du/dx = 1, v = –½ cos2x	M1	condone $v = k$ cos2x soi	v can be inferred from their 'uv'
			$\int_0^{\pi/2} x\sin 2x\,dx = \left[-\frac{1}{2}x\cos 2x\right]_0^{\pi/2} - \int_0^{\pi/2} -\frac{1}{2}\cos 2x\,dx$	A1ft	ft their v = –½ cos2x, ignore limits	
			$= \left[-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\right]_0^{\pi/2}$	A1	[– ½ x cos 2x + ¼ sin 2x] o.e., must be correct at this stage, ignore limits	
			$= -\frac{1}{4}\pi\cos\pi + \frac{1}{4}\sin\pi - (-0\cos 0 + \frac{1}{4}\sin 0) = \frac{1}{4}\pi[-0]$	A1cao	(so dep previous A1)	
			So shaded area = $\pi^3/8 - \pi/4 = \pi(\pi^2 - 2)/8$*	A1 **[7]**	**NB AG** must be from fully correct work	

Question			Answer	Marks	Guidance	
9	(i)		(A) (0, 6) and (1, 4) (B) (−1, 5) and (0, 4)	B1B1 B1B1 **[4]**	Condone P and Q incorrectly labelled (or unlabelled)	
9	(ii)		$\text{f}'(x)=\frac{(x+1).2x-(x^2+3).1}{(x+1)^2}$ $\text{f}'(x) = 0 \Rightarrow 2x\,(x + 1) - (x^2 + 3) = 0$ $\Rightarrow x^2 + 2x - 3 = 0$ $\Rightarrow (x - 1)(x + 3) = 0$ $\Rightarrow x = 1$ or $x = -3$ When $x = -3$, $y = 12/(-2) = -6$ so other TP is (−3, −6)	M1 A1 M1 A1dep B1B1cao **[6]**	Quotient or product rule consistent with their derivatives, condone missing brackets correct expression their derivative = 0 obtaining correct quadratic equation (soi) dep 1st M1 but withhold if denominator also set to zero must be from correct work (but see note re quadratic)	PR: $(x^2+3)(-1)(x+1)^{-2} + 2x(x+1)^{-1}$ If formula stated correctly, allow one substitution error. condone missing brackets if subsequent working implies they are intended Some candidates get $x^2 + 2x + 3$, then realise this should be $x^2 + 2x - 3$, and correct back, but not for every occurrence. Treat this sympathetically. Must be supported, but −3 could be verified by substitution into correct derivative
9	(iii)		$\text{f}(x-1)=\frac{(x-1)^2+3}{x-1+1}$ $=\frac{x^2-2x+1+3}{x-1+1}$ $=\frac{x^2-2x+4}{x}=x-2+\frac{4}{x}$ *	M1 A1 A1 **[3]**	substituting $x - 1$ for both x's in f NB **AG**	allow 1 slip for M1
9	(iv)		$\int_a^b(x-2+\frac{4}{x})\,\text{d}x=\left[\frac{1}{2}x^2-2x+4\ln x\right]_a^b$ $=(\frac{1}{2}b^2-2b+4\ln b)-(\frac{1}{2}a^2-2a+4\ln a)$ Area is $\int_0^1\text{f}(x)\,\text{d}x$ So taking $a = 1$ and $b = 2$ area = (2 − 4 + 4ln 2) − (½ − 2 + 4ln 1) = 4 ln 2 − ½	B1 M1 A1 M1 A1 cao **[5]**	$\left[\frac{1}{2}x^2-2x+4\ln x\right]$ F(b) − F(a) condone missing brackets oe (mark final answer) must be simplified with ln 1 = 0	F must show evidence of integration of at least one term or f(x) = x + 1 − 2 + 4/(x+1) $A=\int_0^1\text{f}(x)\,\text{d}x=\left[\frac{1}{2}x^2-x+4\ln(1+x)\right]_0^1$ M1 = ½ − 1 + 4ln 2 = 4ln2 − ½ A1

GCE

Mathematics (MEI)

Advanced GCE

Unit **4753:** Methods for Advanced Mathematics

Mark Scheme for January 2013

Question			Answer	Marks	Guidance	
1	(i)		$y = e^{-x} \sin 2x$ $\Rightarrow$ $dy/dx = e^{-x}.2\cos 2x + (-e^{-x})\sin 2x$	M1 B1 A1 **[3]**	Product rule $d/dx(\sin 2x) = 2\cos 2x$ Any correct expression	u × their v' + v × their u' but mark final answer
1	(ii)		$dy/dx = 0$ when $2\cos 2x - \sin 2x = 0$ $\Rightarrow$ $2 = \tan 2x$ $\Rightarrow$ $2x = \arctan 2$ $\Rightarrow$ $x = ½ \arctan 2$ *	M1 M1 A1 **[3]**	ft their dy/dx but must eliminate e^{-x} $\sin 2x / \cos 2x = \tan 2x$ used [or $\tan^{-1}$] **NB AG**	derivative must have 2 terms substituting ½ arctan 2 into their deriv M0 (unless $\cos 2x = 1/\sqrt{5}$ and $\sin 2x = 2/\sqrt{5}$ found) must show previous step
2	(i)		$2x + 4y\dfrac{dy}{dx} = 4$ $\Rightarrow$ $\dfrac{dy}{dx} = \dfrac{4-2x}{4y}$	M1 A1 A1 **[3]**	$4y\dfrac{dy}{dx}$ correct equation o.e., but mark final answer	Rearranging for y and differentiating explicitly is M0 Ignore superfluous $dy/dx = \ldots$ unless used subsequently
2	(ii)		$\dfrac{dy}{dx} = 0 \Rightarrow x = 2$ $\Rightarrow$ $4 + 2y^2 = 8 \Rightarrow y^2 = 2$, $y = \sqrt{2}$ or $-\sqrt{2}$	B1dep B1B1 **[3]**	dep correct derivative $\sqrt{2}$, $-\sqrt{2}$	 can isw, penalise inexact answers of ±1.41 or better once only −1 for extra solutions found from using $y = 0$
3			$1 < x < 3 \Rightarrow$ $-1 < x - 2 < 1$ $\Rightarrow$ $\lvert x-2 \rvert < 1$	 B1 B1 **[2]**	oe [or $a = 2$ and $b = 1$]	

