

Edexcel Maths C3

Past Paper Pack

2005-2013

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5. (a) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2 \sin^2 A. \quad (2)$$

- (b) Show that

$$2 \sin 2\theta - 3 \cos 2\theta - 3 \sin \theta + 3 \equiv \sin \theta (4 \cos \theta + 6 \sin \theta - 3). \quad (4)$$

- (c) Express $4 \cos \theta + 6 \sin \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. (4)

- (d) Hence, for $0 \leq \theta < \pi$, solve

$$2 \sin 2\theta = 3(\cos 2\theta + \sin \theta - 1),$$

giving your answers in radians to 3 significant figures, where appropriate. (5)





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Figure 1

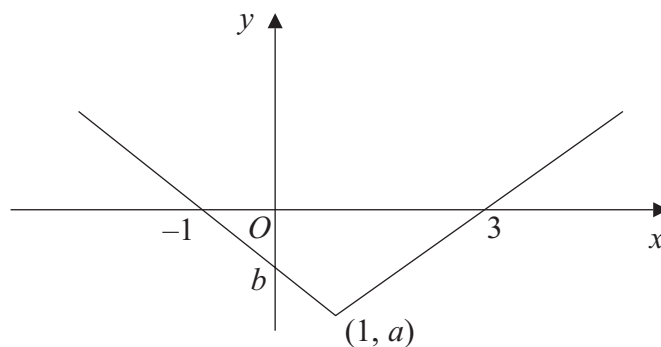


Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the x -axis at $(3, 0)$. The other line meets the x -axis at $(-1, 0)$ and the y -axis at $(0, b)$, $b < 0$.

In separate diagrams, sketch the graph with equation

(a) $y = f(x + 1)$, (2)

(b) $y = f(|x|)$. (3)

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $f(x) = |x - 1| - 2$, find

(c) the value of a and the value of b , (2)

(d) the value of x for which $f(x) = 5x$. (4)







Q7

(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

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1.

Figure 1

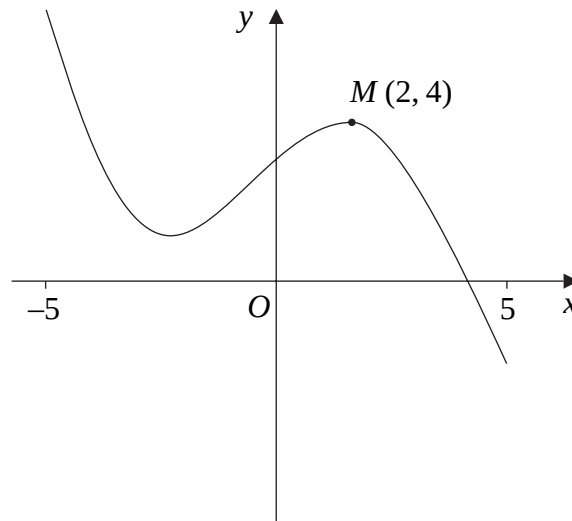


Figure 1 shows the graph of $y = f(x)$, $-5 \leq x \leq 5$.
The point $M(2, 4)$ is the maximum turning point of the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = f(x) + 3$, (2)

(b) $y = |f(x)|$, (2)

(c) $y = f(|x|)$. (3)

Show on each graph the coordinates of any maximum turning points.



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Question 1 continued

Q1

(Total 7 marks)

3



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2. Express

$$\frac{2x^2 + 3x}{(2x+3)(x-2)} - \frac{6}{x^2 - x - 2}$$

as a single fraction in its simplest form.

(7)



3. The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x-coordinate of P is 3.

(5)

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4. (a) Differentiate with respect to x

$$(i) \quad x^2 e^{3x+2}, \quad (4)$$

$$(ii) \quad \frac{\cos(2x^3)}{3x}.$$

(4)

- (b) Given that $x = 4 \sin(2y + 6)$, find $\frac{dy}{dx}$ in terms of x .

(5)

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5. $f(x) = 2x^3 - x - 4.$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}. \quad (3)$$

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{1}{2}\right)},$$

with $x_0 = 1.35$, to find, to 2 decimal places, the values of x_1 , x_2 and x_3 . (3)

The only real root of $f(x) = 0$ is α .

(c) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places. (3)



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- 8.** The functions f and g are defined by

$$f: x \rightarrow 2x + \ln 2, \quad x \in \mathbb{R},$$

$$g: x \rightarrow e^{2x}, \quad x \in \mathbb{R}.$$

- (a) Prove that the composite function gf is

$$\text{gf}: x \rightarrow 4e^{4x}, \quad x \in \mathbb{R}. \quad (4)$$

- (b) In the space provided on page 19, sketch the curve with equation $y = gf(x)$, and show the coordinates of the point where the curve cuts the y -axis.

- (c) Write down the range of gf. (1)

- (d) Find the value of x for which $\frac{d}{dx}[gf(x)] = 3$, giving your answer to 3 significant figures.
- (4)**

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1. (a) Simplify $\frac{3x^2 - x - 2}{x^2 - 1}$.

(3)

(b) Hence, or otherwise, express $\frac{3x^2 - x - 2}{x^2 - 1} - \frac{1}{x(x+1)}$ as a single fraction in its simplest form.

(3)





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Figure 1

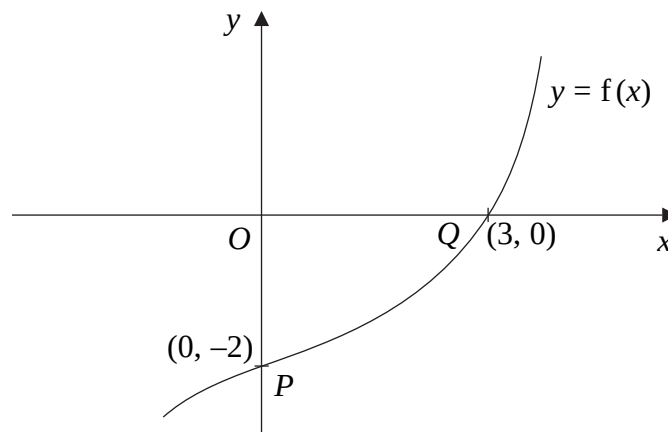


Figure 1 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$, where f is an increasing function of x . The curve passes through the points $P(0, -2)$ and $Q(3, 0)$ as shown.

In separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$, (3)

(b) $y = f^{-1}(x)$, (3)

(c) $y = \frac{1}{2} f(3x)$. (3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.



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5.

The graph shows a curve in the first quadrant of a Cartesian coordinate system. The vertical axis is labeled y and the horizontal axis is labeled x . The origin is marked with O . The curve starts at O , dips below the x -axis to a minimum point labeled P , and then rises to meet the y -axis at $(0, 1)$. The x -axis has a tick mark at $\frac{\pi}{4}$.

$$y = (2x - 1) \tan 2x, \quad 0 \leq x < \frac{\pi}{4}.$$

(a) Show that k satisfies the equation

(6)

$$x_{n+1} = \frac{1}{4}(2 - \sin 4x_n), \quad x_0 = 0.3,$$

(b) Calculate the values of x_1 , x_2 , x_3 and x_4 , giving your answers to 4 decimal places.

(3)

(c) Show that $k = 0.277$, correct to 3 significant figures.

