

# Core Mathematics C3                      For Edexcel Advanced Level

## Paper I

**Time: 1 hour 30 minutes**

### *Instructions and Information*

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Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### *Advice to Candidates*

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You must show sufficient working to make your methods clear to an examiner.

Answers without working may gain no credit.

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1. The function  $f$  is defined by

$$f: x \mapsto \frac{1-2x}{2-x}, \quad x \in \mathbb{R}, \quad x \neq 2.$$

(a) Prove that  $f^{-1}(x) = f(x)$  for all  $x \in \mathbb{R}, x \neq 2$ . (3)

(b) Hence find, in terms of  $k$ ,  $ff(k^2)$ , where  $k^2 \neq 2$ . (2)

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2. The function  $g$  is given by

$$g: x \mapsto \ln |4x - 12|, \quad x \in \mathbb{R}, \quad x \neq 3. \quad (3)$$

(a) Sketch the graph of  $y = g(x)$ .

(b) Find the exact coordinates of all the points at which the curve  $y = g(x)$  meets the coordinate axes. (3)

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3.  $f(x) = x - \frac{1}{x-2} + \frac{5}{x^2+x-6}, \quad x \in \mathbb{R}, \quad x > 2.$

(a) Show that  $f(x) = \frac{x^2+3x-1}{x+3}$ . (5)

(b) Solve the equation

$$f'(x) = \frac{26}{25}. \quad (5)$$


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4. (a) Given  $y = \frac{e^{5x}}{x}$ , find  $\frac{dy}{dx}$  and the value of  $x$  for which  $\frac{dy}{dx} = 0$ . (4)

(b) (i) Given  $x = \sin^2 3y$ , find  $\frac{dx}{dy}$  in terms of  $y$ . (3)

(ii) Evaluate  $\frac{dy}{dx}$  for  $y = \frac{\pi}{12}$ . (3)

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5. (a) Express

$2.5 \sin 2x + 6 \cos 2x$  in the form

$R \sin(2x + \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ , giving your values of  $R$  and  $\alpha$  to 3 decimal places where appropriate. (4)

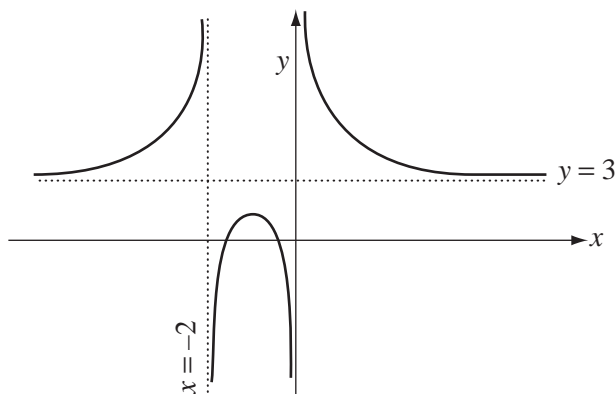
(b) Express  $5 \sin x \cos x - 12 \sin^2 x$  in the form

$a \cos 2x + b \sin 2x + c$ , where  $a$ ,  $b$  and  $c$  are constants to be found. (4)

(c) Hence, using your answer to part (a), deduce the maximum value of

$5 \sin x \cos x - 12 \sin^2 x$ . (2)

6.



The diagram shows the sketch of part of the curve with equation  $y = f(x)$ ,  $x \in \mathbb{R}$ ,  $x \neq 0$ ,  $x \neq -2$ .

The curve has a maximum at  $(-1, 2)$ .

The lines  $y = 3$ ,  $x = -2$  and the  $y$ -axis are asymptotes to the curve as shown.

On separate diagrams sketch the graphs of

(a)  $y = |f(x)|$  (3)

(b)  $y = f(2x)$  (3)

(c)  $y = f(x - 1) - 2$  (4)

In each case state the equations of the new asymptotes and the coordinates of the turning points.

(d) Solve the equation  $f(x - 1) - 2 = 0$  (2)

7. The curve  $C$  has the equation  $y = f(x)$  where

$$f(x) = \frac{1}{2} \ln x + \frac{1}{x^2}, \quad x > 0.$$

$P$  is a stationary point on  $C$ .

- (a) Calculate the  $x$ -coordinate of  $P$ . (4)

- (b) Show that the  $y$ -coordinate of  $P$  can be expressed in the form  $k^{-1} \ln k + k^{-2}$ ,  
where  $k$  is a constant to be found. (2)

The point  $Q$  on  $C$  has  $x$ -coordinate 1.

- (c) Show that the equation to the normal at  $Q$  can be written as

$$Ay + Bx + C = 0$$

where  $A$ ,  $B$  and  $C$  are integers to be found. (3)

8. A cup of tea, initially at boiling point, cools according to Newton's law of cooling so that after  $t$  minutes its temperature,  $T^\circ C$ , is given by

$$T = 15 + 85e^{-\frac{t}{8}}.$$

- (a) Sketch the graph of  $T$  against  $t$ . (3)

- (b) What is the temperature of the tea after 4 minutes? (2)

- (c) How long does it take the tea to cool to  $40^\circ C$ ? (3)

- (d) Find  $\frac{dT}{dt}$  and hence find the value of  $T$  at which the temperature is decreasing at the rate of  $1.7^\circ C$  per minute. (4)

- (e) However long the cup of tea is left to cool down, it never falls below a certain temperature. What temperature is that? (1)

**END**

**TOTAL 75 MARKS**