Question			Answer	Marks	Guidance	
4	(i)		$\theta = a - be^{-kt}$			
			When $t = 0$, $\theta = 15 \Rightarrow 15 = a - b$	M1	$15 = a - b$	must have $e^0 = 1$
			When $t = \infty$, $\theta = 100 \Rightarrow 100 = a$	B1	$a = 100$	
			$\Rightarrow \quad b = 85$	A1cao	$b = 85$	
			When $t = 1$, $\theta = 30 \Rightarrow 30 = 100 - 85e^{-k}$	M1	$30 = a - b\, e^{-k}$	(need not substitute for a and b)
			$\Rightarrow \quad e^{-k} = 70/85$			
			$\Rightarrow \quad -k = \ln(70/85) = -0.194(156\ldots)$	M1	Re-arranging and taking lns	allow $-k = \ln[(a - 30)/b]$ ft on a, b
			$\Rightarrow \quad k = 0.194$	A1	0.19 or better, or $-\ln(70/85)$ oe	mark final ans
				[6]		
4	(ii)		$80 = 100 - 85\, e^{-0.194t}$	M1	ft their values for a, b and k	but must substitute values
			$\Rightarrow \quad e^{-0.194t} = 20/85$			
			$\Rightarrow \quad t = -\ln(4/17) / 0.194 = 7.45$ (min)	A1	art 7.5 or 7 min 30 s or better	
				[2]		
5	(i)		$dF/dv = -25\, v^{-2}$	M1	$d/dv(v^{-1}) = -v^{-2}$ soi	
				A1	$-25\, v^{-2}$ o.e mark final ans	
				[2]		
5	(ii)		When $v = 50$, $dF/dv = -25/50^2$ $(= -0.01)$	B1	$-25/50^2$	
			$\dfrac{dF}{dt} = \dfrac{dF}{dv} . \dfrac{dv}{dt}$	M1	o.e.	e.g. $\dfrac{dF}{dv} = \dfrac{dF}{dt} / \dfrac{dv}{dt}$
			$= -0.01 \times 1.5 = -0.015$	A1cao	o.e. e.g. $-3/200$ isw	
				[3]		

Question			Answer	Marks	Guidance	
6			Let $u = 1 + x \Rightarrow$ $\int_0^3 x(1+x)^{-1/2}\,dx = \int_1^4 (u-1)u^{-1/2}\,du$	M1	$\int (u-1)u^{-1/2}(du)$ *	condone no du, missing bracket, ignore limits
			$= \int_1^4 (u^{1/2} - u^{-1/2})\,du$	A1	$\int (u^{1/2} - u^{-1/2})(du)$	
			$= \left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^4$	A1	$\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]$ o.e.	e.g. $\left[\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2}\right]$; ignore limits
			$= (16/3 - 4) - (2/3 - 2)$	M1dep	upper–lower dep 1st M1 and integration	with correct limits e.g. 1, 4 for u or 0, 3 for x
			$= 2\frac{2}{3}$	A1cao	or $2.\dot{6}$ but must be exact	or using $w = (1+x)^{1/2} \Rightarrow$ $\int \frac{(w^2-1)2w}{w}(dw)$ M1
			OR Let $u = x$, $v' = (1 + x)^{-1/2}$	M1		$= \int 2(w^2-1)(dw)$ A1 $= \left[\frac{2}{3}w^3 - 2w\right]$ A1
			$\Rightarrow$ $u' = 1$, $v = 2(1 + x)^{1/2}$	A1		upper–lower with correct limits (w = 1,2) M1
			$\Rightarrow$ $\int_0^3 x(1+x)^{-1/2}\,dx = \left[2x(1+x)^{1/2}\right]_0^3 - \int_0^3 2(1+x)^{1/2}\,dx$	A1	ignore limits, condone no dx	8/3 A1 cao
			$= \left[2x(1+x)^{1/2} - \frac{4}{3}(1+x)^{3/2}\right]_0^3$	A1	ignore limits	*If $\int_1^4 (u-1)u^{-1/2}\,du$ done by parts:
			$= (2 \times 3 \times 2 - 4 \times 8/3) - (0 - 4/3)$			$2u^{1/2}(u-1) - \int 2u^{1/2}\,du$ A1
			$= 2\frac{2}{3}$	A1cao	or $2.\dot{6}$ but must be exact	$[2u^{1/2}(u-1) - 4u^{3/2}/3]$ A1 substituting correct limits M1 8/3 A1cao
				[5]		