(2)



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7. For the constant k , where $k > 1$, the functions f and g are defined by

$$f: x \mapsto \ln(x + k), \quad x > -k,$$

$$g: x \mapsto |2x - k|, \quad x \in \mathbb{R}.$$

- (a) On separate axes, sketch the graph of f and the graph of g .

On each sketch state, in terms of k , the coordinates of points where the graph meets the coordinate axes.

(5)

- (b) Write down the range of f .

(1)

- (c) Find $fg\left(\frac{k}{4}\right)$ in terms of k , giving your answer in its simplest form.

(2)

The curve C has equation $y = f(x)$. The tangent to C at the point with x -coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

- (d) Find the value of k .

(4)





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8. (a) Given that $\cos A = \frac{3}{4}$, where $270^\circ < A < 360^\circ$, find the exact value of $\sin 2A$. (5)

- (b) (i) Show that $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x$. (3)

Given that

$$y = 3\sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right),$$

- (ii) show that $\frac{dy}{dx} = \sin 2x$. (4)



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Question 8 continued

(Total 12 marks)

Q8

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Figure 1

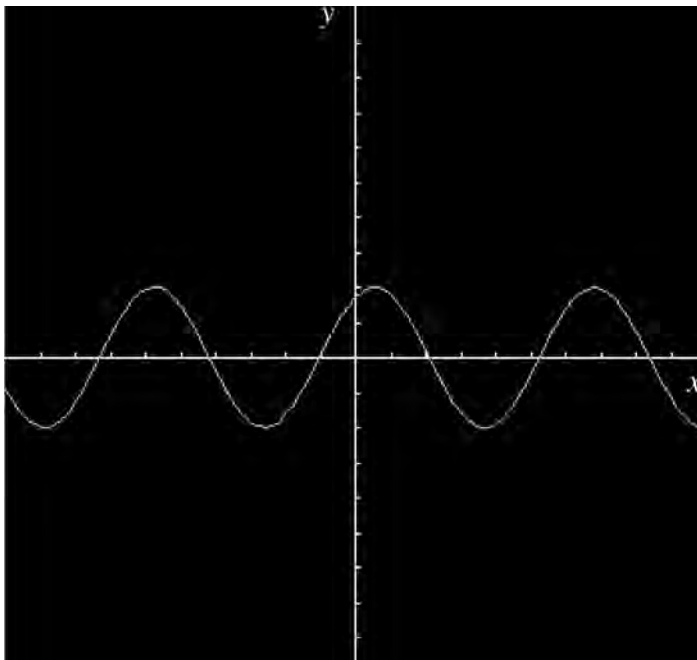


Figure 1 shows an oscilloscope screen.

The curve shown on the screen satisfies the equation

$$y = \sqrt{3} \cos x + \sin x.$$

- (a) Express the equation of the curve in the form $y = R \sin(x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(4)**

- (b) Find the values of x , $0 \leq x < 2\pi$, for which $y = 1$.



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Q5

(Total 8 marks)

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- 6.** The function f is defined by

$$f : x \mapsto \ln(4 - 2x), \quad x < 2 \quad \text{and} \quad x \in \mathbb{R}.$$

- (a) Show that the inverse function of f is defined by

$$f^{-1}: x \mapsto 2 - \frac{1}{2}e^x$$

and write down the domain of f^{-1} .

(4)

- (b) Write down the range of f^{-1} .

(1)

- (c) In the space provided on page 16, sketch the graph of $y = f^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes.

(4)

The graph of $y = x + 2$ crosses the graph of $y = f^{-1}(x)$ at $x = k$.

The iterative formula

$$x_{n+1} = -\frac{1}{2}e^{x_n}, \quad x_0 = -0.3$$

is used to find an approximate value for k .

- (d) Calculate the values of x_1 and x_2 , giving your answers to 4 decimal places.

(2)

- (e) Find the value of k to 3 decimal places.

(2)



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N 2 3 5 8 3 A 0 2 1 2 4

N 2 3 5 8 3 A 0 2 4 2 4

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1. Find the exact solutions to the equations

(a) $\ln x + \ln 3 = \ln 6$,

(2)

(b) $e^x + 3e^{-x} = 4$.

(4)



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3. A curve C has equation

$$y = x^2 e^x.$$

(a) Find $\frac{dy}{dx}$, using the product rule for differentiation.

(3)

(b) Hence find the coordinates of the turning points of C .

(3)

(c) Find $\frac{d^2y}{dx^2}$.

(2)

(d) Determine the nature of each turning point of the curve C .

(2)





5. The functions f and g are defined by

$$f : x \mapsto \ln(2x-1), \quad x \in \mathbb{R}, x > \frac{1}{2},$$

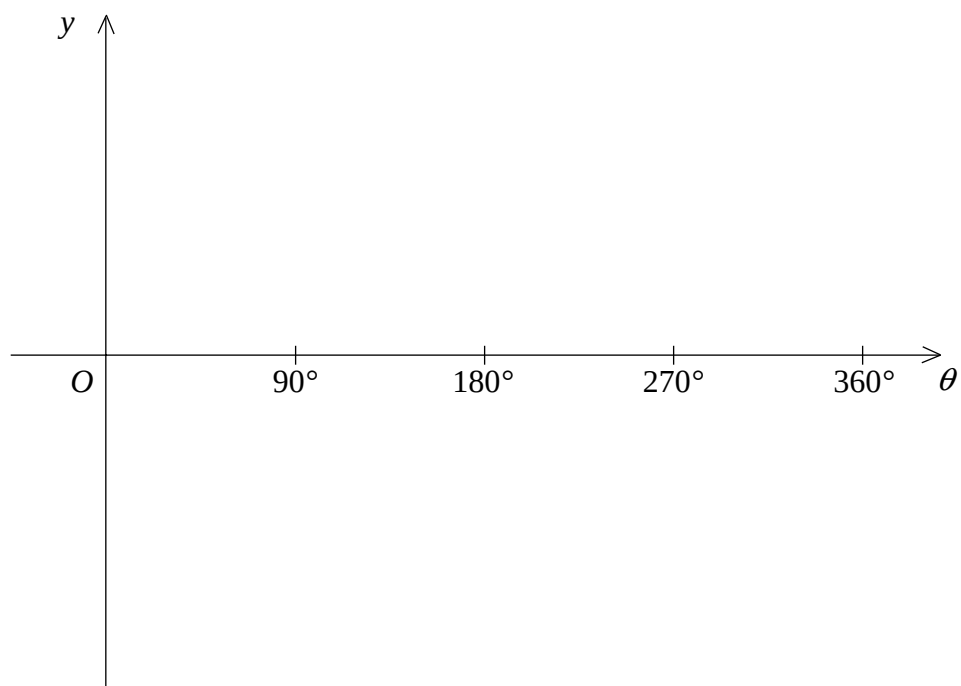
$$g: x \mapsto \frac{2}{x-3}, \quad x \in \mathbb{R}, x \neq 3.$$

- (a) Find the exact value of $fg(4)$. (2)
- (b) Find the inverse function $f^{-1}(x)$, stating its domain. (4)
- (c) Sketch the graph of $y = |g(x)|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the y -axis. (3)
- (d) Find the exact values of x for which $\left| \frac{2}{x-3} \right| = 3$. (3)





Question 7 continued



8. The amount of a certain type of drug in the bloodstream t hours after it has been taken is given by the formula

$$x = De^{-\frac{1}{8}t},$$

where x is the amount of the drug in the bloodstream in milligrams and D is the dose given in milligrams.

