Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3
Advanced Subsidiary

Monday 20 June 2005 – Morning
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N23494A

PMT

1. (*a*) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \tan^2\theta \equiv \sec^2\theta$.

(2)

(*b*) Solve, for $0 \leq \theta < 360°$, the equation

$$2\tan^2\theta + \sec\theta = 1,$$

giving your answers to 1 decimal place.

(6)

---

2. (*a*) Differentiate with respect to x

(i) $3\sin^2 x + \sec 2x$,

(3)

(ii) $\{x + \ln(2x)\}^3$.

(3)

Given that $y = \dfrac{5x^2 - 10x + 9}{(x-1)^2}$, $x \neq 1$,

(*b*) show that $\dfrac{dy}{dx} = -\dfrac{8}{(x-1)^3}$.

(6)

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3. The function f is defined by

$$f: x \mapsto \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}, \quad x > 1.$$

(*a*) Show that $f(x) = \dfrac{2}{x-1}$, $x > 1$.

(4)

(*b*) Find $f^{-1}(x)$.

(3)

The function g is defined by

$$g: x \mapsto x^2 + 5, \quad x \in \mathbb{R}.$$

(*b*) Solve $fg(x) = \frac{1}{4}$.

(3)

---

4. $$f(x) = 3e^x - \tfrac{1}{2}\ln x - 2, \quad x > 0.$$

(*a*) Differentiate to find $f'(x)$.

(3)

The curve with equation $y = f(x)$ has a turning point at P. The x-coordinate of P is α.

(*b*) Show that $\alpha = \tfrac{1}{6}e^{-\alpha}$.

(2)

The iterative formula

$$x_{n+1} = \tfrac{1}{6}e^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for α.

(*c*) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(2)

(*d*) By considering the change of sign of $f'(x)$ in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places.

(2)

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5. (*a*) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A.$$

(2)

(*b*) Show that

$$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv \sin\theta(4\cos\theta + 6\sin\theta - 3).$$

(4)

(*c*) Express $4\cos\theta + 6\sin\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \tfrac{1}{2}\pi$.

(4)

(*d*) Hence, for $0 \le \theta < \pi$, solve

$$2\sin 2\theta = 3(\cos 2\theta + \sin\theta - 1),$$

giving your answers in radians to 3 significant figures, where appropriate.

(5)

6.

Figure 1

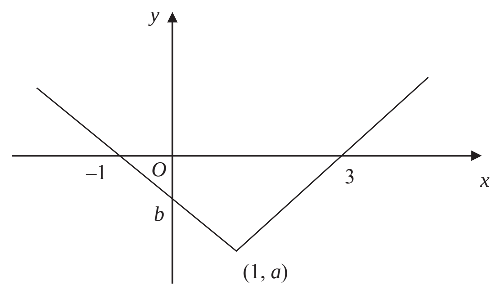

Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the x-axis at $(3, 0)$. The other line meets the x-axis at $(-1, 0)$ and the y-axis at $(0, b)$, $b < 0$.

In separate diagrams, sketch the graph with equation

(*a*) $y = f(x + 1)$,

(2)

(*b*) $y = f(|x|)$.

(3)

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $f(x) = |x - 1| - 2$, find

(*c*) the value of a and the value of b,

(2)

(*d*) the value of x for which $f(x) = 5x$.

(4)

---

7. A particular species of orchid is being studied. The population p at time t years after the study started is assumed to be

$$p = \frac{2800ae^{0.2t}}{1 + ae^{0.2t}}, \text{ where } a \text{ is a constant.}$$

Given that there were 300 orchids when the study started,

(*a*) show that $a = 0.12$,

(3)

(*b*) use the equation with $a = 0.12$ to predict the number of years before the population of orchids reaches 1850.

(4)

(*c*) Show that $p = \dfrac{336}{0.12 + e^{-0.2t}}$.

(1)

(*d*) Hence show that the population cannot exceed 2800.

(2)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6665

Edexcel GCE

Core Mathematics C3

Advanced Level

Monday 23 January 2006 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, other name and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

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N23495A

1.

Figure 1

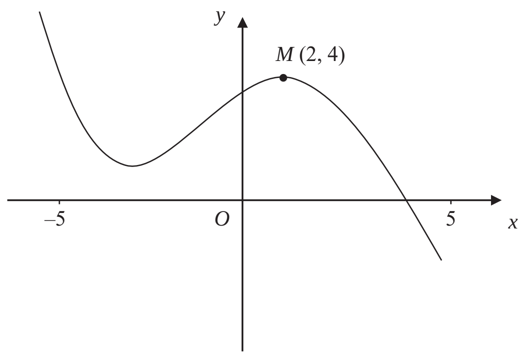

Figure 1 shows the graph of $y = f(x)$, $-5 \le x \le 5$.

The point M (2, 4) is the maximum turning point of the graph.