Question			Answer	Marks	Guidance	
7	(i)		$3^5 + 2 = 245$ [which is not prime]	M1 A1 **[2]**	Attempt to find counter-example correct counter-example identified	If A0, allow M1 for $3^n + 2$ correctly evaluated for 3 values of n
7	(ii)		$(3^0 = 1)$, $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, … so units digits cycle through 1, 3, 9, 7, 1, 3, … so cannot be a '5'. **OR** 3^n is not divisible by 5 all numbers ending in '5' are divisible by 5. so its last digit cannot be a '5'	M1 A1 B1 B1 **[2]**	Evaluate 3^n for n = 0 to 4 or 1 to 5 must state conclusion for B2	allow just final digit written
8	(i)		translation in the x–direction of $\pi/4$ to the right translation in y-direction of 1 unit up.	M1 A1 M1 A1 **[4]**	allow 'shift', 'move' oe (eg using vector) allow 'shift', 'move' oe (eg using vector)	If just vectors given withhold **one** 'A' mark only 'Translate $\begin{pmatrix}\pi/4\\1\end{pmatrix}$' is 4 marks; if this is followed by an additional incorrect transformation, SC M1M1A1A0 $\begin{pmatrix}\pi/4\\1\end{pmatrix}$ only is M2A1A0

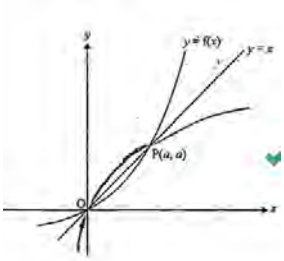
Question		Answer	Marks	Guidance	
8	(ii)	$g(x) = \frac{2\sin x}{\sin x + \cos x}$			(Can deal with num and denom separately)
		$g'(x) = \frac{(\sin x + \cos x)2\cos x - 2\sin x(\cos x - \sin x)}{(\sin x + \cos x)^2}$	M1	Quotient (or product) rule consistent with their derivs	$\frac{vu' - uv'}{v^2}$; allow one slip, missing brackets
		$= \frac{2\sin x\cos x + 2\cos^2 x - 2\sin x\cos x + 2\sin^2 x}{(\sin x + \cos x)^2}$	A1	Correct expanded expression (could leave the '2' as a factor)	$\frac{uv' - vu'}{v^2}$ is M0. Condone $\cos x^2$, $\sin x^2$
		$= \frac{2\cos^2 x + 2\sin^2 x}{(\sin x + \cos x)^2} = \frac{2(\cos^2 x + \sin^2 x)}{(\sin x + \cos x)^2}$			
		$= \frac{2}{(\sin x + \cos x)^2}$ *	A1	**NB AG**	must take out 2 as a factor or state $\sin^2 x + \cos^2 x = 1$
		When $x = \pi/4$, $g'(\pi/4) = 2/(1/\sqrt{2} + 1/\sqrt{2})^2$	M1	substituting $\pi/4$ into correct deriv	
		$= 1$	A1		
		$f'(x) = \sec^2 x$	M1	o.e., e.g. $1/\cos^2 x$	
		$f'(0) = \sec^2(0) = 1$, [so gradient the same here]	A1		
			[7]		

Question			Answer	Marks	Guidance	
8	(iii)		$\int_0^{\pi/4} f(x)\,dx = \int_0^{\pi/4} \frac{\sin x}{\cos x}\,dx$ let $u = \cos x$, $du = -\sin x\,dx$ when $x = 0$, $u = 1$, when $x = \pi/4$, $u = 1/\sqrt{2}$			
			$= \int_1^{1/\sqrt{2}} -\frac{1}{u}\,du$	M1	substituting to get $\int -1/u\ (du)$	ignore limits here, condone no du but not dx allow $\int 1/u\ .-du$
			$= \int_{1/\sqrt{2}}^{1} \frac{1}{u}\,du$ *	A1	NB **AG**	but for A1 must deal correctly with the −ve sign by interchanging limits
			$= [\ln u]_{1/\sqrt{2}}^{1}$	M1	$[\ln u]$	
			$= \ln 1 - \ln(1/\sqrt{2})$ $= \ln\sqrt{2} = \ln 2^{1/2} = \frac{1}{2}\ln 2$	A1 **[4]**	$\ln\sqrt{2}$, $\frac{1}{2}\ln 2$ or $-\ln(1/\sqrt{2})$	mark final answer
8	(iv)		Area = area in part **(iii)** translated up 1 unit.	M1	soi from $\pi/4$ added	or $\int_{\pi/4}^{\pi/2} (1+\tan(x-\pi/4))\,dx = [x + \ln\sec(x-\pi/4)]_{\pi/4}^{\pi/2}$ $= \pi/2 + \ln\sqrt{2} - \pi/4 = \pi/4 + \ln\sqrt{2}$ B2
			So $= \frac{1}{2}\ln 2 + 1 \times \pi/4 = \frac{1}{2}\ln 2 + \pi/4$.	A1cao **[2]**	oe (as above)	
9	(i)		At P(a, a) g$(a) = a$ so $\frac{1}{2}(e^a - 1) = a$ $\Rightarrow \quad e^a = 1 + 2a$ *	B1 **[1]**	**NB AG**	
9	(ii)		$A = \int_0^a \frac{1}{2}(e^x - 1)\,dx$	M1	correct integral and limits	limits can be implied from subsequent work
			$= \frac{1}{2}[e^x - x]_0^a$	B1	integral of $e^x - 1$ is $e^x - x$	
			$= \frac{1}{2}(e^a - a - e^0)$	A1		
			$= \frac{1}{2}(1 + 2a - a - 1) = \frac{1}{2}a$ *	A1	**NB AG**	
			area of triangle $= \frac{1}{2}a^2$	B1		
			area between curve and line $= \frac{1}{2}a^2 - \frac{1}{2}a$	B1cao **[6]**	mark final answer	

