MEI

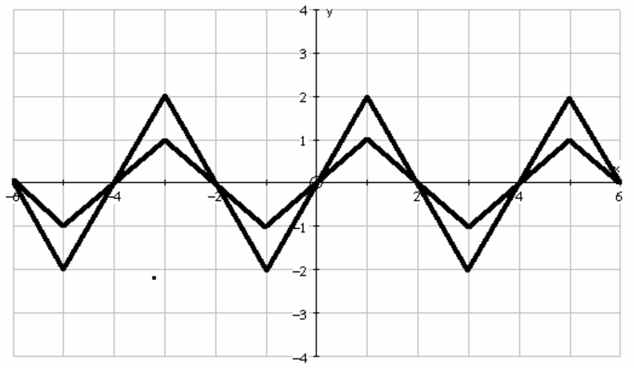
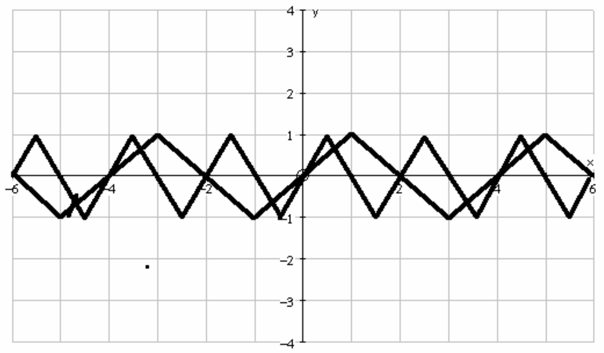
Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

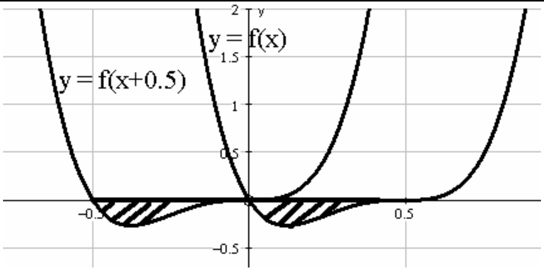
METHODS OF ADVANCED MATHEMATICS, C3

Practice Paper C3-C

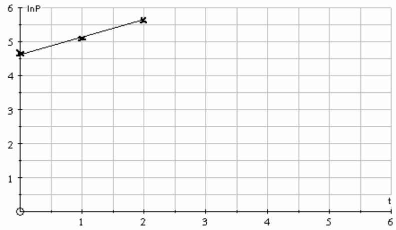
MARK SCHEME

Qu		Answer	Mark	Comment
Section A				
1		Take n = 11; the result is divisible by 11. (n = 10 is the smallest number)	M1A1 **2**	
2	**(i)**	$\mathrm{f}(-x)$ $= \left\|(-x)^3\right\|$ $= \left\|-x^3\right\|$ $= \left\|x^3\right\|$ $= \mathrm{f}(x)$ This argument holds regardless of whether x is positive, negative or zero.	M1 A1 **2**	Using $-x$
	(ii)	$\mathrm{g}(-x)$ $= (-x-1)^3$ g(1) ≠ g(-1) (say), so the function is not odd.	M1 A1 **2**	
3	**(i)**		B1 B1 **2**	One for the correct amplitude and one for a function of the right period correctly placed.
	(ii)		B1 B1 **2**	One for a function of the right period and one for the correct amplitude correctly placed.
4		$V = \frac{4}{3}\pi r^3$ $\Rightarrow \frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$ and $\frac{\mathrm{d}r}{\mathrm{d}t} = 2$ $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}r}.\frac{\mathrm{d}r}{\mathrm{d}t}$ $= 4\pi \times 10^2 \times 2$ $= 800\pi$ $(= 2513.27...)$ The rate of increase of V is 2500 cm ^{3}/sec, to 2sf (say).	M1 B1 B1 A1 A1 **5**	B1 for number, B1 for units.

5	**(i)**	$x^2+y^2=25$ $\Rightarrow 2x+2y\dfrac{dy}{dx}=0$ $\Rightarrow \dfrac{dy}{dx}=-\dfrac{x}{y}$	M1A1 E1 **3**	
	(ii)	Gradient of normal = $\dfrac{4}{3}$ $\Rightarrow y-4=\dfrac{4}{3}(x-3)$ $\Rightarrow 3y=4x$ Or equivalent.	B1 M1 A1 **3**	
6	**(a)**	$\int x\cos 2x dx$ $=\dfrac{x}{2}\sin 2x-\int\dfrac{1}{2}\sin 2x dx$ $=\dfrac{x}{2}\sin 2x+\dfrac{1}{4}\cos 2x+c$	M1 A1 A1 **3**	
	(b)	$\int_2^3\dfrac{x}{x^2+1}dx$ $u=x^2+1\Rightarrow\dfrac{du}{dx}=2x$ $x=2\Rightarrow u=5,\ x=3\Rightarrow u=10$ $=\dfrac{1}{2}\int_5^{10}\dfrac{1}{u}du=\dfrac{1}{2}\left[\ln u\right]_5^{10}=\dfrac{1}{2}\ln 2$	M1 A1 A1 A1 A1 **5**	Substitution Integral in u Limits Integral Answer
7		For turning point $y=e^{-x}\sin x$ $\Rightarrow\dfrac{dy}{dx}=-e^{-x}\sin x+e^{-x}\cos x$ $=0$ when $\sin x=\cos x\Rightarrow x=\dfrac{\pi}{4}$ (in this range) $\Rightarrow$ $OA'=\dfrac{\pi}{4}$ For intersection: $e^{-x}\sin x=e^{-x}\Rightarrow\sin x=1$ $\Rightarrow x=\dfrac{\pi}{2}$(in this range) $\Rightarrow OB'=\dfrac{\pi}{2}\Rightarrow A'B'=\dfrac{\pi}{4}$	M1 A1 M1 A1 M1 A1 A1 **7**	Product = 0 Solving