Sketch, on separate diagrams, the graphs of

(*a*) $y = f(x) + 3$, **(2)**

(*b*) $y = |f(x)|$, **(2)**

(*c*) $y = f(|x|)$. **(3)**

Show on each graph the coordinates of any maximum turning points.

---

2. Express

$$\frac{2x^2 + 3x}{(2x+3)(x-2)} - \frac{6}{x^2 - x - 2}$$

as a single fraction in its simplest form. **(7)**

---

3. The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x-coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants. **(5)**

---

4. (*a*) Differentiate with respect to x

(i) $x^2 e^{3x+2}$, **(4)**

(ii) $\dfrac{\cos(2x^3)}{3x}$. **(4)**

(*b*) Given that $x = 4\sin(2y + 6)$, find $\dfrac{dy}{dx}$ in terms of x. **(5)**

5.

$$f(x) = 2x^3 - x - 4.$$

(*a*) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}.$$

(3)

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

(*b*) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{1}{2}\right)},$$

with $x_0 = 1.35$, to find, to 2 decimal places, the value of x_1, x_2 and x_3. **(3)**

The only real root of $f(x) = 0$ is α.

(*c*) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places. **(3)**

---

6. $$\mathrm{f}(x) = 12\cos x - 4\sin x.$$

Given that $\mathrm{f}(x) = R\cos(x + \alpha)$, where $R \geq 0$ and $0 \leq \alpha \leq 90°$,

(*a*) find the value of R and the value of α.

(4)

(*b*) Hence solve the equation

$$12\cos x - 4\sin x = 7$$

for $0 \leq x < 360°$, giving your answers to one decimal place.

(5)

(*c*) (i) Write down the minimum value of $12\cos x - 4\sin x$.

(1)

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs.

(2)

---

7. (*a*) Show that

(i) $\dfrac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \quad x \neq (n - \tfrac{1}{4})\pi,\ n \in \mathbb{Z},$

(2)

(ii) $\tfrac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \tfrac{1}{2}$.

(3)

(*b*) Hence, or otherwise, show that the equation

$$\cos\theta\left(\frac{\cos 2\theta}{\cos\theta + \sin\theta}\right) = \frac{1}{2}$$

can be written as

$$\sin 2\theta = \cos 2\theta.$$

(3)

(*c*) Solve, for $0 \leq \theta < 2\pi$,

$$\sin 2\theta = \cos 2\theta,$$

giving your answers in terms of π.

(4)

---

8. The functions f and g are defined by

$$\mathrm{f}: x \mapsto 2x + \ln 2, \quad x \in \mathbb{R},$$

$$\mathrm{g}: x \mapsto e^{2x}, \quad x \in \mathbb{R}.$$

(*a*) Prove that the composite function gf is

$$\mathrm{gf}: x \mapsto 4e^{4x}, \quad x \in \mathbb{R}.$$

(4)

(*b*) Sketch the curve with equation $y = \mathrm{gf}(x)$, and show the coordinates of the point where the curve cuts the y-axis.

(1)

(*c*) Write down the range of gf.

(1)

(*d*) Find the value of x for which $\dfrac{\mathrm{d}}{\mathrm{d}x}[\mathrm{gf}(x)] = 3$, giving your answer to 3 significant figures.

(4)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Monday 12 June 2006 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N23581A

1. (*a*) Simplify $\dfrac{3x^2 - x - 2}{x^2 - 1}$.

(3)

(*b*) Hence, or otherwise, express $\dfrac{3x^2 - x - 2}{x^2 - 1} - \dfrac{1}{x(x+1)}$ as a single fraction in its simplest form.

(3)

---

2. Differentiate, with respect to x,

(a) $e^{3x} + \ln 2x$,

(3)

(*b*) $(5 + x^2)^{\frac{3}{2}}$.

(3)

---

3. **Figure 1**

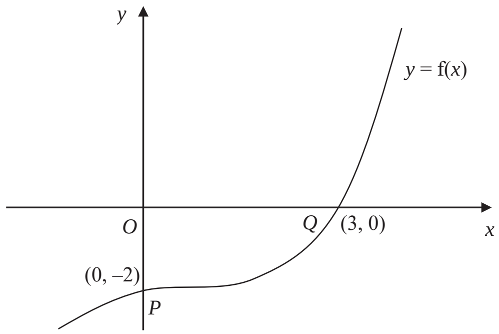

Figure 1 shows part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$, where f is an increasing function of x. The curve passes through the points $P(0, -2)$ and $Q(3, 0)$ as shown.

In separate diagrams, sketch the curve with equation

(*a*) $y = |\mathrm{f}(x)|$,

(3)

(*b*) $y = \mathrm{f}^{-1}(x)$,

(3)

(*c*) $y = \frac{1}{2}\mathrm{f}(3x)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

4. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T °C, t minutes after it enters the liquid, is given by

$$T = 400\mathrm{e}^{-0.05t} + 25, \quad t \geq 0.$$

(*a*) Find the temperature of the ball as it enters the liquid.