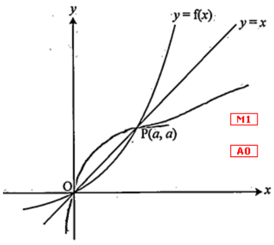
Question			Answer	Marks	Guidance	
9	(iii)		$y = ½(e^x – 1)$ swap x and y $x = ½ (e^y − 1)$			
			$\Rightarrow$ $2x = e^y – 1$	M1	Attempt to invert – one valid step	merely swapping x and y is not 'one step'
			$\Rightarrow$ $2x + 1 = e^y$	A1		
			$\Rightarrow$ $\ln(2x + 1) = y$ *	A1	$y = \ln(2x + 1)$ or $g(x) = \ln(2x + 1)$ **AG**	apply a similar scheme if they start with g(x) and invert to get f(x).
			$\Rightarrow$ $g(x) = \ln(2x + 1)$			or g f(x) = g((e^x – 1)/2) M1
			Sketch: recognisable attempt to reflect in $y = x$	M1	through O and (a, a)	= $\ln(1 + e^x − 1) = \ln(e^x)$ A1 = x A1
			Good shape	A1	no obvious inflexion or TP, extends to third quadrant, without gradient becoming too negative	similar scheme for fg See appendix for examples
				[5]		
9	(iv)		$f'(x) = ½ e^x$	B1		
			$g'(x) = 2/(2x + 1)$	M1	$1/(2x + 1)$ (or $1/u$ with $u = 2x + 1$) …	
				A1	… × 2 to get $2/(2x + 1)$	
			$g'(a) = 2/(2a + 1)$, $f'(a) = ½ e^a$	B1	either $g'(a)$ or $f'(a)$ correct soi	
			so $g'(a) = 2/e^a$ or $f'(a) = ½ (2a+1)$ $= 1/(½ e^a)$ $= (2a + 1)/2$ [$= 1/f'(a)$] [$= 1/g'(a)$]	M1 A1	substituting $e^a = 1 + 2a$ establishing $f'(a) = 1/ g'(a)$	either way round
			tangents are reflections in $y = x$	B1	must mention tangents	
				[7]		

APPENDIX 1

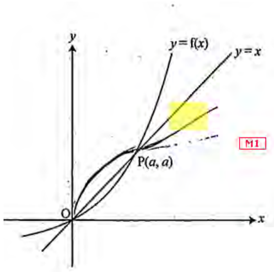
Exemplar marking of 9(iii)



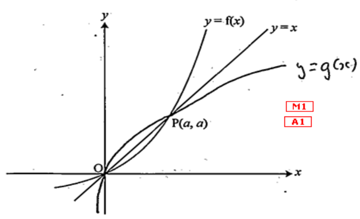
M1A1



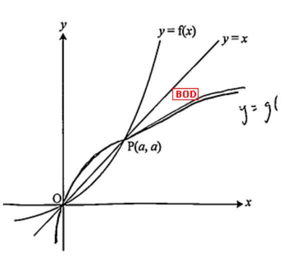
M1A0 – infl



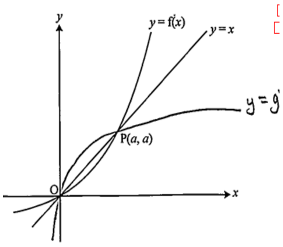
M1A0 –infl, no 3rd quadrant



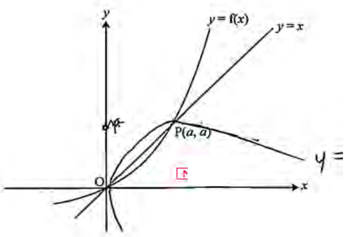
M1A1 (slight infl – condone)



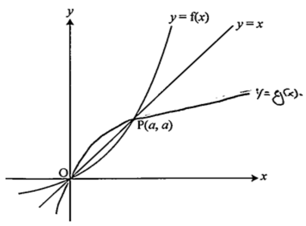
M1A1 (slight infl)



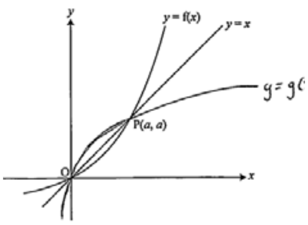
M1A0 (tp, just)



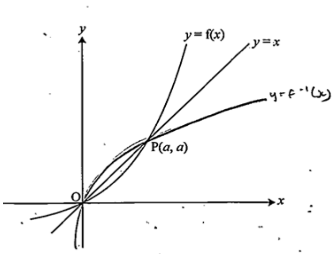
M1A0



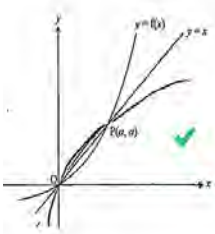
bod M1A1 (slight infl)



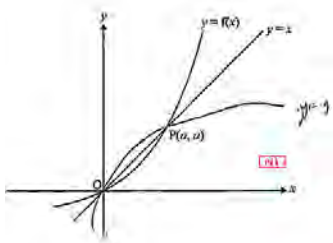
bod M1A1 (condone TP)