Section B				
8	**(i)**	$\frac{dy}{dx} = 5x.6(2x-1)^2 + 5(2x-1)^3$ $= 5(2x-1)^2(8x-1)$ Where the gradient is zero, $x = \frac{1}{2}$ (double root, where the curve touches the x-axis) or $x = \frac{1}{8}$ (at S).	M1A1 M1 A1 **4**	
	(ii)	Area has magnitude $\left\lvert \int_0^{\frac{1}{2}} 5x(2x-1)^3 dx \right\rvert$ $= \left\lvert \int_0^{\frac{1}{2}} (40x^4 - 60x^3 + 30x^2 - 5x) dx \right\rvert$ $= \left\lvert \left[8x^5 - 15x^4 + 10x^3 - \frac{5x^2}{2} \right]_0^{\frac{1}{2}} \right\rvert$ $= \left\lvert \frac{1}{4} - \frac{15}{16} + \frac{10}{8} - \frac{5}{8} \right\rvert$ $= \left\lvert -\frac{1}{16} \right\rvert$ So area is $\frac{1}{16}$ units^2. .	B1 M1A1 A2 A1 **6**	(or at end if the -ve sign is explained) Expand or integrate by parts A1 for some terms right *Alternatively by substitution*
	(iii)	$f(x+0.5)$ $= 5(x+0.5)(2[x+0.5]-1)^2$ $= 40x^3(x+0.5)$	M1 A1 **2**	
	(iv)	$\int_{-\frac{1}{2}}^0 40x^3(x+0.5)\,dx$ $= \int_{-\frac{1}{2}}^0 (40x^4 + 20x^3)\,dx$ $= \left[8x^5 + 5x^4\right]_{-\frac{1}{2}}^0 = 0 - \left(-\frac{8}{32} + \frac{5}{16}\right) = -\frac{1}{16}$	M1 A1 A1 A1 **4**	Multiply out Terms Both terms
	(v)	 The area representing the one integral is a translation of that representing the other, so their values are equal.	B1 E1 **2**	

<table>
<tr><td>9</td><td>(i)</td><td>$P = P_0 e^{kt}$
$\Rightarrow \ln P = \ln P_0 + kt$
So if y is identified with ln*P*, *m* with *k* and *x* with *t*, we have $y = mx + c$; gradient *k*, intercept lnP_0.</td><td>M1A1

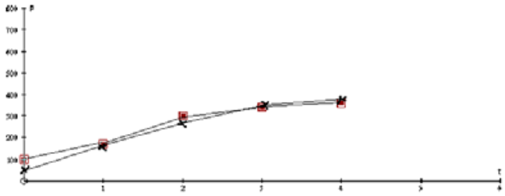
E1
3</td><td></td></tr>
<tr><td></td><td>(ii)</td><td><table><tr><td>*T*</td><td>*P*</td><td>ln *P* (to 3dp)</td></tr><tr><td>0</td><td>100</td><td>4.61</td></tr><tr><td>1</td><td>170</td><td>5.14</td></tr><tr><td>2</td><td>300</td><td>5.70</td></tr></table>


Gradient estimate (about)
$k = 0.55$;
Intercept (about) 4.61;
So P_0 = exp(4.61) = 100,
to 2 sf.</td><td>B1

B1
B1

M1
A1
B1
6</td><td>for table of values
points plotted
Straight line

Gradient

P_0</td></tr>
<tr><td></td><td>(iii)</td><td><table><tr><td>t</td><td>P (to 1 dp)</td></tr><tr><td>0</td><td>52.2</td></tr><tr><td>1</td><td>170</td></tr><tr><td>2</td><td>287.8</td></tr><tr><td>3</td><td>336.1</td></tr><tr><td>4</td><td>357.4</td></tr></table>
</td><td>M1
A1

M1
A1
5</td><td>For 1st curve

For 2nd curve</td></tr>
<tr><td></td><td>(iv)</td><td>$375 = 150\arctan(t-1) + 170$
$\Rightarrow t = 1 + \tan\frac{205}{150}$
$= 5.83....$
So the population will exceed 375 in 6 years.</td><td>B1

M1
A1

B1
4</td><td></td></tr>
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