(1)

(*b*) Find the value of t for which $T = 300$, giving your answer to 3 significant figures.

(4)

(*c*) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in °C per minute to 3 significant figures.

(3)

(*d*) From the equation for temperature T in terms of t, given above, explain why the temperature of the ball can never fall to 20 °C.

(1)

5. **Figure 2**

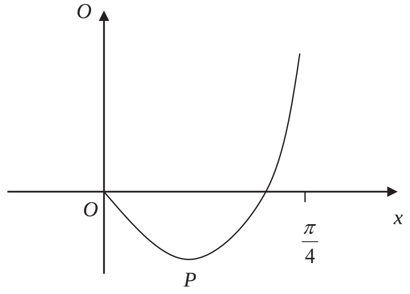

Figure 2 shows part of the curve with equation

$$y = (2x - 1)\tan 2x, \quad 0 \le x < \frac{\pi}{4}.$$

The curve has a minimum at the point P. The x-coordinate of P is k.

(*a*) Show that k satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

(6)

The iterative formula

$$x_{n+1} = \tfrac{1}{4}(2 - \sin 4x_n), \quad x_0 = 0.3,$$

is used to find an approximate value for k.

(*b*) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimals places.

(3)

(*c*) Show that $k = 0.277$, correct to 3 significant figures.

(2)

---

6. (*a*) Using $\sin^2\theta + \cos^2\theta \equiv 1$, show that the $\operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$.

(2)

(*b*) Hence, or otherwise, prove that

$$\operatorname{cosec}^4\theta - \cot^4\theta \equiv \operatorname{cosec}^2\theta + \cot^2\theta.$$

(2)

(*c*) Solve, for $90° < \theta < 180°$,

$$\operatorname{cosec}^4\theta - \cot^4\theta = 2 - \cot\theta.$$

(6)

---

7. For the constant k, where $k > 1$, the functions f and g are defined by

$$\mathrm{f}: x \mapsto \ln(x + k), \quad x > -k,$$

$$\mathrm{g}: x \mapsto |2x - k|, \quad x \in \mathbb{R}.$$

(*a*) On separate axes, sketch the graph of f and the graph of g.

On each sketch state, in terms of k, the coordinates of points where the graph meets the coordinate axes.

(5)

(*b*) Write down the range of f.

(1)

(*c*) Find $\mathrm{fg}\left(\frac{k}{4}\right)$ in terms of k, giving your answer in its simplest form.

(2)

The curve C has equation $y = \mathrm{f}(x)$. The tangent to C at the point with x-coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

(*d*) Find the value of k.

(4)

PMT

8. (*a*) Given that $\cos A = \frac{3}{4}$, where $270° < A < 360°$, find the exact value of $\sin 2A$.

(5)

(*b*) (i) Show that $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x$.

(3)

Given that

$$y = 3\sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right),$$

(ii) show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \sin 2x$.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6665/01

Edexcel GCE

Core Mathematics C3
Advanced Level

Thursday 18 January 2007 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

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Information for Candidates

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There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

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N23583A

PMT

1. (*a*) By writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(5)

(*b*) Given that $\sin\theta = \frac{\sqrt{3}}{4}$, find the exact value of $\sin 3\theta$.

(2)

---

2. $$f(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}, \quad x \neq -2.$$

(*a*) Show that $f(x) = \frac{x^2 + x + 1}{(x+2)^2}$, $x \neq -2$.

(4)

(*b*) Show that $x^2 + x + 1 > 0$ for all values of x.

(3)

(*c*) Show that $f(x) > 0$ for all values of x, $x \neq -2$.

(1)

---

3. The curve C has equation $x = 2\sin y$.

(*a*) Show that the point $P\left(\sqrt{2}, \frac{\pi}{4}\right)$ lies on C.

(1)

(*b*) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ at P.

(4)

(*c*) Find an equation of the normal to C at P. Give your answer in the form $y = mx + c$, where m and c are exact constants.

(4)

---

4. (i) The curve C has equation $y = \frac{x}{9 + x^2}$.

Use calculus to find the coordinates of the turning points of C.

(6)

(ii) Given that $y = (1 + e^{2x})^{\frac{3}{2}}$, find the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}\ln 3$.

(5)

5.

Figure 1

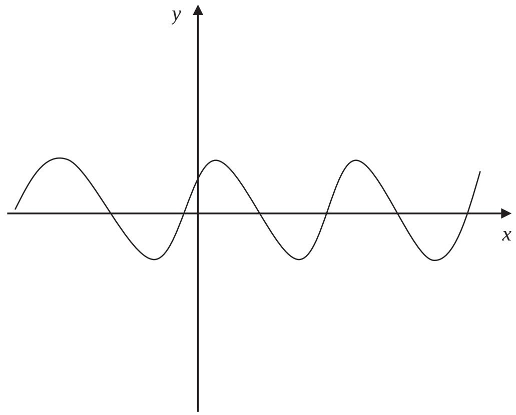

Figure 1 shows an oscilloscope screen.