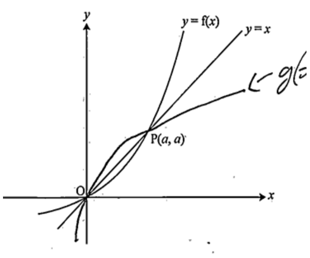
bod M1A1, (see 3rd quad)



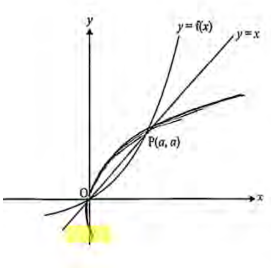
M1A1



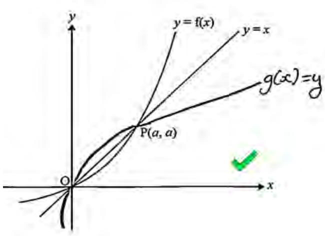
M1A0 (TP, 3rd quad)



M1A1



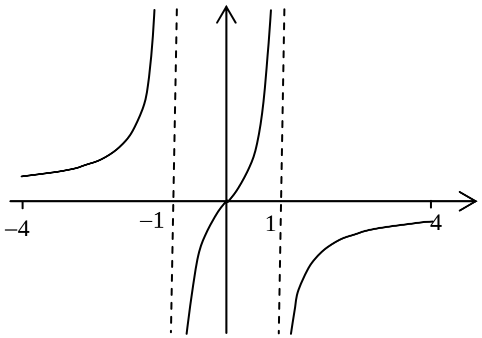
M1A0 (see 3rd q)



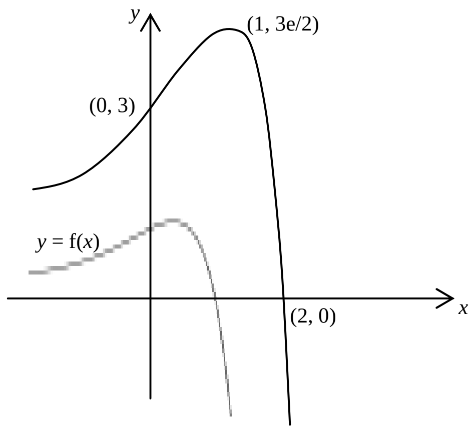
M1A0 (3rd q bends back)

Question			Answer	Marks	Guidance
1	(i)		$a = ½$	B1	or 0.5
			$b = 1$	B1	
				[2]	
1	(ii)		$½\,\lvert x + 1\rvert = \lvert x\rvert$		
			$\Rightarrow\ ½(x + 1) = x$,	M1	o.e. ft their a ($\neq 0$), b (but allow recovery to correct values) or verified by subst $x = 1$, $y = 1$ into $y = ½\lvert x + 1\rvert$ and $y = \lvert x\rvert$
			$\Rightarrow x = 1$, $y = 1$	A1	unsupported answers M0A0
			or $½(x + 1) = -x$,	M1	o.e., ft their a. b ; or verified by subst $(-1/3, 1/3)$ into $y = ½\lvert x + 1\rvert$ and $y = \lvert x\rvert$
			$\Rightarrow x = -1/3$, $y = 1/3$	A1	or 0.33, −0.33 or better unsupported answers M0A0
			or		
			$¼(x + 1)^2 = x^2$	M1	ft their a and b
			$\Rightarrow 3x^2 - 2x - 1 = 0$	M1ft	obtaining a quadratic = 0,ft their previous line, but must have an x^2 term
			$\Rightarrow x = -1/3$ or 1	A1	SC3 for (1, 1) (−1/3, 1/3) and one or more additional points
			$y = 1/3$ or 1	A1	
				[4]	
2	(i)		$n^3 - n = n(n^2 - 1)$	B1	two correct factors
			$= n(n - 1)(n + 1)$	B1	
				[2]	
2	(ii)		$n - 1$, n and $n + 1$ are consecutive integers	B1	
			so at least one is even, and one is div by 3	B1	
			[$\Rightarrow$ $n^3 - n$ is div by 6]	**[2]**	
3	(i)		Range is $-1 \le y \le 3$	M1	−1, 3
				A1	$-1 \le y \le 3$ or $-1 \le f(x) \le 3$ or $[-1, 3]$ (not −1 to 3, $-1 \le x \le 3$, $-1 < y < 3$ etc)
				[2]	

