Core Mathematics C3   For Edexcel
Advanced Level

Paper G
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

1. The third and fifth terms of an arithmetic progression are $\ln 18$ and $\ln 162$ respectively.

Find (*a*) the exact value of the common difference, **(4)**

(*b*) the first term. **(2)**

2. Express
$$\frac{2x+5}{4x^2-9} \times \frac{2x^2-x-3}{2(x+1)}$$
as a single fraction in its simplest form. **(6)**

3. The function f is defined by f: $x \mapsto \cos x, \quad 0 \le x \le \pi,$

and the function g is defined by g: $x \mapsto x + \dfrac{\pi}{2}, \quad x \ge 0.$

(*a*) What is the range of f(x)? **(2)**

(*b*) What is the domain of fg(x)? **(3)**

(*c*) What is the range of fg(x)? **(2)**

4. (*a*) Given that $\sin x = \dfrac{7}{25}$ and that the angle x is obtuse,

find the exact value $\sin 2x$. **(5)**

(*b*) Prove that $\tan\left(\dfrac{\pi}{4} - \theta\right) = \dfrac{1-\tan\theta}{1+\tan\theta}$. **(3)**

5. The function f is even and has a domain $\mathbb{R}$.
$$f(x) = a - x, \quad \text{for} \quad 0 \le x \le a$$
$$\text{and} \quad f(x) = 2x - 2a, \quad \text{for} \quad x > a,$$
where a is a positive constant

(*a*) Sketch the curve with equation $y = f(x)$ showing the coordinates of all the points at which the curve meets the axes. **(4)**

(*b*) Find, in terms of a, the values of f$(2a)$ and f$(-2a)$. **(2)**

(*c*) Find the values of x for which f$(x) = \dfrac{1}{2}a$. **(3)**

6. Show that

$$y = e^{2x}(\cos 3x + \sin 3x)$$

satisfies the equation $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 13y = 0.$ **(11)**

7. The curve C has the equation $y = f(x)$, where

$$f(x) = 4x - \ln x, \qquad x > 0.$$

The point A is a stationary point on C.

(*a*) Calculate the coordinates of A and determine the nature of the stationary point. **(5)**

B is the point on the curve C whose x coordinate is 1.

(*b*) Find the equations of both the tangent and normal to the curve at the point B. **(6)**

The tangent at B meets the x-axis at P, and the normal at B meets the x-axis at Q.

(*c*) Find the area of the $\triangle PBQ$. **(3)**

8. (*a*) Given $f(x) = 5\sin x + 12\cos x$,

find $f(x)$ in the form $R\sin(x + \alpha)$ where R and α are positive constants to be found. $(0 < \alpha < 90°)$ **(4)**

Hence or otherwise find,

(*b*) the minimum value of $f(x) + 4$, **(2)**

(*c*) the smallest positive value of x for which this minimum value occurs, **(3)**

(*d*) the values of x, $0 < x < 360°$, which satisfy the equation

$$5\sin x + 12\cos x = 6.$$ **(5)**

END **TOTAL 75 MARKS**