The curve on the screen satisfies the equation $y = \sqrt{3}\cos x + \sin x$.

(*a*) Express the equation of the curve in the form $y = R\sin(x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(4)

(*b*) Find the values of x, $0 \leq x < 2\pi$, for which $y = 1$.

(4)

---

6. The function f is defined by

$$f : x \mapsto \ln(4 - 2x), \quad x < 2 \text{ and } x \in \mathbb{R}.$$

(*a*) Show that the inverse function of f is defined by

$$f^{-1} : x \mapsto 2 - \tfrac{1}{2}e^{x}$$

and write down the domain of f^{-1}.

(4)

(*b*) Write down the range of f^{-1}.

(1)

(*c*) Sketch the graph of $y = f^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes.

(4)

The graph of $y = x + 2$ crosses the graph of $y = f^{-1}(x)$ at $x = k$.

The iterative formula

$$x_{n+1} = -\tfrac{1}{2}e^{x_n}, \quad x_0 = -0.3,$$

is used to find an approximate value for k.

(*d*) Calculate the values of x_1 and x_2, giving your answer to 4 decimal places.

(2)

(*e*) Find the values of k to 3 decimal places.

(2)

7.

$$f(x) = x^4 - 4x - 8.$$

(*a*) Show that there is a root of $f(x) = 0$ in the interval $[-2, -1]$.

(3)

(*b*) Find the coordinates of the turning point on the graph of $y = f(x)$.

(3)

(*c*) Given that $f(x) = (x - 2)(x^3 + ax^2 + bx + c)$, find the values of the constants a, b and c.

(3)

(*d*) Sketch the graph of $y = f(x)$.

(3)

(*e*) Hence sketch the graph of $y = |f(x)|$.

(1)

---

8. (i) Prove that

$$\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x.$$

(3)

(ii) Given that

$$y = \arccos x, \quad -1 \leq x \leq 1 \quad \text{and} \quad 0 \leq y \leq \pi,$$

(*a*) express $\arcsin x$ in terms of y.

(2)

(*b*) Hence evaluate $\arccos x + \arcsin x$. Give your answer in terms of π.

(1)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Thursday 14 June 2007 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

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Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. Find the exact solutions to the equations

(*a*) $\ln x + \ln 3 = \ln 6$,

(2)

(*b*) $e^x + 3e^{-x} = 4$.

(4)

2.

$$f(x) = \frac{2x+3}{x+2} - \frac{9+2x}{2x^2+3x-2}, \quad x > \tfrac{1}{2}.$$

(*a*) Show that $f(x) = \dfrac{4x-6}{2x-1}$.

(7)

(*b*) Hence, or otherwise, find $f'(x)$ in its simplest form.

(3)

---

3. A curve *C* has equation $y = x^2e^x$.

(*a*) Find $\dfrac{dy}{dx}$, using the product rule for differentiation.

(3)

(*b*) Hence find the coordinates of the turning points of *C*.

(3)

(*c*) Find $\dfrac{d^2y}{dx^2}$.

(2)

(*d*) Determine the nature of each turning point of the curve *C*.

(2)

PMT

4. $$f(x) = -x^3 + 3x^2 - 1.$$

(*a*) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \sqrt{\left(\frac{1}{3-x}\right)}.$$

(2)

(*b*) Starting with $x_1 = 0.6$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{1}{3-x_n}\right)}$$

to calculate the values of x_2, x_3 and x_4, giving all your answers to 4 decimal places.

(2)

(*c*) Show that $x = 0.653$ is a root of $f(x) = 0$ correct to 3 decimal places.

(3)

---

5. The functions f and g are defined by

$$f : x \mapsto \ln(2x-1), \quad x \in \mathbb{R}, \quad x > \tfrac{1}{2},$$

$$g : x \mapsto \frac{2}{x-3}, \quad x \in \mathbb{R}, \quad x \neq 3.$$

(*a*) Find the exact value of fg(4).

(2)

(*b*) Find the inverse function $f^{-1}(x)$, stating its domain.

(4)

(*c*) Sketch the graph of $y = |g(x)|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the y-axis.

(3)

(*d*) Find the exact values of x for which $\left|\frac{2}{x-3}\right| = 3$.

(3)

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6. (*a*) Express $3 \sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(4)

(*b*) Hence find the greatest value of $(3 \sin x + 2 \cos x)^4$.

(2)

(*c*) Solve, for $0 < x < 2\pi$, the equation

$$3 \sin x + 2 \cos x = 1,$$

giving your answers to 3 decimal places.

(5)

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7. (*a*) Prove that

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2 \operatorname{cosec} 2\theta, \quad \theta \neq 90n°.$$

(4)

(*b*) Sketch the graph of $y = 2 \operatorname{cosec} 2\theta$ for $0° < \theta < 360°$.

(2)

(*c*) Solve, for $0° < \theta < 360°$, the equation

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 3$$

giving your answers to 1 decimal place.