A dose of 10 mg of the drug is given.

- (a) Find the amount of the drug in the bloodstream 5 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 10 mg is given after 5 hours.

- (b) Show that the amount of the drug in the bloodstream 1 hour after the second dose is 13.549 mg to 3 decimal places.

(2)

No more doses of the drug are given. At time T hours after the second dose is given, the amount of the drug in the bloodstream is 3 mg.

- (c) Find the value of T .

(3)



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Question 8 continued

Q8

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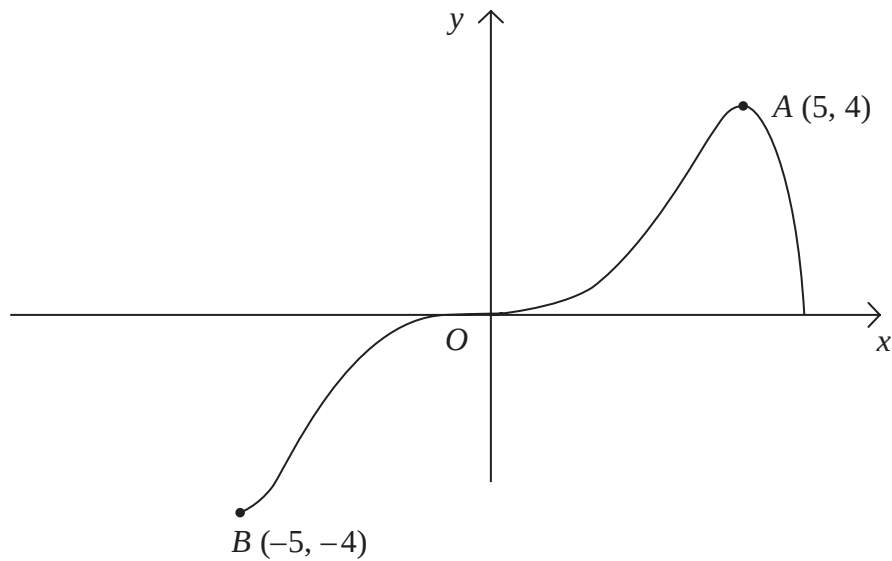


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$.
The curve passes through the origin O and the points $A(5, 4)$ and $B(-5, -4)$.

In separate diagrams, sketch the graph with equation

(a) $y = |f(x)|$, (3)

(b) $y = f(|x|)$, (3)

(c) $y = 2f(x+1)$. (4)

On each sketch, show the coordinates of the points corresponding to A and B .



Leave
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H 2 6 3 1 5 R B 0 9 2 4

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5. The radioactive decay of a substance is given by

$$R = 1000e^{-ct}, \quad t \geq 0.$$

where R is the number of atoms at time t years and c is a positive constant.

- (a) Find the number of atoms when the substance started to decay. **(1)**

It takes 5730 years for half of the substance to decay.

- (b) Find the value of c to 3 significant figures. (4)

- (c) Calculate the number of atoms that will be left when $t = 22\,920$. (2)

- (d) In the space provided on page 13, sketch the graph of R against t . (2)

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6. (a) Use the double angle formulae and the identity

$$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(4)

- (b) (i) Prove that

$$\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} \equiv 2 \sec x, \quad x \neq (2n+1)\frac{\pi}{2}.$$

(4)

- (ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} = 4.$$

(3)

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Leave
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7. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x, \quad -\pi \leq x \leq \pi.$$

The point $A(0, 4)$ lies on C .

- (a) Find an equation of the normal to the curve C at A .

(5)

- (b) Express y in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 significant figures.

(4)

- (c) Find the coordinates of the points of intersection of the curve C with the x -axis. Give your answers to 2 decimal places.

(4)

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Leave
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- 8.** The functions f and g are defined by

$$f : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}$$

$$g: x \mapsto \frac{3}{x} - 4, \quad x > 0, \quad x \in \mathbb{R}$$

- (a) Find the inverse function f^{-1} .

(2)

- (b) Show that the composite function gf is

$$\text{gf} : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

- (c) Solve $gf(x) = 0$.

(2)

- (d) Use calculus to find the coordinates of the stationary point on the graph of $y = gf(x)$.

(5)

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Leave
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Q8

(Total 13 marks)**TOTAL FOR PAPER: 75 MARKS****END**

- $$y = 4e^{2x+1}.$$

(a) Find, in terms of $\ln 2$, the x -coordinate of P .

(2)

- (4)



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$$f(x) = 5\cos x + 12\sin x$$

Given that $f(x) = R \cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

(a) find the value of R and the value of α to 3 decimal places.

(4)

(b) Hence solve the equation

$$5\cos x + 12\sin x = 6$$

for $0 \leq x < 2\pi$.

(5)

(c) (i) Write down the maximum value of $5\cos x + 12\sin x$.

(1)

(ii) Find the smallest positive value of x for which this maximum value occurs.

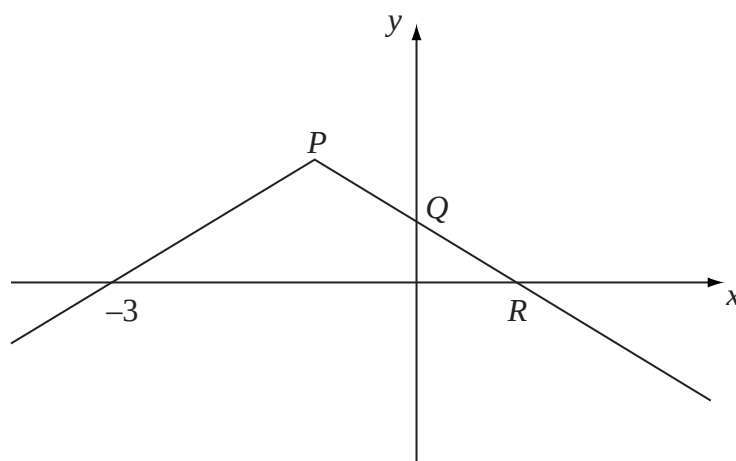
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- (2)

- (6)**

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6. (a) Differentiate with respect to x ,

$$(i) \quad e^{3x}(\sin x + 2 \cos x), \quad (3)$$

$$(ii) \quad x^3 \ln(5x+2).$$

(3)

Given that $y = \frac{3x^2 + 6x - 7}{(x+1)^2}$, $x \neq -1$,

(b) show that $\frac{dy}{dx} = \frac{20}{(x+1)^3}$. (5)

(c) Hence find $\frac{d^2y}{dx^2}$ and the real values of x for which $\frac{d^2y}{dx^2} = -\frac{15}{4}$. (3)



