# Worked Solutions

## Edexcel C3 Paper F

**1.** (*a*) $\dfrac{2}{x+3} - \dfrac{1}{(x+3)(x+4)}$

$\dfrac{2(x+4)-1}{(x+3)(x+4)}$

$\dfrac{2x+7}{(x+3)(x+4)}$ **(4)**

(*b*) $2x + 7 = 0 \Rightarrow x = -3\frac{1}{2}$ **(2)**

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**2.** (*a*) $\mathrm{f}^{-1} : x \mapsto \dfrac{2}{x} + 3, \qquad x \in \mathbb{R}, \qquad x \neq 0$

let $y = \dfrac{2}{x-3}$

$yx - 3y = 2$

$yx = 2 + 3y$

$x = \dfrac{2}{y} + 3$

$\therefore \mathrm{f}^{-1}: x \to \dfrac{2}{x} + 3,\ x \in \mathbb{R},\ x \neq 0$ **(4)**

(*b*) $\mathrm{f}(4) = \dfrac{2}{4-3} = 2$

$\mathrm{ff}^{-1}(7) = 7$ **(2)**

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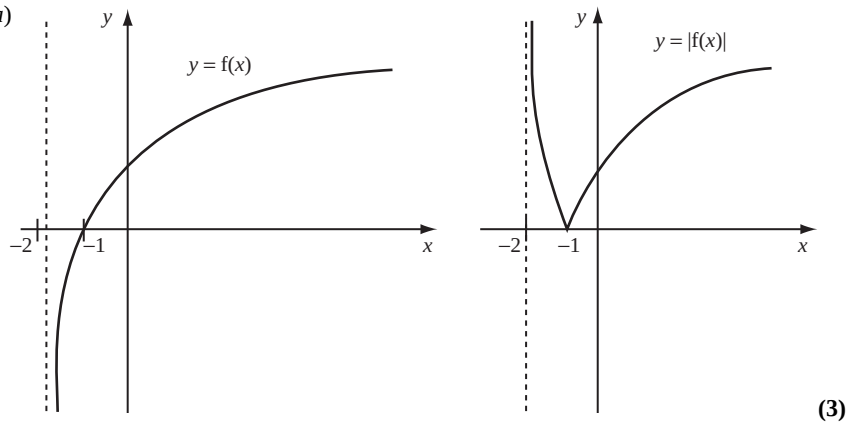
**3.** (*a*)




**(3)**

(*b*) The line $y = x$ only cuts the graph once. **(1)**

(*c*) $\left.\begin{array}{l}\ln(2+2) - 2 = -0.6137\\ \ln(1+2) - 1 = 0.098\end{array}\right\}$ change of sign $\therefore$ root lies in interval $[1, 2]$

**(2)**

(*d*) $x_1 = 1.0986,\ x_2 = 1.1309,\ x_3 = 1.1413,\ x_4 = 1.1446,\ x_5 = 1.1457$

solution 1.146 (3 d.p.). **(3)**

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**4.** (*a*) $x^2 \cdot \dfrac{1}{x} + 2x \ln x = x(1 + 2\ln x)$ **(4)**

(*b*) $2\cos 3x \cdot (-\sin 3x) \cdot 3 = -6 \sin 3x \cos 3x$ **(3)**

(*c*) $\dfrac{x \cdot \cos x - \sin x}{x^2}$ **(3)**

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**5.** (*a*) $\text{R.H.S} = \dfrac{1-\tan^2\theta}{2\tan\theta} \qquad (\times \tan^2\theta)$

$\equiv \dfrac{1}{\tan 2\theta}$

$\equiv \cot 2\theta$ (**5**)

(*b*) $\cot^2\theta - 1 = 2\cot\theta$

$\therefore \cot 2\theta = 1 \Rightarrow \tan 2\theta = 1$

$2\theta = \dfrac{\pi}{4}, \quad \dfrac{5\pi}{4}, \quad \dfrac{9\pi}{4}, \quad \dfrac{13\pi}{4}$

$\therefore \theta = \dfrac{\pi}{8}, \quad \dfrac{5\pi}{8}, \quad \dfrac{9\pi}{8}, \quad \dfrac{13\pi}{8}$ **(4)**

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**6.** (*a*) $e^{2x} + 6 = 5e^x$

$e^{2x} - 5e^x + 6 = 0$

$(e^x - 3)(e^x - 2) = 0$ (**3**)

(*b*) $e^x = 3 \Rightarrow x = \ln 3$ (or 1.0986 4 d.p.)

and $e^x = 2$

$\Rightarrow x = \ln 2$ (or 0.6931 to 4 d.p.) (**4**)

(*c*) $e^{2(x+1)} - 5e^{x+1} + 6 = 0$

$\left.\begin{array}{l} e^{x+1} = 3 \\ e^{x+1} = 2 \end{array}\right\} \Rightarrow \begin{array}{l} x = \ln 3 - 1 \quad (\text{or } 0.0986 \quad 4 \text{ d.p.}) \\ x = \ln 2 - 1 \quad (\text{or } -0.3069 \quad 4 \text{ d.p.}) \end{array}$ **(4)**

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**7.** (*a*) $25\left(\dfrac{7}{25}\sin x + \dfrac{24}{25}\cos x\right) \equiv R(\cos\alpha \sin x + \sin\alpha \cos x)$

$\Rightarrow R = 25$ and $\tan\alpha = \dfrac{24}{7} \Rightarrow \alpha = 73.7°$ (1 d.p.) **(4)**

(*b*) $25\sin(x + 73.7) = 15$

$\sin(x + 73.7) = \dfrac{3}{5}$

$x + 73.7 = 36.9, \quad 143.1, \quad 360 + 36.9$

$x = 69.4°, \quad 323.2°$ **(4)**

(*c*) $15\sec x - 7\tan x = 24$

$\Rightarrow 15 - 7\sin x = 24\cos x.$ **(2)**

(*d*) 25, $x = 16.3$ **(2)**

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**8.** (*a*) $\dfrac{dx}{dy} = \cos y$ **(2)**

(*b*) $y = \dfrac{\pi}{4} \qquad \cos y = \dfrac{1}{\sqrt{2}} \therefore$ grad. of tgt $= \sqrt{2}$ **(3)**

(*c*) $y - \dfrac{\pi}{4} = \sqrt{2}\left(x - \dfrac{1}{\sqrt{2}}\right)$ **(2)**

(*d*) $y = 0, \ -\dfrac{\pi}{4} = \sqrt{2}x - 1$

$\sqrt{2}x = 1 - \dfrac{\pi}{4}$

$x = \dfrac{1 - \frac{\pi}{4}}{\sqrt{2}} = \dfrac{4-\pi}{4\sqrt{2}}$ **(3)**

(*e*) area $= \left(\dfrac{4-\pi}{4\sqrt{2}}\right) \cdot \dfrac{\pi}{8}$ **(2)**

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