MEI

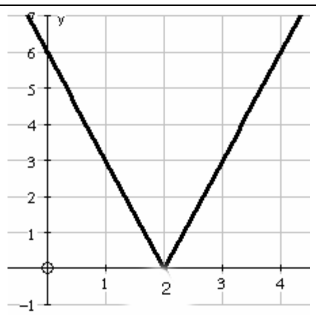
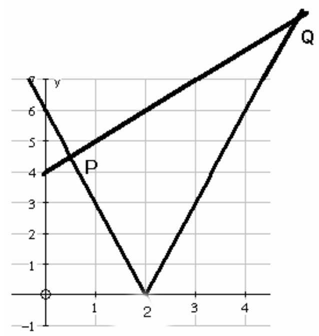
Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

METHODS OF ADVANCED MATHEMATICS, C3

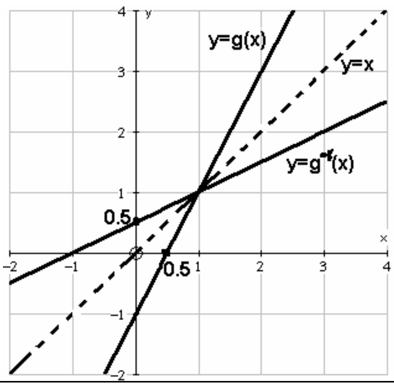
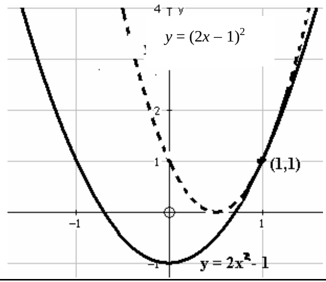
Practice Paper C3-D

MARK SCHEME

Qu		Answer	Mark	Comment
Section A				
1	**(i)**	$y^2 = 4x + 7 \Rightarrow 2y.\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{y}$	M1 A1 **2**	
	(ii)	$x = \frac{1}{4}\left(y^2 - 7\right) \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{4}.2y = \frac{y}{2} = \frac{1}{\frac{2}{y}}$	B1 M1 A1 **3**	
2	**(i)**	$\mathrm{e}^{2x} + 2 + \mathrm{e}^{-2x}$	B1 **1**	
	(ii)	$= \int (e^{2x} + 2 + e^{-2x})dx$ $= \frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + c$	B1 B1 B1 **3**	One for each exponential term, one for both 2x and constant.
3	**(i)**		B1 B1 **2**	One for two half lines; one for correct orientation and meeting at $(2, 0)$.
	(ii)	 At P $-3x + 6 = x + 4$ $\Rightarrow \quad x = \frac{1}{2}$ At Q $3x - 6 = x + 4$ $\Rightarrow \quad x = 5$ The solution is $x = \frac{1}{2}, 5$. As shown on graph	M1 A1 A1 E1 **4**	
4		$\int x \sin 3x \,\mathrm{d}x; \quad u = x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1, \ \frac{\mathrm{d}v}{\mathrm{d}x} = \sin 3x \Rightarrow v = -\frac{1}{3}\cos x$ $= -\frac{1}{3}x\cos 3x + \int \frac{1}{3}\cos 3x \,\mathrm{d}x$ $= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + c$	M1 A1 M1 A1 **4**	Choice of u $-\frac{1}{3}\cos 3x$ Correct form c must be seen

5		$t = \ln\sqrt{\frac{5}{(x-3)}}$ $= -\frac{1}{2}\ln\frac{(x-3)}{5}$ $\Rightarrow -2t = \ln\frac{(x-3)}{5}$ $\Rightarrow \mathrm{e}^{-2t} = \frac{(x-3)}{5} \Rightarrow x = 5\mathrm{e}^{-2t} + 3$	M1 M1 A1 A1 **4**	Rules of logs Change to exponentials
6	**(i)**	P(1,0)	B1 **1**	
	(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x.\frac{1}{x} - 1.\ln x}{x^2}$ $= \frac{1-\ln x}{x^2}$ At Q, gradient is zero, so $x = \mathrm{e}$. Q is $(\mathrm{e}, \frac{1}{\mathrm{e}})$. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{x^2(-\frac{1}{x}) - (1-\ln x).2x}{x^4}$ $= \frac{-3+2\ln x}{x^3}$ When $x = \mathrm{e}$, this is –ve, so Q is a maximum.	M1 A1 M1 A1 M1 A1 **6**	quotient rule = 0 Or equivalent methods For $^1/_\mathrm{e}$.
7	**(i)**	$A = \pi r^2 \Rightarrow \frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$ $\Rightarrow \frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}r}.\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{2} \times 2\pi r = \pi r$	M1 A1 M1 A1 **4**	
	(ii)	So when r = 6 , $\frac{\mathrm{d}A}{\mathrm{d}t} = 6\pi\ (= 15.707...)$. The area increases at 15.7 $\mathrm{km^2h^{-1}}$, to 3sf.	M1 A1 **2**	Substituting