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Question 7 continued

Q7

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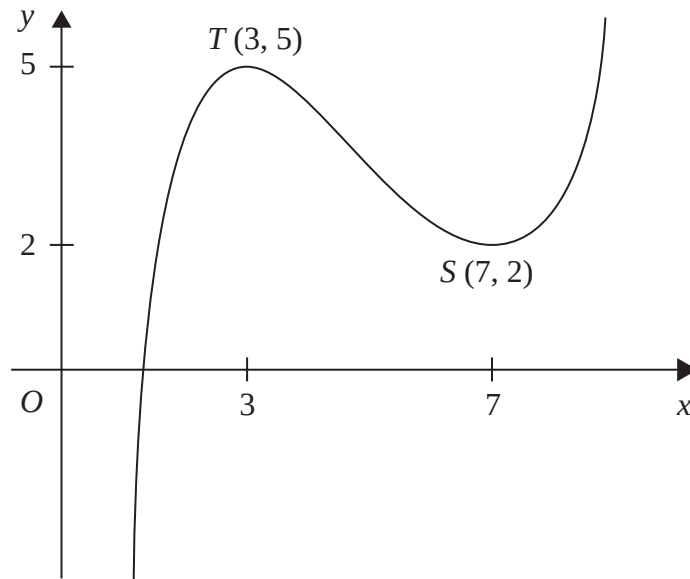


Figure 1

Figure 1 shows the graph of $y = f(x)$, $1 < x < 9$.
The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 2f(x) - 4$, (3)

(b) $y = |f(x)|$. (3)

Indicate on each diagram the coordinates of any turning points on your sketch.



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- 6.** (a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta.$$

(4)

- (ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3 \theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π .

(5)

- (b) Using $\sin(\theta - \alpha) = \sin \theta \cos \alpha - \cos \theta \sin \alpha$, or otherwise, show that

$$\sin 15^\circ = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)

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Leave
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7. $f(x) = 3xe^x - 1$

The curve with equation $y = f(x)$ has a turning point P .

- (a) Find the exact coordinates of P .

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

- (b) Use the iterative formula

$$x_{n+1} = \frac{1}{3} e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1 , x_2 and x_3 .

(3)

- (c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

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8. (a) Express $3 \cos \theta + 4 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90^\circ$.

(4)

- (b) Hence find the maximum value of $3 \cos \theta + 4 \sin \theta$ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3 \cos(15t)^\circ + 4 \sin(15t)^\circ,$$

where t is the time in hours from midday and $0 \leq t < 24$.

- (c) Calculate the minimum temperature of the warehouse as given by this model.

(2)

- (d) Find the value of t when this minimum temperature occurs.

(3)

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Question 8 continued

Q8

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3. Rabbits were introduced onto an island. The number of rabbits, P , t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \quad t \in \mathbb{R}, t \geq 0$$

- (a) Write down the number of rabbits that were introduced to the island. (1)

- (b) Find the number of years it would take for the number of rabbits to first exceed 1000. (2)

- (c) Find $\frac{dP}{dt}$.

- (d) Find P when $\frac{dP}{dt} = 50$. (3)

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4. (i) Differentiate with respect to x

$$(a) \quad x^2 \cos 3x \quad (3)$$

$$(b) \quad \frac{\ln(x^2+1)}{x^2+1} \tag{4}$$

(ii) A curve C has the equation

$$y = \sqrt[3]{4x+1}, \quad x > -\frac{1}{4}, \quad y > 0$$

The point P on the curve has x -coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a , b and c are integers.

(6)

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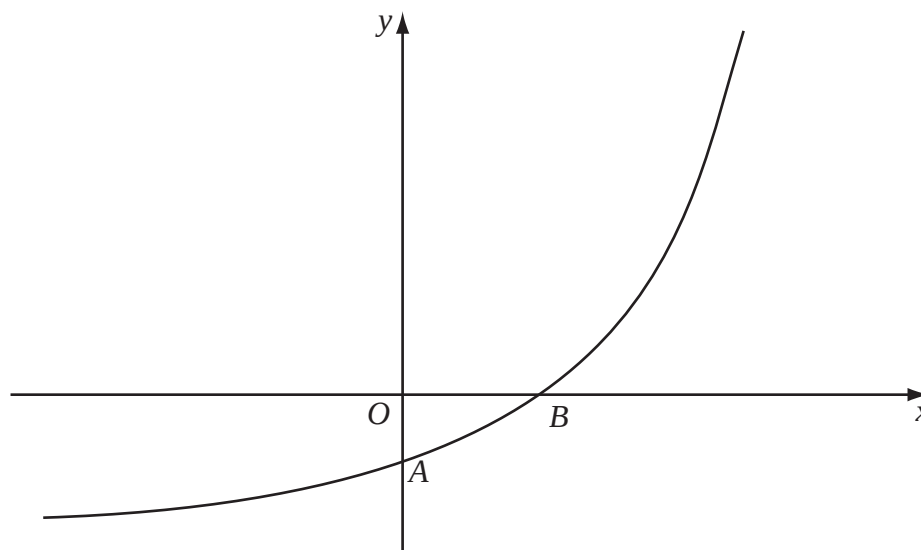


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0, 1-k)$ and $B(\frac{1}{2} \ln k, 0)$, where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$, (3)

(b) $y = f^{-1}(x)$. (2)

Show on each sketch the coordinates, in terms of k , of each point at which the curve meets or cuts the axes.

Given that $f(x) = e^{2x} - k$,

(c) state the range of f , (1)

(d) find $f^{-1}(x)$, (3)

(e) write down the domain of f^{-1} . (1)



Leave
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6. (a) Use the identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A \quad (2)$$

The curves C_1 and C_2 have equations

$$C_1: y = 3\sin 2x$$

$$C_2: y = 4\sin^2 x - 2\cos 2x$$

- (b) Show that the x -coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2 \quad (3)$$

- (c) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$, giving the value of α to 2 decimal places.

(3)

- (d) Hence find, for $0 \leq x < 180^\circ$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2$$

giving your answers to 1 decimal place.

(4)

[illegible]

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7. The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R}, x \neq -4, x \neq 2$$

(a) Show that $f(x) = \frac{x-3}{x-2}$ (5)

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R}, x \neq \ln 2$$

(b) Differentiate $g(x)$ to show that $g'(x) = \frac{e^x}{(e^x - 2)^2}$ (3)

(c) Find the exact values of x for which $g'(x) = 1$ (4)

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Leave
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4. (i) Given that $y = \frac{\ln(x^2 + 1)}{x}$, find $\frac{dy}{dx}$.

(4)

(ii) Given that $x = \tan y$, show that $\frac{dy}{dx} = \frac{1}{1+x^2}$.

(5)

[illegible]

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5. Sketch the graph of $y = \ln|x|$, stating the coordinates of any points of intersection with the axes.

(3)

6.

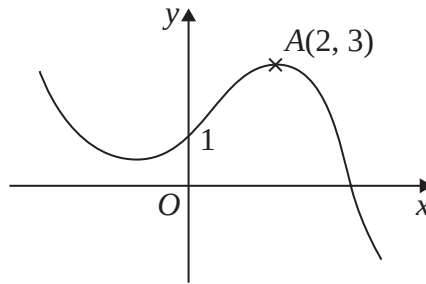
**Figure 1**

Figure 1 shows a sketch of the graph of $y = f(x)$.

The graph intersects the y -axis at the point $(0, 1)$ and the point $A(2, 3)$ is the maximum turning point.

Sketch, on separate axes, the graphs of

- (i) $y = f(-x) + 1$,
- (ii) $y = f(x + 2) + 3$,
- (iii) $y = 2f(2x)$.