(6)

PMT

8. The amount of a certain type of drug in the bloodstream t hours after it has been taken is given by the formula

$$x = De^{-\frac{1}{8}t},$$

where x is the amount of the drug in the bloodstream in milligrams and D is the dose given in milligrams.

A dose of 10 mg of the drug is given.

(*a*) Find the amount of the drug in the bloodstream 5 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 10 mg is given after 5 hours.

(*b*) Show that the amount of the drug in the bloodstream 1 hour after the second dose is 13.549 mg to 3 decimal places.

(2)

No more doses of the drug are given. At time T hours after the second dose is given, the amount of the drug in the bloodstream is 3 mg.

(*c*) Find the value of T.

(3)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6665/01

Edexcel GCE

Core Mathematics C3
Advanced Level

Thursday 17 January 2008 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

H26315RBA

1. Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants a, b, c, d and e.

(4)

---

2. A curve C has equation

$$y = e^{2x} \tan x, \quad x \neq (2n + 1)\frac{\pi}{2}.$$

(*a*) Show that the turning points on C occur where $\tan x = -1$.

(6)

(*b*) Find an equation of the tangent to C at the point where $x = 0$.

(2)

3.

$$f(x) = \ln(x + 2) - x + 1, \qquad x > -2, x \in \mathbb{R}.$$

(*a*) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$.

(2)

(*b*) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \quad x_0 = 2.5,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 5 decimal places.

(3)

(*c*) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

4.

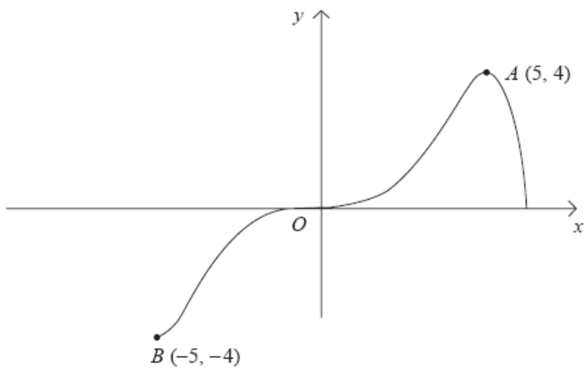

Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$.

The curve passes through the origin O and the points $A(5, 4)$ and $B(-5, -4)$.

In separate diagrams, sketch the graph with equation

(*a*) $y = |f(x)|$,

(3)

(*b*) $y = f(|x|)$,

(3)

(*c*) $y = 2f(x + 1)$.

(4)

On each sketch, show the coordinates of the points corresponding to A and B.

---

5. The radioactive decay of a substance is given by

$$R = 1000e^{-ct}, \quad t \geq 0.$$

where R is the number of atoms at time t years and c is a positive constant.

(*a*) Find the number of atoms when the substance started to decay.
(1)

It takes 5730 years for half of the substance to decay.

(*b*) Find the value of c to 3 significant figures.
(4)

(*c*) Calculate the number of atoms that will be left when $t = 22\,920$.
(2)

(*d*) Sketch the graph of R against t .
(2)

---

6. (*a*) Use the double angle formulae and the identity

$$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.
(4)

(*b*) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2\sec x, \qquad x \neq (2n+1)\frac{\pi}{2}.$$
(4)

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4.$$
(3)

---

7. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x, \qquad -\pi \leq x \leq \pi .$$

The point $A(0, 4)$ lies on C.

(*a*) Find an equation of the normal to the curve C at A.
(5)

(*b*) Express y in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 significant figures.
(4)

(*c*) Find the coordinates of the points of intersection of the curve C with the x-axis.
Give your answers to 2 decimal places.
(4)

---

8. The functions f and g are defined by

$$f : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}.$$

$$g : x \mapsto \frac{3}{x} - 4, \quad x > 0, \ x \in \mathbb{R}.$$

(*a*) Find the inverse function f^{-1}.
(2)

(*b*) Show that the composite function gf is

$$gf : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$
(4)

(*c*) Solve $gf(x) = 0$.
(2)

(*d*) Use calculus to find the coordinates of the stationary point on the graph of $y = gf(x)$.
(5)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Subsidiary Level

Friday 6 June 2008 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

N30745A

1. The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y-coordinate of P is 8.

(a) Find, in terms of ln 2, the x-coordinate of P. **(2)**

(b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found. **(4)**

---

2. $$f(x) = 5\cos x + 12\sin x.$$

Given that $f(x) = R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

(a) find the value of R and the value of α to 3 decimal places. **(4)**

(b) Hence solve the equation

$$5\cos x + 12\sin x = 6$$

for $0 \leq x < 2\pi$. **(5)**

(c) (i) Write down the maximum value of $5\cos x + 12\sin x$. **(1)**

(ii) Find the smallest positive value of x for which this maximum value occurs. **(2)**

3.