Section B				
8	**(i)**	$P(-1,0)$	B1 **1**	
	(ii)	$y = x\sqrt{1+x}$ $= x(1+x)^{\frac{1}{2}}$ $\Rightarrow \frac{dy}{dx} = 1.(1+x)^{\frac{1}{2}} + x.\frac{1}{2}(1+x)^{-\frac{1}{2}}$ $= \frac{2(1+x)+x}{2(1+x)^{\frac{1}{2}}}$ $= \frac{3x+2}{2\sqrt{1+x}}$	M1 A1 M1 E1 **4**	Product rule Any correct expression Combining fractions
	(iii)	At a turning point, gradient is zero. $x = -\frac{2}{3}$ there. Then $y = -\frac{2}{3}\sqrt{\frac{1}{3}}$ $= -\frac{2\sqrt{3}}{9}$ These are the coordinates of the TP. At P the gradient is undefined.	M1 A1 A1 B1 **4**	Accept a simplified form with √3 in the bottom. Accept reference to infinity or to vertical.
	(iv)	$\int_{-1}^{0} x\sqrt{1+x}dx$ $= \int_0^1 (u-1)u^{\frac{1}{2}}du, \quad u = 1+x$ $\Rightarrow \frac{du}{dx} = 1$ $= \int_0^1 (u^{\frac{3}{2}} - u^{\frac{1}{2}})du$ $= \left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right]_0^1$ $= -\frac{4}{15}$ The magnitude represents the area bounded by the curve and axis between P and O. It is negative because the curve is below the axis (except at the end points).	M1 E1 M1A1 A1 B1 B1 **7**	Integral in *u* Change of limits Integrating One for geometry and one for sign.
	(v)	$y = x\sqrt{1+x}\sin 2x$ $\Rightarrow \frac{dy}{dx} = \frac{3x+2}{2\sqrt{1+x}}\sin 2x + x\sqrt{1+x}.2\cos 2x$	M1A1 **2**	

9	**(i)**	Range of f is $[0,\infty)$, of g $(-\infty,\infty)$. f has no inverse because (say) for any value of f > 0 there are 2 corresponding values of x	B1 B1 E1 **3**	
	(ii)	$y = 2x - 1$ $\Rightarrow x = \frac{1}{2}y + \frac{1}{2}$ $\Rightarrow g^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$ 	M1 A1 B1 B1 **4**	One mark for one line, and one mark for second correctly related
	(iii)	$gf(x) = 2x^2 - 1$ $fg(x) = (2x-1)^2$	B1 B1 **2**	
	(iv)	$2x^2 - 1 = (2x-1)^2$ $\Rightarrow 0 = 2x^2 - 4x + 2$ $\Rightarrow 0 = 2(x-1)^2$ $\Rightarrow x = 1$ 	M1 A1 B1 B1 **4**	$y = (2x-1)^2$ $y = (2x-1)^2$
	(v)	$f(x+a) = (x+a)^2$ $g^2(x) = 2(2x-1) - 1 = 4x - 3$ $f(x+a) = g^2(x) \Rightarrow (x+a)^2 = 4x - 3$ $\Rightarrow x^2 + (2a-4)x + a^2 + 3 = 0$ $\Rightarrow$ There are two roots to this equation if $(2a-4)^2 > 4(a^2+3)$ i.e. $4a^2 - 16a + 16 > 4a^2 + 12$ $\Rightarrow 16a < 4 \Rightarrow a < \frac{1}{4}$	B1 M1 M1 A1 A1 **5**	Both fns correct Equating Using $b^2 - 4ac$ Correct inequality Result