On each sketch, show the coordinates of the point at which your graph intersects the y -axis and the coordinates of the point to which A is transformed.

(9)

Leave
blank**Question 6 continued**

N 3 5 3 8 1 A 0 1 5 2 8

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7. (a) By writing $\sec x$ as $\frac{1}{\cos x}$, show that $\frac{d(\sec x)}{dx} = \sec x \tan x$.

(3)

Given that $y = e^{2x} \sec 3x$,

- (b) find $\frac{dy}{dx}$.

(4)

The curve with equation $y = e^{2x} \sec 3x$, $-\frac{\pi}{6} < x < \frac{\pi}{6}$, has a minimum turning point at (a, b) .

- (c) Find the values of the constants a and b , giving your answers to 3 significant figures.

(4)

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Leave
blank**Question 7 continued**

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8. Solve

$$\operatorname{cosec}^2 2x - \cot 2x = 1$$

for $0 \leq x \leq 180^\circ$.

(7)



Q9

END

Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Tuesday 15 June 2010 – Morning

Time: 1 hour 30 minutes

Examiner's use only

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Team Leader's use only

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Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

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Check that you have the correct question paper.

Answer ALL the questions.

You must write your answer to each question in the space following the question.

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There are 8 questions in this question paper. The total mark for this paper is 75.

There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

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4. The function f is defined by

$$f : x \mapsto |2x - 5|, \quad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(2)

(b) Solve $f(x) = 15 + x$.

(3)

The function g is defined by

$$g : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leq x \leq 5$$

(c) Find $fg(2)$.

(2)

(d) Find the range of g .

(3)



Leave
blank**Question 4 continued**

5.

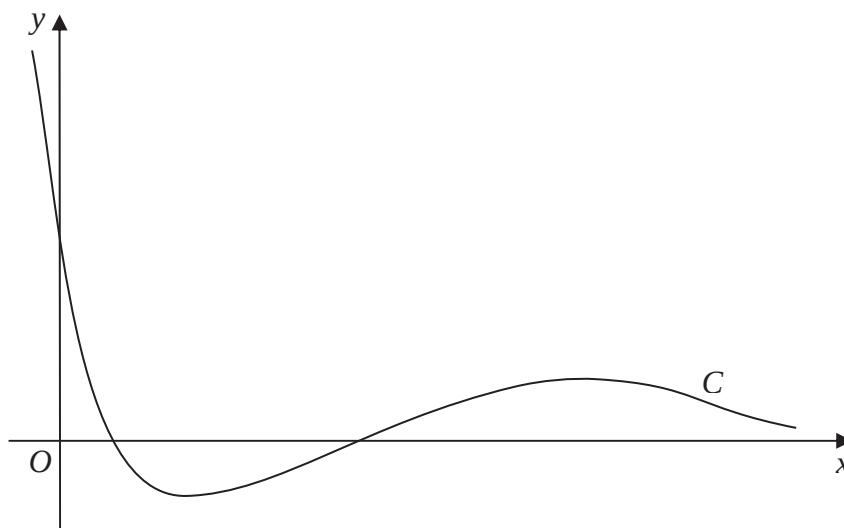


Figure 1 shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

- (a) Find the coordinates of the point where C crosses the y -axis. (1)
- (b) Show that C crosses the x -axis at $x = 2$ and find the x -coordinate of the other point where C crosses the x -axis. (3)
- (c) Find $\frac{dy}{dx}$. (3)
- (d) Hence find the exact coordinates of the turning points of C . (5)

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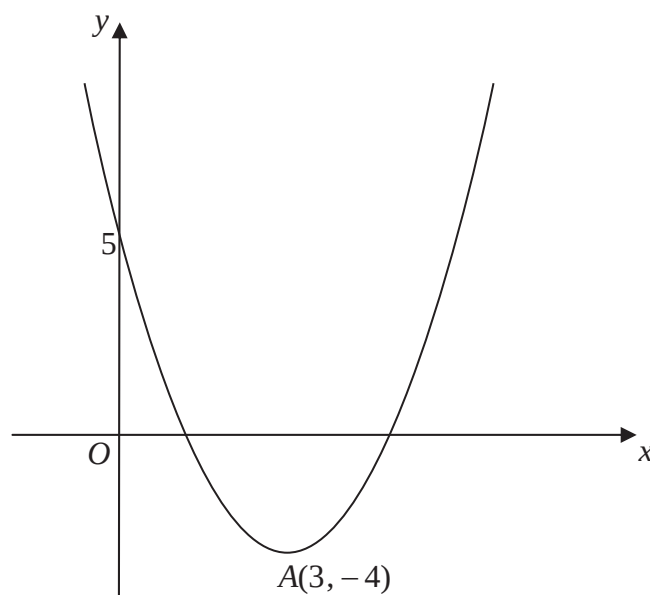
**Figure 2**

Figure 2 shows a sketch of the curve with the equation $y = f(x)$, $x \in \mathbb{R}$.
The curve has a turning point at $A(3, -4)$ and also passes through the point $(0, 5)$.

- (a) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |f(x)|$,

(ii) $y = 2f(\frac{1}{2}x)$.

(4)

- (b) Sketch the curve with equation

$$y = f(|x|)$$

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y -axis.

(3)

The curve with equation $y = f(x)$ is a translation of the curve with equation $y = x^2$.

- (c) Find $f(x)$.

(2)

- (d) Explain why the function f does not have an inverse.

(1)

Leave
blank**Question 6 continued**

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7. (a) Express $2 \sin \theta - 1.5 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

- (b) (i) Find the maximum value of $2 \sin \theta - 1.5 \cos \theta$.

- (ii) Find the value of θ , for $0 \leq \theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H=6+2\sin\left(\frac{4\pi t}{25}\right)-1.5\cos\left(\frac{4\pi t}{25}\right), \quad 0\leq t<12,$$

where t hours is the number of hours after midday.

- (c) Calculate the maximum value of H predicted by this model and the value of t , to 2 decimal places, when this maximum occurs.

(3)

- (d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)

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Leave
blank**Question 7 continued**

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Question 4 continued

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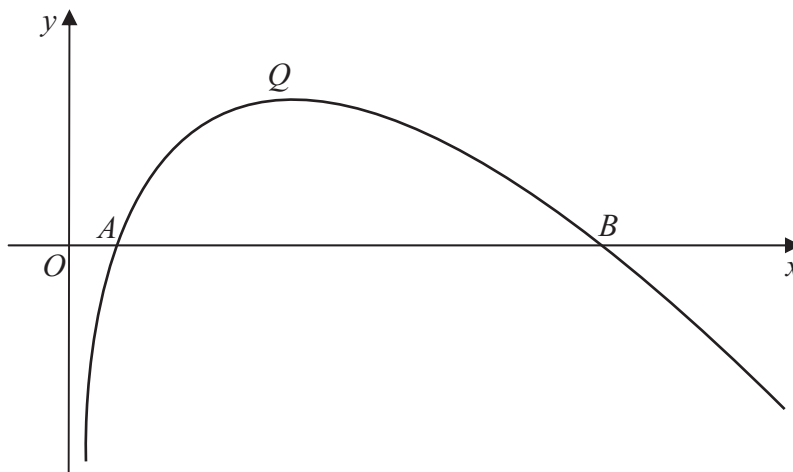
**Figure 1**

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (8-x)\ln x, \quad x > 0$$

The curve cuts the x -axis at the points A and B and has a maximum turning point at Q , as shown in Figure 1.

