

**Worked Solutions**

**Edexcel C3 Paper J**

$$1. \frac{(2x-5)(2x+5)}{x(x+1)} \times \frac{(x+2)(x+1)}{(2x-5)(x+2)} = \frac{2x+5}{x} \quad (6)$$


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$$2. (a) \frac{dx}{dy} = \sec^2 y = 1 + x^2$$

$$\frac{dy}{dx} = \frac{1}{1+x^2} \quad (4)$$

$$(b) \frac{d^2y}{dx^2} = \frac{-2x}{(1+x^2)^2}$$

$$(1+x^2) \frac{(-2x)}{(1+x^2)^2} + 2x \cdot \frac{1}{1+x^2} = 0 \quad (3)$$


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$$3. (a) n = 1, T_1 = \ln p \quad (1)$$

$$(b) T_2 = \ln(pq) = \ln p + \ln q$$

$$\therefore d = \ln pq - \ln p = \ln q \quad (2)$$

$$(c) S_n = \frac{n}{2} (\ln p + \ln pq^{n-1})$$

$$= \frac{n}{2} (\ln p + \ln p + (n-1) \ln q) = \frac{n}{2} (2 \ln p + (n-1) \ln q) \quad (3)$$


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$$4. (a) \left. \begin{array}{l} f(1) = 2 + \ln 3 - 5 = -1.901 \\ f(2) = 4 + \ln 6 - 5 = 0.79 \end{array} \right\} \begin{array}{l} \text{change in sign} \Rightarrow \\ \text{root in interval} \end{array} \quad (2)$$

$$(b) 5 - \ln 3x = 2x \quad \therefore x = \frac{1}{2}(5 - \ln 3x) \quad (2)$$

$$(c) x_1 = 1.74796, \quad x_2 = 1.67147, \quad x_3 = 1.69384, \quad x_4 = 1.68719 \quad (2)$$

$$(d) \alpha = 1.689(3 \text{ d.p.}) \quad (1)$$


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$$5. (a) \cos 2x \cos 60 + \sin 2x \sin 60 = 2 \sin 2x \cos 30 + 2 \cos 2x \sin 30$$

$$\cos 2x \cdot \frac{1}{2} + \sin 2x \cdot \frac{\sqrt{3}}{2} = 2 \sin 2x \cdot \frac{\sqrt{3}}{2} + 2 \cos 2x \cdot \frac{1}{2}$$

$$-\cos 2x = \sqrt{3} \sin 2x$$

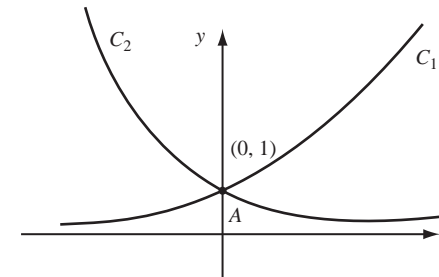
$$\tan 2x = -\frac{1}{\sqrt{3}} \quad (5)$$

$$(b) 2x = 150, 330$$

$$x = 75, 165 \quad (3)$$


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6. (a)



$C_2$  is steeper than  $C_1$

(b) A is at (0, 1)

$$(c) C_1 : \frac{dy}{dx} = \frac{1}{2}e^{\frac{1}{2}x}, \text{ at } x=0, \quad \frac{dy}{dx} = \frac{1}{2}$$

$$C_2 : \frac{dy}{dx} = -2e^{-2x}, \text{ at } x=0, \quad \frac{dy}{dx} = -2$$

gradient of normal to  $C_2 = +\frac{1}{2}$  (product = -1)

$$\therefore \text{gradient of tangent to } C_1 = \frac{1}{2} = \text{gradient of normal to } C_2 \quad (6)$$


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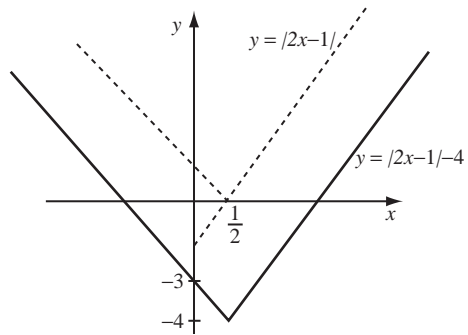
7. (a)  $7(3x + 1)^6 \times 3$  (3)

(b)  $\frac{d}{dx} \left( \frac{1}{2} \ln(4x + 1) \right) = \frac{1}{2} \left( \frac{4}{4x + 1} \right)$  (3)

(c)  $-7 \sin 7x$  (3)

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8. (a)



(b)  $|2x - 1| - 4 = 3$  (2)

$2x - 1 - 4 = 3, x = 4$  or  $-2x + 1 - 4 = 3 \quad x = -3$  (3)

(c)  $g(x) = x^2 - 8x + 16 + 1 \quad g(x) = (x - 4)^2 + 1 \quad g(x) \geq 1$  (3)

(d)  $f(3) = 5 - 4 = 1 \quad gf(3) = g(1) = 10$  (2)

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9. (a) (i)  $9 \cos \theta - 40 \sin \theta$   
 $= R \cos \theta \cos \alpha - R \sin \theta \sin \alpha$

$R^2 = 9^2 + 40^2 \Rightarrow R = 41$

$\tan \alpha = \frac{40}{9} \Rightarrow \alpha = 1.35^\circ$  (4)

(ii)  $\cos(\theta + 1.35) = \frac{6}{41}$

$\theta + 1.35 = 1.42$

$\theta = 0.07^\circ$  (3)

(b)  $13 + \frac{10}{\tan \theta} = 3 \tan \theta$

$3 \tan^2 \theta - 13 \tan \theta - 10 = 0$  (2)

$(3 \tan \theta + 2)(\tan \theta - 5) = 0$

$\tan \theta = 5$

or  $-\frac{2}{3}$  (no solution)

$\tan \theta = 5 \Rightarrow \theta = 1.37^\circ$  (5)

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