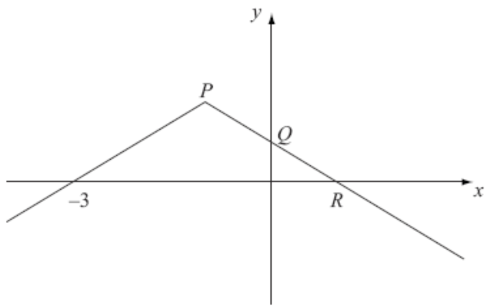

Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$,

The graph consists of two line segments that meet at the point P.

The graph cuts the y-axis at the point Q and the x-axis at the points $(-3, 0)$ and R.

Sketch, on separate diagrams, the graphs of

(*a*) $y = |\mathrm{f}(x)|$, **(2)**

(*b*) $y = \mathrm{f}(-x)$. **(2)**

Given that $\mathrm{f}(x) = 2 - |x + 1|$,

(*c*) find the coordinates of the points P, Q and R, **(3)**

(*d*) solve $\mathrm{f}(x) = \frac{1}{2}x$. **(5)**

4. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{2(x-1)}{x^2 - 2x - 3} - \frac{1}{x-3}, \quad x > 3.$$

(*a*) Show that $\mathrm{f}(x) = \dfrac{1}{x+1}$, $x > 3$. **(4)**

(*b*) Find the range of f. **(2)**

(*c*) Find $\mathrm{f}^{-1}(x)$. State the domain of this inverse function. **(3)**

The function g is defined by

$$\mathrm{g}: x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(*d*) Solve $\mathrm{fg}(x) = \dfrac{1}{8}$. **(3)**

5. (*a*) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$. **(2)**

(*b*) Solve, for $0 \le \theta < 180°$, the equation

$$2\cot^2\theta - 9\operatorname{cosec}\theta = 3,$$

giving your answers to 1 decimal place. **(6)**

PMT

6. (*a*) Differentiate with respect to x,

(i) $e^{3x}(\sin x + 2\cos x)$,

(3)

(ii) $x^3 \ln(5x + 2)$.

(3)

Given that $y = \dfrac{3x^2 + 6x - 7}{(x+1)^2}$, $x \neq -1$,

(*b*) show that $\dfrac{dy}{dx} = \dfrac{20}{(x+1)^3}$.

(5)

(*c*) Hence find $\dfrac{d^2y}{dx^2}$ and the real values of x for which $\dfrac{d^2y}{dx^2} = -\dfrac{15}{4}$.

(3)

7.

$$f(x) = 3x^3 - 2x - 6.$$

(*a*) Show that f $(x) = 0$ has a root, α, between $x = 1.4$ and $x = 1.45$.

(2)

(*b*) Show that the equation f $(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(3)

(*c*) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(*d*) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6665/01

Edexcel GCE

Core Mathematics C3

Advanced Subsidiary

Thursday 15 January 2009 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

H31123A

1. (*a*) Find the value of $\frac{dy}{dx}$ at the point where $x = 2$ on the curve with equation

$$y = x^2 \sqrt{(5x - 1)}.$$

(6)

(*b*) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x.

(4)

---

2. $$f(x) = \frac{2x+2}{x^2 - 2x - 3} - \frac{x+1}{x-3}.$$

(*a*) Express f(x) as a single fraction in its simplest form.

(4)

(*b*) Hence show that $f'(x) = \frac{2}{(x-3)^2}$.

(3)

3.

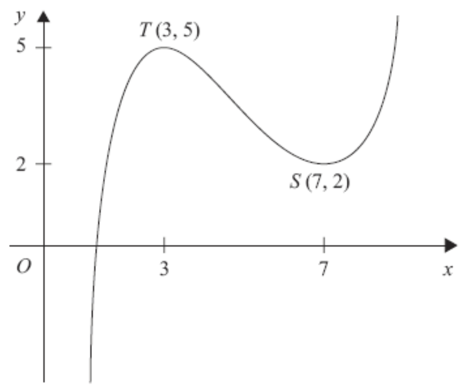

Figure 1

Figure 1 shows the graph of $y = f(x)$, $1 < x < 9$.

The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(*a*) $y = 2f(x) - 4$,

(3)

(*b*) $y = |f(x)|$.

(3)

Indicate on each diagram the coordinates of any turning points on your sketch.

---

4. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(6)

---

5. The functions f and g are defined by

$$f : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R},$$

$$g : x \mapsto e^{x^2}, \quad x \in \mathbb{R}.$$

(*a*) Write down the range of g.
(1)

(*b*) Show that the composite function fg is defined by

$$fg : x \mapsto x^2 + 3e^{x^2}, \quad x \in \mathbb{R}.$$

(2)

(*c*) Write down the range of fg.
(1)

(*d*) Solve the equation $\frac{d}{dx}\left[fg(x)\right] = x(xe^{x^2} + 2)$.

(6)

---

6. (*a*) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3 \theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π.