(a) Write down the coordinates of A and the coordinates of B . (2)

(b) Find $f'(x)$. (3)

(c) Show that the x -coordinate of Q lies between 3.5 and 3.6 (2)

(d) Show that the x -coordinate of Q is the solution of

$$x = \frac{8}{1 + \ln x} \quad (3)$$

To find an approximation for the x -coordinate of Q , the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(e) Taking $x_0 = 3.55$, find the values of x_1 , x_2 and x_3 .
Give your answers to 3 decimal places. (3)



This image shows a full page of blank, lined paper. It features approximately 28 horizontal blue or grey lines spaced evenly apart, typical of notebook paper. The lines extend across the entire width of the page, leaving small margins at the top and bottom. There are no vertical lines, text, or other markings on the page.



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6. The function f is defined by

$$f: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R}, \quad x \neq 5$$

(a) Find $f^{-1}(x)$.

(3)

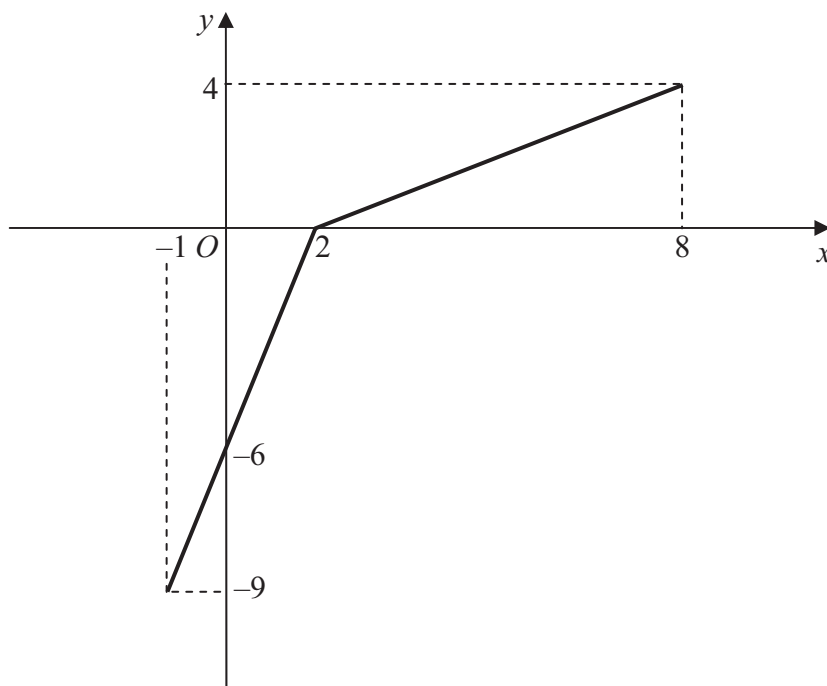


Figure 2

The function g has domain $-1 \leq x \leq 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = g(x)$.

(b) Write down the range of g .

(1)

(c) Find $gg(2)$.

(2)

(d) Find $fg(8)$.

(2)

(e) On separate diagrams, sketch the graph with equation

(i) $y = |g(x)|,$

(ii) $y = g^{-1}(x).$

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(f) State the domain of the inverse function g^{-1} .

(1)



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Question 6 continued





$$f(x) = 2 \sin (x^2) + x - 2, \quad 0 \leq x < 2\pi$$

(a) Show that $f(x)=0$ has a root α between $x=0.75$ and $x=0.85$

(2)

The equation $f(x)=0$ can be written as $x=\left[\arcsin(1-0.5x)\right]^{\frac{1}{2}}$.

(b) Use the iterative formula

$$x_{n+1} = \left[\arcsin(1 - 0.5x_n) \right]^{\frac{1}{2}}, \quad x_0 = 0.8$$

to find the values of x_1 , x_2 and x_3 , giving your answers to 5 decimal places.

(3)

(c) Show that $\alpha = 0.80157$ is correct to 5 decimal places.

(3)



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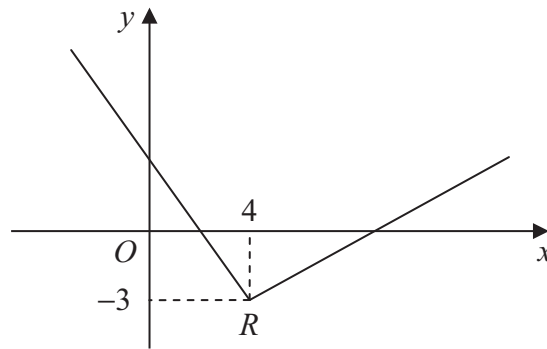
**Figure 1**

Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $R(4, -3)$, as shown in Figure 1.

Sketch, on separate diagrams, the graphs of

(a) $y = 2f(x+4)$, (3)

(b) $y = |f(-x)|$. (3)

On each diagram, show the coordinates of the point corresponding to R .





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Question 6 continued





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8. (a) Express $2\cos 3x - 3\sin 3x$ in the form $R\cos(3x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures. (4)

$$f(x) = e^{2x} \cos 3x$$

- (b) Show that $f'(x)$ can be written in the form

$$f'(x) = R e^{2x} \cos(3x + \alpha)$$

where R and α are the constants found in part (a).

- (c) Hence, or otherwise, find the smallest positive value of x for which the curve with equation $y = f(x)$ has a turning point. (3)



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Question 8 continued

(Total 12 marks)

Q8

TOTAL FOR PAPER: 75 MARKS

END



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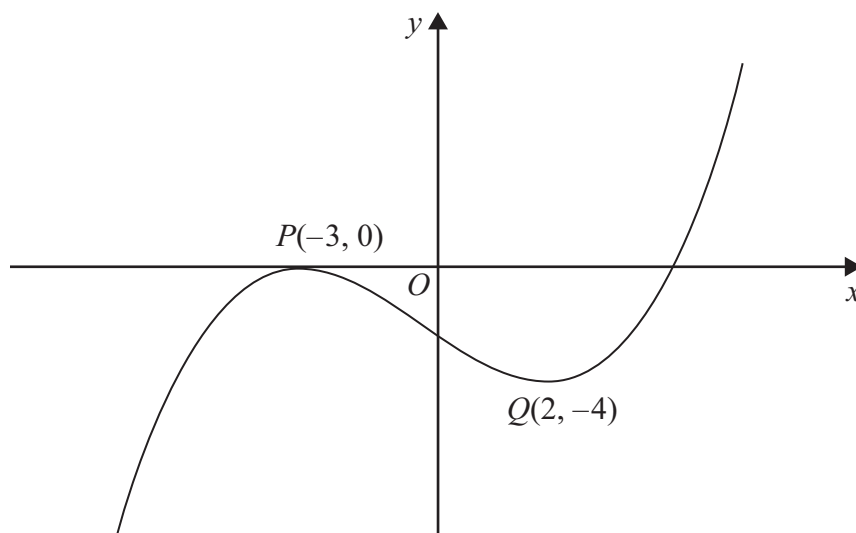
**Figure 1**

Figure 1 shows the graph of equation $y = f(x)$.