Question			Answer	Marks	Guidance
3	(ii)		$y = 1 - 2\sin x \quad x \leftrightarrow y$		[can interchange x and y at any stage]
			$x = 1 - 2\sin y \Rightarrow x - 1 = -2\sin y$	M1	attempt to re-arrange
			$\Rightarrow \quad \sin y = (1 - x)/2$	A1	o.e. e.g. $\sin y = (x - 1)/(-2)$ (or $\sin x = (y - 1)/(-2)$)
			$\Rightarrow \quad y = \arcsin\,[(1 - x)/2]$	A1	or $f^{-1}(x) = \arcsin\,[(1 - x)/2]$, not x or $f^{-1}(y) = \arcsin[1 - y)/2]$ (viz must have swapped x and y for final 'A' mark).
				[3]	$\arcsin\,[(x - 1)/-2]$ is A0
3	(iii)		$f'(x) = -2\cos x$	M1	condone $2\cos x$
			$\Rightarrow \quad f'(0) = -2$	A1	cao
			$\Rightarrow \quad$ gradient of $y = f^{-1}(x)$ at $(1, 0) = -½$	A1	not $1/-2$
				[3]	
4			$V = \pi h^2 \Rightarrow dV/dh = 2\pi h \Rightarrow$	M1A1	if derivative $2\pi h$ seen without $dV/dh = \ldots$ allow M1A0
			$dV/dt = dV/dh \times dh/dt$	M1	soi ; o.e. – any correct statement of the chain rule using V, h and t – condone use of a letter other than t for time here
			$dV/dt = 10$	B1	soi; if a letter other than t used (and not defined) B0
			$dh/dt = 10/(2\pi \times 5) = 1/\pi$	A1	or 0.32 or better, mark final answer
				[5]	
5			$y = \ln\left(\sqrt{\dfrac{2x-1}{2x+1}}\right) = \dfrac{1}{2}(\ln(2x-1) - \ln(2x+1))$	M1	use of $\ln(a/b) = \ln a - \ln b$
				M1	use of $\ln\sqrt{c} = ½ \ln c$
			$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{1}{2}\left(\dfrac{2}{2x-1} - \dfrac{2}{2x+1}\right)$	A1	o.e.; correct expression (if this line of working is missing, M1M1A0A0)
			$= \dfrac{1}{2x-1} - \dfrac{1}{2x+1}$ *	A1	**NB AG**
				[4]	for alternative methods, see additional solutions

Question			Answer	Marks	Guidance
6			$\int_0^{\pi/2} \frac{\sin 2x}{3+\cos 2x}\,\mathrm{d}x = \left[-\frac{1}{2}\ln(3+\cos 2x)\right]_0^{\pi/2}$	M1 A2	$k \ln(3 + \cos 2x)$ $-\frac{1}{2}\ln(3 + \cos 2x)$
			or $u = 3 + \cos 2x$, $\mathrm{d}u = -2\sin 2x\,\mathrm{d}x$	M1	o.e. e.g. $\mathrm{d}u/\mathrm{d}x = -2\sin 2x$ or if $v = \cos 2x$, $\mathrm{d}v = -2\sin 2x\mathrm{d}x$ o.e. condone $2\sin 2x\,\mathrm{d}x$
			$\int_0^{\pi/2} \frac{\sin 2x}{3+\cos 2x}\,\mathrm{d}x = \int_4^2 -\frac{1}{2u}\,\mathrm{d}u$	A1	$\int -\frac{1}{2u}\,\mathrm{d}u$, or if $v = \cos 2x$, $\int -\frac{1}{2(3+v)}\,\mathrm{d}v$
			$= \left[-\frac{1}{2}\ln u\right]_4^2$	A1	$[-\frac{1}{2}\ln u]$ or $[-\frac{1}{2}\ln(3 + v)]$ ignore incorrect limits
			$= -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 4$	A1	from correct working o.e. e.g. $-\frac{1}{2}\ln(3+\cos(2.\pi/2)) + \frac{1}{2}\ln(3 + \cos(2.0))$
			$= \frac{1}{2}\ln(4/2)$		o.e. required step for final A1, must have evaluated to 4 and 2 at this stage
			$= \frac{1}{2}\ln 2$ *	A1	**NB AG**
				[5]	
7	(i)		$\mathrm{f}(-x) = \frac{2(-x)}{1-(-x)^2}$	M1	substituting $-x$ for x in $\mathrm{f}(x)$
			$= -\frac{2x}{1-x^2} = -\mathrm{f}(x)$	A1	
				[2]	
7	(ii)			M1 A1 **[2]**	Recognisable attempt at a half turn rotation about O Good curve starting from $x = -4$, asymptote $x = -1$ shown on graph. (Need not state – 4 and – 1 explicitly as long as graph is reasonably to scale.) Condone if curve starts to the left of $x = -4$.

Question			Answer	Marks	Guidance
8	(i)		(1, 0) and (0, 1)	B1B1 **[2]**	$x = 0, y = 1$; $y = 0, x = 1$
8	(ii)		$f'(x) = 2(1-x)e^{2x} - e^{2x}$ $= e^{2x}(1-2x)$ $f'(x) = 0$ when $x = ½$ $y = ½ e$	B1 M1 A1 M1dep A1cao B1 **[6]**	$d/dx(e^{2x}) = 2e^{2x}$ product rule consistent with their derivatives correct expression, so $(1-x)e^{2x} - e^{2x}$ is B0M1A0 setting their derivative to 0 dep 1st M1 $x = ½$ allow $½ e^1$ isw
8	(iii)		$A = \int_0^1 (1-x)e^{2x}dx$ $u = (1-x)$, $u' = -1$, $v' = e^{2x}$, $v = ½ e^{2x}$ $\Rightarrow A = \left[\frac{1}{2}(1-x)e^{2x}\right]_0^1 - \int_0^1 \frac{1}{2}e^{2x}.(-1)dx$ $= \left[\frac{1}{2}(1-x)e^{2x} + \frac{1}{4}e^{2x}\right]_0^1$ $= ¼ e^2 - ½ - ¼$ $= ¼ (e^2 - 3)$ *	B1 M1 A1 A1 A1cao **[5]**	correct integral and limits; condone no dx (limits may be seen later) u, u', v', v, all correct; or if split up $u = x$, $u' = 1$, $v' = e^{2x}$, $v = ½ e^{2x}$ condone incorrect limits; or, from above, ...$\left[\frac{1}{2}xe^{2x}\right]_0^1 - \int_0^1 \frac{1}{2}e^{2x}dx$ o.e. if integral split up; condone incorrect limits **NB AG**