(5)

(*b*) Using $\sin(\theta - \alpha) = \sin \theta \cos \alpha - \cos \theta \sin \alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)

7.
$$f(x) = 3xe^x - 1.$$

The curve with equation $y = f(x)$ has a turning point P.

(*a*) Find the exact coordinates of P.

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$.

(*b*) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}.$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3.

(3)

(*c*) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

---

8. (*a*) Express $3 \cos \theta + 4 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$.

(4)

(*b*) Hence find the maximum value of $3 \cos \theta + 4 \sin \theta$ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3 \cos(15t)° + 4 \sin(15t)°,$$

where t is the time in hours from midday and $0 \leq t < 24$.

(*c*) Calculate the minimum temperature of the warehouse as given by this model.

(2)

(*d*) Find the value of t when this minimum temperature occurs.

(3)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Subsidiary Level

Thursday 11 June 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Orange or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

H36264A

1.

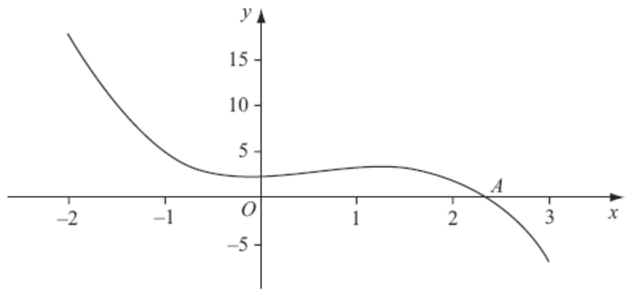

Figure 1

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x-axis at the point A where $x = \alpha$.

To find an approximation to α, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4.
Give your answers to 3 decimal places where appropriate.
(3)

(b) Show that $\alpha = 2.359$ correct to 3 decimal places.
(3)

2. (a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$.
(2)

(b) Solve, for $0 \leq \theta < 360°$, the equation

$$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2.$$

(6)

PMT

3. Rabbits were introduced onto an island. The number of rabbits, P, t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \quad t \in \mathbb{R}, \quad t \geq 0.$$

(*a*) Write down the number of rabbits that were introduced to the island.

(1)

(*b*) Find the number of years it would take for the number of rabbits to first exceed 1000.

(2)

(*c*) Find $\frac{dP}{dt}$.

(2)

(*d*) Find P when $\frac{dP}{dt} = 50$.

(3)

---

4. (i) Differentiate with respect to x

(*a*) $x^2 \cos 3x$,

(3)

(*b*) $\frac{\ln(x^2+1)}{x^2+1}$.

(4)

(ii) A curve C has the equation

$$y = \sqrt{(4x+1)}, \quad x > -\tfrac{1}{4}, \quad y > 0.$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

5.

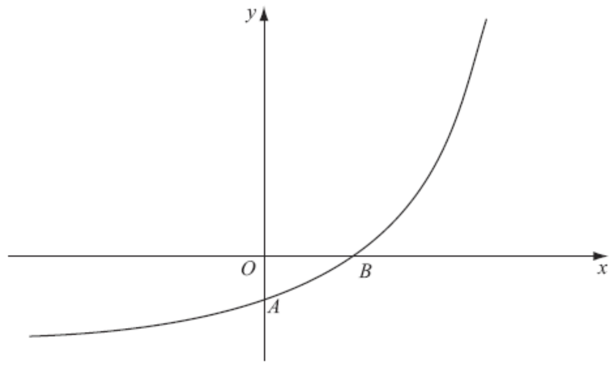

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0, 1 - k)$ and $B(\frac{1}{2}\ln k, 0)$, where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(*a*) $y = |f(x)|$,

(3)

(*b*) $y = f^{-1}(x)$.

(2)

Show on each sketch the coordinates, in terms of k, of each point at which the curve meets or cuts the axes.

Given that $f(x) = e^{2x} - k$,

(*c*) state the range of f,

(1)

(*d*) find $f^{-1}(x)$,

(3)

(*e*) write down the domain of f^{-1}.

(1)

PMT

6. (*a*) Use the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A$$

(2)

The curves C_1 and C_2 have equations

$$C_1 : y = 3\sin 2x$$
$$C_2 : y = 4\sin^2 x - 2\cos 2x$$

(*b*) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$

(3)

(*c*) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

(*d*) Hence find, for $0 \leq x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2,$$

giving your answers to 1 decimal place.

(4)

---

7. The function f is defined by

$$\mathrm{f}(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R},\ x \neq -4,\ x \neq 2.$$

(*a*) Show that $\mathrm{f}(x) = \dfrac{x-3}{x-2}$.

(5)

The function g is defined by

$$\mathrm{g}(x) = \frac{\mathrm{e}^x - 3}{\mathrm{e}^x - 2}, \quad x \in \mathbb{R},\ x \neq \ln 2.$$

(*b*) Differentiate g(x) to show that $\mathrm{g}'(x) = \dfrac{\mathrm{e}^x}{(\mathrm{e}^x - 2)^2}$.

