

Worked Solutions

Edexcel C3 Paper H

1. $x^2 - y^2 = (x - y)(x + y)$

Assume both x and y are even then $x - y = 2m$ where m is an integer.

$$x + y = 2n \quad \text{where } n \text{ is an integer.}$$

$$\therefore x^2 - y^2 = 4mn \neq 18$$

Assume both x and y are odd then $x - y = 2m'$

$$x + y = 2n'$$

$$\therefore x^2 - y^2 = 4m'n' \neq 18$$

Assume x odd, y even then $x - y = 2m_1 + 1$ odd

$$x + y = 2m_2 + 1 \quad \text{odd}$$

$$\therefore x^2 - y^2 = 4m_1m_2 + 2(m_1 + m_2) + 1 \text{ which is odd } \therefore \text{contradiction.} \quad (6)$$

2.
$$\frac{(2x-1)(x+3)}{(2x-1)(2x+1)} \times \frac{x(2x+1)}{(x+3)(x+5)} = \frac{x}{x+5} \quad (6)$$

3. (a) $y = 2x - 1 \quad \therefore 2x = y + 1$

$$f^{-1}: x \mapsto \frac{x+1}{2}, \quad x \in \mathbb{R} \quad (2)$$

(b) $gf^{-1}(x) = 3\left(\frac{x+1}{2}\right)^2 + 1 \quad (2)$

(c) $3\left(\frac{x+1}{2}\right)^2 + 1 = \frac{7}{4}$
 $3(x+1)^2 + 4 = 7$
 $(x+1)^2 = 1$
 $x = 0 \text{ or } -2 \quad (3)$

4. (a) x must be obtuse.

$$\therefore \tan x < 0, \sin x > 0$$

$$\cos^2 x = \frac{25}{169}$$

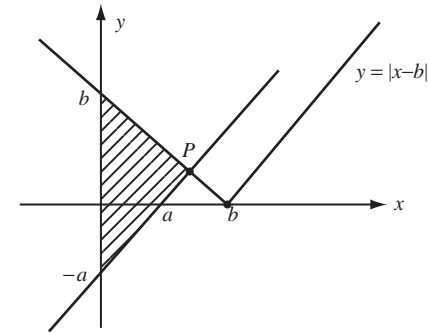
$$\tan^2 x = \sec^2 x - 1 = \frac{169}{25} - 1 = \frac{144}{25}$$

$$\tan x = -\frac{12}{5} \quad (4)$$

(b) $\operatorname{cosec} 2x = \frac{1}{2 \sin x \cos x}$

$$= \frac{1}{2 \cdot \frac{12}{13} \cdot \frac{-5}{13}} = \frac{-169}{120} \quad (4)$$

5. (a)



(b) at $P \quad x - a = -(x - b)$

$$x - a = -x + b$$

$$x = \frac{a+b}{2}$$

$$\text{Area} = \frac{1}{2}(b+a)\left(\frac{a+b}{2}\right) = \frac{(a+b)^2}{4} \quad (4)$$

6. $y = \ln(3 - x)$
 $y = \ln(x + 2) - 2$
 $\ln(3 - x) = \ln(x + 2) - 2$
 $\ln(x + 2) - \ln(3 - x) = 2$
 $\frac{x + 2}{3 - x} = e^2$
 $x + 2 = 3e^2 - xe^2$
 $x(1 + e^2) = 3e^2 - 2$
 $x = 2.40$ (2 d.p.)
 $y = -0.52$ (2 d.p.)

(8)

7. (a) (i) $x \cdot \frac{3}{3x + 4} + \ln(3x + 4)$

(3)

(ii) $5(3x^2 + 7)^4 \cdot 6x$

$30x(3x^2 + 7)^4$

(3)

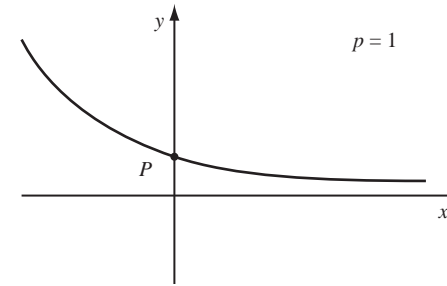
(b) $\frac{d}{dx} \left(\frac{\cos x}{\sin x} \right)$

$= \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x}$

$= \frac{-1}{\sin^2 x} = -\operatorname{cosec}^2 x$

(4)

8. (a)



(2)

(b) $\frac{dy}{dx} = -\frac{1}{2}e^{-\frac{1}{2}x}$ at $x = q$: $\frac{dy}{dx} = -\frac{1}{2}e^{-\frac{1}{2}q}$

gradient of normal $= 2e^{\frac{1}{2}q}$

Equation of normal is $y = 2e^{\frac{1}{2}q}x$

pt. Q lies on curve $\therefore e^{-\frac{1}{2}q} = 2e^{\frac{1}{2}q}q$

$1 = 2e^q q$

$\therefore q$ satisfies $2e^x x - 1 = 0$

(4)

$2e^x x = 1$

$x = \frac{1}{2e^x}$

(1)

$x_1 = 0.33516, x_2 = 0.35761, x_3 = 0.34967, x_4 = 0.35246$

$q = 0.352$ (3 d.p.)

(3)

9. (a) (i) $R^2 = 9^2 + 40^2$

$$R = 41$$

$$\tan \alpha = \frac{9}{40}$$

$$\alpha = 12.68$$

(ii) $41 \cos(\theta + 12.68) = 4$ (4)

$$\theta + 12.68 = 84.40$$

$$\theta = 71.7(1 \text{ d.p.})$$
 (3)

(b) $6 \sin^2 \theta - \sin \theta - 2 = 0$

$$(2 \sin \theta + 1)(3 \sin \theta - 2) = 0$$

$$\sin \theta = -\frac{1}{2}, \text{ no solutions}$$

$$\sin \theta = \frac{2}{3}, \quad \theta = 41.8^\circ, 138.2^\circ$$
 (5)