The points $P(-3, 0)$ and $Q(2, -4)$ are stationary points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 3f(x + 2)$

(3)

(b) $y = |f(x)|$

(3)

On each diagram, show the coordinates of any stationary points.



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4. The point P is the point on the curve $x = 2 \tan\left(y + \frac{\pi}{12}\right)$ with y -coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P .

(7)



P 4 0 0 8 4 A 0 1 0 2 4







A graph of a function $y = f(x)$ is shown on a Cartesian coordinate system. The curve passes through the origin O , has a local maximum at point P , and intersects the x -axis at another point. The segment of the curve from the origin to point P is labeled C .



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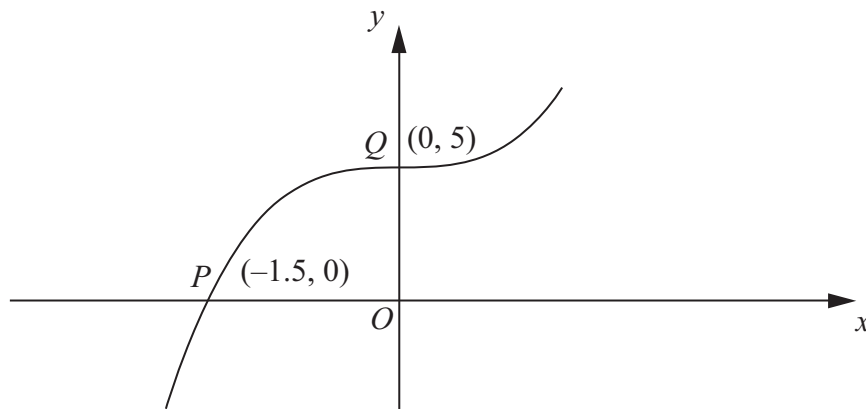
**Figure 2**

Figure 2 shows part of the curve with equation $y = f(x)$
The curve passes through the points $P(-1.5, 0)$ and $Q(0, 5)$ as shown.

On separate diagrams, sketch the curve with equation

(a) $y = |f(x)|$ (2)

(b) $y = f(|x|)$ (2)

(c) $y = 2f(3x)$ (3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.



Leave
blank**Question 4 continued**

P 4 0 6 8 6 R A 0 1 3 3 2

P 4 0 6 8 6 R A 0 2 0 3 2

(i) $x^{\frac{1}{2}} \ln(3x)$

(ii) $\frac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form.

(6)

(b) Given that $x = 3 \tan 2y$ find $\frac{dy}{dx}$ in terms of x .

(5)

Question 8 continued

Q8

END

$$g(x) = e^{x-1} + x - 6$$

- $$x = \ln(6 - x) + 1, \quad x < 6$$

$$x_{n+1} = \ln(6 - x_n) + 1, \quad x_0 = 2$$

(b) Calculate the values of x_1 , x_2 and x_3 to 4 decimal places.

(c) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places.

A Cartesian coordinate system with a horizontal x-axis and a vertical y-axis. The origin is labeled O . A curve, labeled $y = f(x)$, is plotted. The curve passes through the point $P(-3, 0)$ on the x-axis and the point $Q(0, 2)$ on the y-axis. The curve is concave down and increases from left to right.



P 4 1 4 8 6 A 0 6 2 8

Leave
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P 4 1 4 8 6 A 0 7 2 8







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Centre No.						Paper Reference							Surname	Initial(s)	
Candidate No.						6	6	6	5	/	0	1	R	Signature	

Paper Reference(s)

6665/01R

Edexcel GCE

Core Mathematics C3

Advanced

Thursday 13 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

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Team Leader's use only

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Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

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1. Express

$$\frac{3x+5}{x^2+x-12} - \frac{2}{x-3}$$

as a single fraction in its simplest form.

(4)



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2.

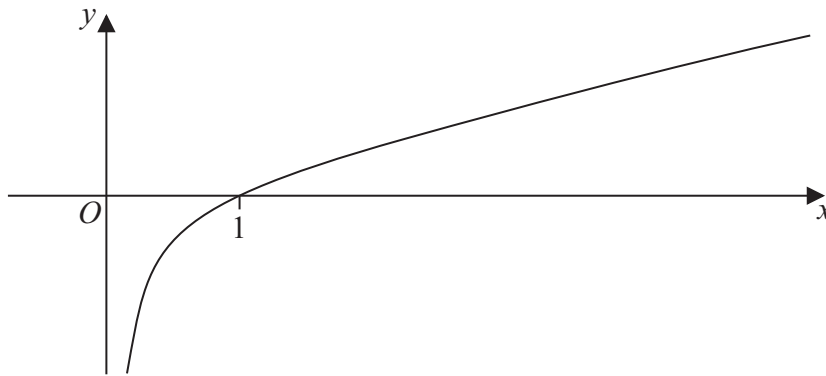
**Figure 1**

Figure 1 shows a sketch of the curve with equation $y = f(x)$, $x > 0$, where f is an increasing function of x . The curve crosses the x -axis at the point $(1, 0)$ and the line $x = 0$ is an asymptote to the curve.

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$, $x > 0$ (2)

(b) $y = |f(x)|$, $x > 0$ (3)

Indicate clearly on each sketch the coordinates of the point at which the curve crosses or meets the x -axis.





P 4 2 9 5 3 A 0 1 0 2 8





7.

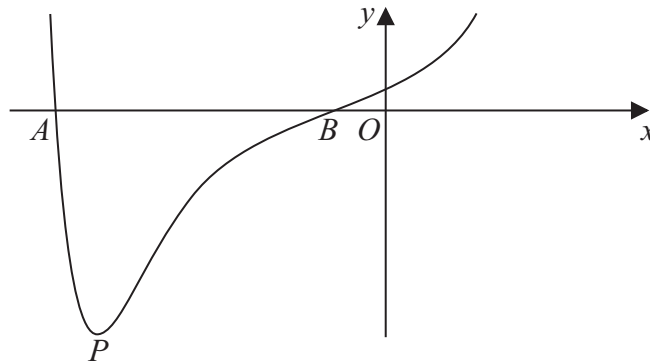


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = (x^2 + 3x + 1)e^{x^2}$$

The curve cuts the x -axis at points A and B as shown in Figure 2.

- (a) Calculate the x coordinate of A and the x coordinate of B , giving your answers to 3 decimal places. (2)

- (b) Find $f'(x)$. (3)

The curve has a minimum turning point at the point P as shown in Figure 2.

- (c) Show that the x coordinate of P is the solution of

$$x = -\frac{3(2x^2 + 1)}{2(x^2 + 2)}$$
(3)

- (d) Use the iteration formula

$$x_{n+1} = -\frac{3(2x_n^2 + 1)}{2(x_n^2 + 2)}, \quad \text{with } x_0 = -2.4,$$

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 3 decimal places. (3)

The x coordinate of P is α .