Question			Answer	Marks	Guidance
8	(iv)		$g(x) = 3f(\frac{1}{2}x) = 3(1 - \frac{1}{2}x)e^x$	B1	o.e; mark final answer
				B1	through (2,0) and (0,3) – condone errors in writing coordinates (e.g. (0,2)).
				B1dep	reasonable shape, dep previous B1
				B1	TP at (1, 3e/2) or (1, 4.1) (or better). (Must be evidence that $x = 1$, $y = 4.1$ is indeed the TP − appearing in a table of values is not enough on its own.)
				[4]	
8	(v)		$6 \times \frac{1}{4}(e^2 - 3)$ $[= 3(e^2 - 3)/2]$	B1	o.e. mark final answer
				[1]	

Question			Answer	Marks	Guidance
9	(i)		$a = ½$	B1	allow $x = ½$
				[1]	
9	(ii)		$y^3 = \dfrac{x^3}{2x-1}$		
			$\Rightarrow \quad 3y^2\dfrac{dy}{dx} = \dfrac{(2x-1)3x^2 - x^3.2}{(2x-1)^2}$	B1	$3y^2 dy/dx$
				M1	Quotient (or product) rule consistent with their derivatives; $(v\,du + u\,dv)/v^2$ M0
				A1	correct RHS expression – condone missing bracket
			$= \dfrac{6x^3 - 3x^2 - 2x^3}{(2x-1)^2} = \dfrac{4x^3 - 3x^2}{(2x-1)^2}$	A1	
			$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{4x^3 - 3x^2}{3y^2(2x-1)^2}$ *	A1	**NB AG** penalise omission of bracket in QR at this stage
			$dy/dx = 0$ when $4x^3 - 3x^2 = 0$	M1	
			$\Rightarrow x^2(4x - 3) = 0$, $x = 0$ or ¾	A1	if in addition $2x - 1 = 0$ giving $x = ½$, A0
			$y^3 = (¾)^3/½ = 27/32$,	M1	must use $x = ¾$; if (0, 0) given as an additional TP, then A0
			$y = 0.945$ (3sf)	A1	can infer M1 from answer in range 0.94 to 0.95 inclusive
				[9]	

Question			Answer	Marks	Guidance
9	(iii)		$u = 2x - 1 \Rightarrow du = 2dx$		
			$\int \frac{x}{\sqrt[3]{2x-1}}dx = \int \frac{\frac{1}{2}(u+1)}{u^{1/3}}\frac{1}{2}du$	M1	$\frac{\frac{1}{2}(u+1)}{u^{1/3}}$ if missing brackets, withhold A1
				M1	× ½ d*u* condone missing d*u* here, but withhold A1
			$= \frac{1}{4}\int \frac{u+1}{u^{1/3}}du = \frac{1}{4}\int (u^{2/3} + u^{-1/3})du$ *	A1	**NB AG**
			area $= \int_1^{4.5} \frac{x}{\sqrt[3]{2x-1}}dx$	M1	correct integral and limits – may be inferred from a change of limits andP their attempt to integrate (their) ¼ $(u^{2/3} + u^{-1/3})$
			when $x = 1$, $u = 1$, when $x = 4.5$, $u = 8$	A1	$u = 1, 8$ (or substituting back to *x*'s and using 1 and 4.5)
			$= \frac{1}{4}\int_1^8 (u^{2/3} + u^{-1/3})du$		
			$= \frac{1}{4}\left[\frac{3}{5}u^{5/3} + \frac{3}{2}u^{2/3}\right]_1^8$	B1	$\left[\frac{3}{5}u^{5/3} + \frac{3}{2}u^{2/3}\right]$ o.e. e.g. [$u^{5/3}/(5/3) + u^{2/3}/(2/3)$]
			$= \frac{1}{4}\left[\frac{96}{5} + 6 - \frac{3}{5} - \frac{3}{2}\right]$	A1	o.e. correct expression (may be inferred from a correct final answer)
			$= 5\frac{31}{40} = 5.775$ or $\frac{231}{40}$	A1	cao, must be exact; mark final answer
				[8]	

Additional Solutions

Question			Answer	Marks	Guidance
5		(1)	$y=\ln\left(\sqrt{\frac{2x-1}{2x+1}}\right)=\frac{1}{2}\ln\left(\frac{2x-1}{2x+1}\right)$	M1	$\ln\sqrt{c} = ½ \ln c$ used
			$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\frac{1}{\left(\frac{2x-1}{2x+1}\right)}\frac{(2x+1)2-(2x-1)2}{(2x+1)^2}$	A2	fully correct expression for the derivative
			$=\frac{1}{2}\frac{2x+1}{2x-1}\frac{4}{(2x+1)^2}=\frac{2}{(2x-1)(2x+1)}$		
			$\frac{1}{2x-1}-\frac{1}{2x+1}=\frac{2x+1-(2x-1)}{(2x-1)(2x+1)}$		
			$=\frac{2}{(2x-1)(2x+1)}$		
			$\frac{1}{2x-1}-\frac{1}{2x+1}=\frac{2x+1-(2x-1)}{(2x-1)(2x+1)}=\frac{2}{(2x-1)(2x+1)}$	A1	simplified and shown to be equivalent to $\frac{1}{2x-1}-\frac{1}{2x+1}$
				[4]	
5		(2)	$y=\ln\left(\sqrt{\frac{2x-1}{2x+1}}\right)=\ln\sqrt{2x-1}-\ln\sqrt{2x+1}$	M1	$\ln(a/b) = \ln a - \ln b$ used
			$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sqrt{2x-1}}\frac{1}{2}(2x-1)^{-1/2}.2-\frac{1}{\sqrt{2x+1}}\frac{1}{2}(2x+1)^{-1/2}.2$	A2	fully correct expression
			$=\frac{1}{2x-1}-\frac{1}{2x+1}$	A1	simplified and shown to be equivalent to $\frac{1}{2x-1}-\frac{1}{2x+1}$
				[4]	

