

# Core Mathematics C3                      For Edexcel Advanced Level

## Paper J

**Time: 1 hour 30 minutes**

### *Instructions and Information*

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Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### *Advice to Candidates*

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You must show sufficient working to make your methods clear to an examiner.

Answers without working may gain no credit.

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1. Simplify

$$\frac{4x^2 - 25}{x^2 + x} \div \frac{2x^2 - x - 10}{x^2 + 3x + 2}. \quad (6)$$


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2. (a) Given  $x = \tan y$ , find  $\frac{dx}{dy}$  and hence find  $\frac{dy}{dx}$  in terms of  $x$ . (4)

(b) Show that  $(1 + x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$ . (3)

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3. The  $n^{\text{th}}$  term of an arithmetic progression is  $\ln(pq^{n-1})$  where  $p$  and  $q$  are positive integers.

(a) What is the first term of the sequence? (1)

(b) Show that the common difference is  $\ln q$ . (2)

(c) Find, in terms of  $\ln p$ ,  $\ln q$  and  $n$ , the sum of the first  $n$  terms of the series formed by the terms of the sequence. (3)

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4. The root of the equation  $f(x) = 0$ , where

$$f(x) = 2x + \ln 3x - 5$$

is to be estimated by using an iterative formula.

(a) Show that the root  $\alpha$ , such that  $f(\alpha) = 0$ , lies in the interval  $[1, 2]$ . (2)

(b) Show that  $f(x) = 0$  can be rewritten as

$$x = \frac{1}{2}(5 - \ln 3x). \quad (2)$$

(c) Use the iteration

$$x_{n+1} = \frac{1}{2}(5 - \ln 3x_n) \quad \text{with} \quad x_0 = 1.5,$$

to obtain the values of  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$ . (2)

(d) Give the value of  $\alpha$  correct to 3 decimal places. (1)

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5. (a) Given that

$$\cos(2x - 60) = 2 \sin(2x + 30),$$

prove that  $\tan 2x = -\frac{1}{\sqrt{3}}$ . (5)

- (b) Using the result from part (a), find two values of  $x$ ,  $0 < x < 180^\circ$ , which satisfy the equation

$$2 \sin(2x + 30) - \cos(2x - 60) = 0. \quad (3)$$


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6. (a) On the same axes sketch the graphs of  $C_1$ ,  $y = e^{\frac{1}{2}x}$ , and  $C_2$ ,  $y = e^{-2x}$ .

The graphs intersect at the point  $A$ . (4)

(b) State the coordinates of  $A$ . (1)

(c) Prove that the tangent at point  $A$  to the curve  $C_1$  is the normal to the curve  $C_2$  at the same point. (5)

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7. Differentiate with respect to  $x$ ,

(a)  $(3x + 1)^7$ , (3)

(b)  $\ln \sqrt{(4x + 1)}$ , (3)

(c)  $\cos 7x$ . (3)

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8. The function  $f$  is defined by

$$f: x \mapsto |2x - 1| - 4, \quad x \in \mathbb{R}.$$

(a) Sketch the graph of  $y = f(x)$ . (2)

(b) Solve the equation  $f(x) = 3$ . (3)

The function  $g$  is defined by

$$g: x \mapsto x^2 - 8x + 17, \quad x \geq 0.$$

(c) Find the range of  $g$ . (3)

(d) Find  $gf(3)$ . (2)

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9. (a) (i) Express

$9 \cos \theta - 40 \sin \theta$  in the form

$$R \cos(\theta + \alpha) \quad \text{where } R > 0 \quad \text{and} \quad 0 < \alpha < \frac{\pi}{2}. \quad (4)$$

(ii) Hence solve the equation

$$9 \cos \theta - 40 \sin \theta = 6,$$

$$\text{for } 0 < \theta < \frac{\pi}{2}, \text{ giving your answer to 2 decimal places.} \quad (3)$$

(b) Solve the equation

$$13 + 10 \cot \theta = 3 \tan \theta,$$

$$\text{for } 0 < \theta < \frac{\pi}{2}, \text{ giving your answer to 2 decimal places.} \quad (5)$$

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**END**

**TOTAL 75 MARKS**