- (e) By choosing a suitable interval, prove that $\alpha = -2.43$ to 2 decimal places. (2)



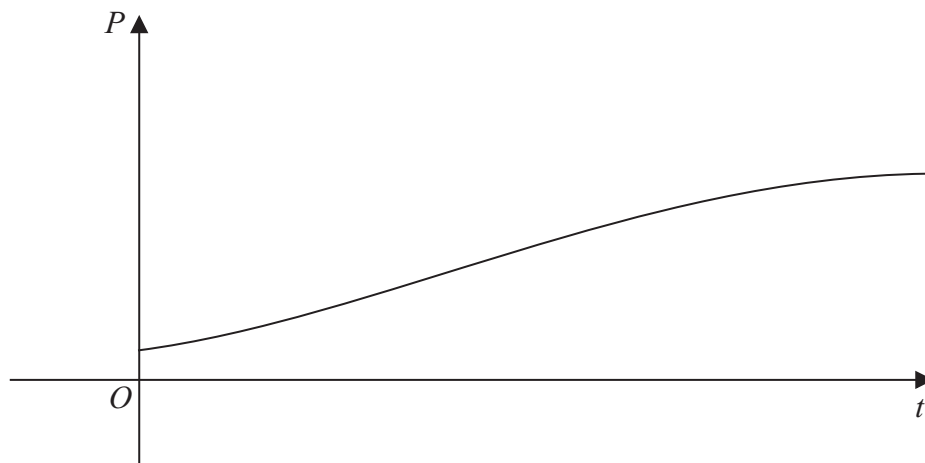
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Question 7 continued



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8.

**Figure 3**

The population of a town is being studied. The population P , at time t years from the start of the study, is assumed to be

$$P = \frac{8000}{1 + 7e^{-kt}}, \quad t \geq 0,$$

where k is a positive constant.

The graph of P against t is shown in Figure 3.

Use the given equation to

(a) find the population at the start of the study, (2)

(b) find a value for the expected upper limit of the population. (1)

Given also that the population reaches 2500 at 3 years from the start of the study,

(c) calculate the value of k to 3 decimal places. (5)

Using this value for k ,

(d) find the population at 10 years from the start of the study, giving your answer to 3 significant figures. (2)

(e) Find, using $\frac{dP}{dt}$, the rate at which the population is growing at 10 years from the start of the study. (3)





$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2$$

(4)

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2. Given that

$$f(x) = \ln x, \quad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$,

(ii) $y = |f(x)|$,

(iii) $y = -f(x - 4)$.

Show, on each diagram, the point where the graph meets or crosses the x -axis.
In each case, state the equation of the asymptote.

(7)





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- 6.** Find algebraically the exact solutions to the equations

$$(a) \quad \ln(4 - 2x) + \ln(9 - 3x) = 2 \ln(x + 1), \quad -1 < x < 2 \quad (5)$$

(b) $2^x e^{3x+1} = 10$

Give your answer to (b) in the form $\frac{a + \ln b}{c + \ln d}$ where a, b, c and d are integers.



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7. The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = f(x)$ is shown in Figure 1.

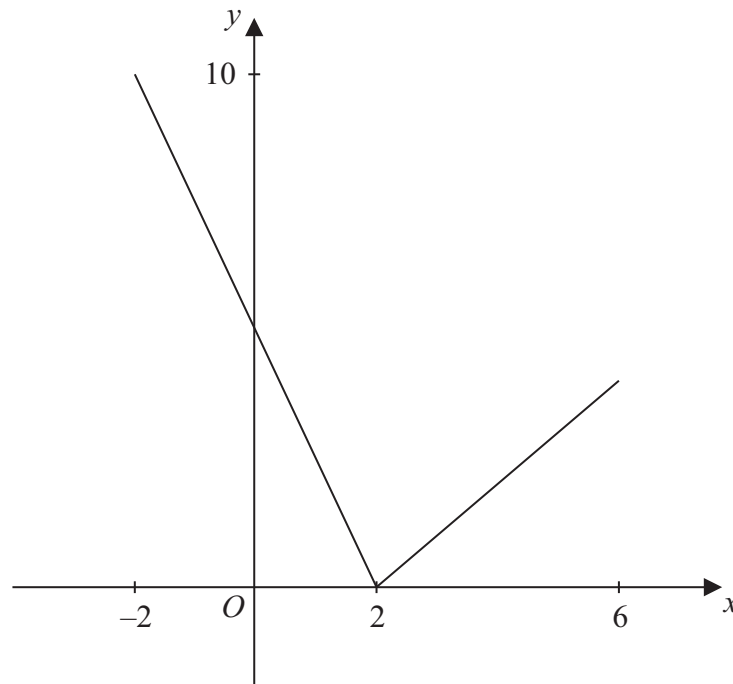


Figure 1

- (a) Write down the range of f . (1)
- (b) Find $ff(0)$. (2)

The function g is defined by

$$g : x \rightarrow \frac{4 + 3x}{5 - x}, \quad x \in \mathbb{R}, \quad x \neq 5$$

- (c) Find $g^{-1}(x)$ (3)
- (d) Solve the equation $gf(x) = 16$ (5)





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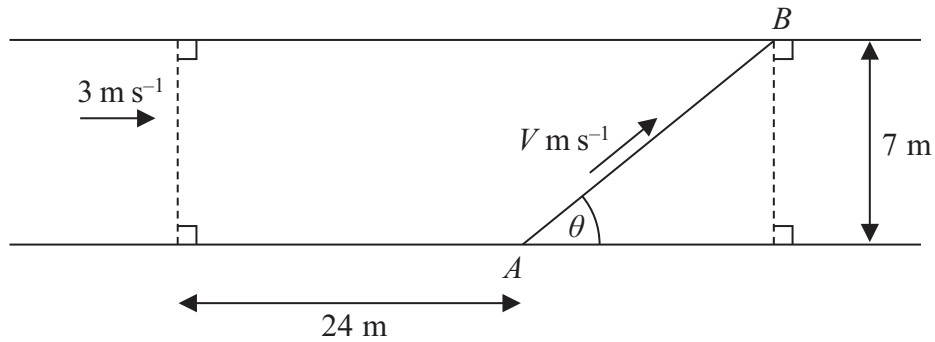


Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1} .

Kate is 24 m ahead of John when she starts to cross the road from the fixed point A.

John passes her as she reaches the other side of the road at a variable point B, as shown in Figure 2.

Kate's speed is $V \text{ m s}^{-1}$ and she moves in a straight line, which makes an angle θ , $0 < \theta < 150^\circ$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24 \sin \theta + 7 \cos \theta}, \quad 0 < \theta < 150^\circ$$

- (a) Express $24 \sin \theta + 7 \cos \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90^\circ$, giving the value of α to 2 decimal places.

(3)

Given that θ varies,

- (b) find the minimum value of V .

(2)

Given that Kate's speed has the value found in part (b),

- (c) find the distance AB .

(3)

Given instead that Kate's speed is 1.68 m s^{-1} ,

- (d) find the two possible values of the angle θ , given that $0 < \theta < 150^\circ$.

(6)





Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

f(x)	f'(x)
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2} x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \times 2 \times \dots \times r} x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

$$\text{The trapezium rule: } \int_a^b y \, dx \approx \frac{1}{2} h \{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}, \text{ where } h = \frac{b-a}{n}$$

Core Mathematics C1

Mensuration

$$\text{Surface area of sphere} = 4\pi r^2$$

$$\text{Area of curved surface of cone} = \pi r \times \text{slant height}$$

Arithmetic series

$$u_n = a + (n - 1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$$