Question			Answer	Marks	Guidance
5		(3)	$y=\ln\left(\sqrt{\dfrac{2x-1}{2x+1}}\right)$		
			$\dfrac{dy}{dx}=\dfrac{1}{\sqrt{\dfrac{2x-1}{2x+1}}}\dfrac{1}{2}\left(\dfrac{2x-1}{2x+1}\right)^{-1/2}\dfrac{(2x+1)2-(2x-1)2}{(2x+1)^2}$ or	M1	$\dfrac{1}{u}\times$ their u' where $u=\sqrt{\dfrac{2x-1}{2x+1}}$ or $\dfrac{\sqrt{2x-1}}{\sqrt{2x+1}}$ (any attempt at u' will do)
			$\dfrac{1}{\dfrac{\sqrt{2x-1}}{\sqrt{2x+1}}}\dfrac{\sqrt{2x+1}.\frac{1}{2}.2(2x-1)^{-1/2}-\sqrt{2x-1}.\frac{1}{2}.2(2x+1)^{-1/2}}{\sqrt{2x+1}^2}$	A2	o.e. any completely correct expression for the derivative
			$=\dfrac{1}{2}\left(\dfrac{2x+1}{2x-1}\right)\dfrac{4}{(2x+1)^2}=\dfrac{2}{(2x-1)(2x+1)}$		or $=\dfrac{\sqrt{2x+1}}{\sqrt{2x-1}}\dfrac{(2x+1)-(2x-1)}{(2x+1)^{3/2}(2x-1)^{1/2}}=\dfrac{2}{(2x+1)(2x-1)}$
			$\dfrac{1}{2x-1}-\dfrac{1}{2x+1}=\dfrac{(2x+1)-(2x-1)}{(2x-1)(2x+1)}=\dfrac{2}{(2x-1)(2x+1)}$	A1 **[4]**	simplified and correctly shown to be equivalent to $\dfrac{1}{2x-1}-\dfrac{1}{2x+1}$
9	(ii)	(1)	$y=\dfrac{x}{(2x-1)^{1/3}}$		
			$\Rightarrow\dfrac{dy}{dx}=\dfrac{(2x-1)^{1/3}.1-x.(1/3)(2x-1)^{-2/3}.2}{(2x-1)^{2/3}}$	M1 A1	quotient rule or product rule on y − allow one slip correct expression for the derivative
			$=\dfrac{6x-3-2x}{3(2x-1)^{4/3}}=\dfrac{4x-3}{3(2x-1)^{4/3}}$	M1 A1	factorising or multiplying top and bottom by $(2x-1)^{2/3}$
			$=\dfrac{(4x-3)x^2}{3y^2(2x-1)^{2/3}(2x-1)^{4/3}}=\dfrac{4x^3-3x^2}{3y^2(2x-1)^2}$	A1	establishing equivalence with given answer **NB AG**

Question			Answer	Marks	Guidance
9	(ii)	(2)	$y=\left(\dfrac{x^3}{(2x-1)}\right)^{1/3}$		
			$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{3}\left(\dfrac{x^3}{(2x-1)}\right)^{-2/3}\dfrac{(2x-1).3x^2-x^3.2}{(2x-1)^2}$	B1	$\dfrac{1}{3}\left(\dfrac{x^3}{(2x-1)}\right)^{-2/3}\times\ldots$
				M1A1	$\ldots\times\dfrac{(2x-1).3x^2-x^3.2}{(2x-1)^2}$
			$=\dfrac{1}{3}\dfrac{4x^3-3x^2}{x^2(2x-1)^{4/3}}=\dfrac{4x-3}{3(2x-1)^{4/3}}$	A1	
			$=\dfrac{(4x-3)x^2}{3y^2(2x-1)^{2/3}(2x-1)^{4/3}}=\dfrac{4x^3-3x^2}{3y^2(2x-1)^2}$	A1	establishing equivalence with given answer **NB AG**
9	(ii)	(3)	$y^3(2x-1)=x^3$		
			$3y^2\dfrac{dy}{dx}(2x-1)+y^3.2=3x^2$	B1	$\mathrm{d}/\mathrm{d}x(y^3)=3y^2(\mathrm{d}y/\mathrm{d}x)$
				M1	product rule on $y^3(2x-1)$ or $2xy^3$
				A1	correct equation
			$\dfrac{dy}{dx}=\dfrac{3x^2-2y^3}{3y^2(2x-1)}$		
			$=\dfrac{3x^2-2\dfrac{x^3}{(2x-1)}}{3y^2(2x-1)}$	M1	subbing for $2y^3$
			$=\dfrac{3x^2(2x-1)-2x^3}{3y^2(2x-1)^2}=\dfrac{6x^3-3x^2-2x^3}{3y^2(2x-1)^2}=\dfrac{4x^3-3x^2}{3y^2(2x-1)^2}$	A1	**NB AG**