(3)

(*c*) Find the exact values of x for which $\mathrm{g}'(x) = 1$

(4)

---

8. (*a*) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$.

(1)

(*b*) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8\cos x = 0.$$

giving your answers to 2 decimal places.

(5)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Wednesday 20 January 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

N35381A

1. Express

$$\frac{x+1}{3x^2-3}-\frac{1}{3x+1}$$

as a single fraction in its simplest form.

(4)

2.
$$\text{f}(x)=x^3+2x^2-3x-11$$

(*a*) Show that $\text{f}(x)=0$ can be rearranged as

(2)

$$x=\sqrt{\left(\frac{3x+11}{x+2}\right)},\quad x\neq -2.$$

The equation $\text{f}(x)=0$ has one positive root α.

The iterative formula $x_{n+1}=\sqrt{\left(\frac{3x_n+11}{x_n+2}\right)}$ is used to find an approximation to α.

(*b*) Taking $x_1=0$, find, to 3 decimal places, the values of x_2, x_3 and x_4.

(3)

(*c*) Show that $\alpha = 2.057$ correct to 3 decimal places.

(3)

---

3. (*a*) Express $5\cos x - 3\sin x$ in the form $R\cos(x+\alpha)$, where $R>0$ and $0<\alpha<\frac{1}{2}\pi$.

(4)

(*b*) Hence, or otherwise, solve the equation

$$5\cos x-3\sin x=4$$

for $0\leq x<2\pi$, giving your answers to 2 decimal places.

(5)

---

4. (i) Given that $y=\frac{\ln(x^2+1)}{x}$, find $\frac{\text{d}y}{\text{d}x}$.

(4)

(ii) Given that $x=\tan y$, show that $\frac{\text{d}y}{\text{d}x}=\frac{1}{1+x^2}$.

(5)

---

5. Sketch the graph of $y = \ln |x|$, stating the coordinates of any points of intersection with the axes.
(3)

6.

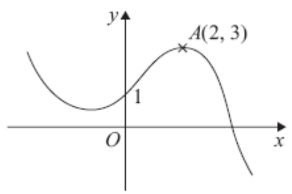

Figure 1

Figure 1 shows a sketch of the graph of $y = \mathrm{f}(x)$.

The graph intersects the y-axis at the point (0, 1) and the point $A(2, 3)$ is the maximum turning point.

Sketch, on separate axes, the graphs of

(i) $y = \mathrm{f}(-x) + 1$,

(ii) $y = \mathrm{f}(x + 2) + 3$,

(iii) $y = 2\mathrm{f}(2x)$.

On each sketch, show the coordinates of the point at which your graph intersects the y-axis and the coordinates of the point to which A is transformed.
(9)

---

7. (*a*) By writing $\sec x$ as $\dfrac{1}{\cos x}$, show that $\dfrac{\mathrm{d}(\sec x)}{\mathrm{d}x} = \sec x \tan x$.
(3)

Given that $y = \mathrm{e}^{2x} \sec 3x$,

(*b*) find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.
(4)

The curve with equation $y = \mathrm{e}^{2x} \sec 3x$, $-\dfrac{\pi}{6} < x < \dfrac{\pi}{6}$, has a minimum turning point at (a, b).

(*c*) Find the values of the constants a and b, giving your answers to 3 significant figures.
(4)

---

8. Solve

$$\operatorname{cosec}^2 2x - \cot 2x = 1$$

for $0 \leq x \leq 180°$.
(7)

---

9. (i) Find the exact solutions to the equations

(*a*) $\ln(3x - 7) = 5$,
(3)

(*b*) $3^x \mathrm{e}^{7x+2} = 15$.
(5)

(ii) The functions f and g are defined by

$$\mathrm{f}(x) = \mathrm{e}^{2x} + 3, \qquad x \in \mathbb{R},$$

$$\mathrm{g}(x) = \ln(x - 1), \qquad x \in \mathbb{R}, \quad x > 1.$$

(*a*) Find f^{-1} and state its domain.
(4)

(*b*) Find fg and state its range.
(3)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Tuesday 15 June 2010 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

H35385A

1. (*a*) Show that

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan \theta.$$

(2)

(*b*) Hence find, for $-180° \leq \theta < 180°$, all the solutions of

$$\frac{2\sin 2\theta}{1+\cos 2\theta} = 1.$$

Give your answers to 1 decimal place.

(3)

---

2. A curve C has equation

$$y = \frac{3}{(5-3x)^2}, \qquad x \neq \frac{5}{3}.$$

The point P on C has x-coordinate 2.

Find an equation of the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

---

3. $$\text{f}(x) = 4\,\text{cosec}\,x - 4x + 1, \qquad \text{where } x \text{ is in radians.}$$

(*a*) Show that there is a root α of $\text{f}(x) = 0$ in the interval [1.2, 1.3].

(2)

(*b*) Show that the equation $\text{f}(x) = 0$ can be written in the form

$$x = \frac{1}{\sin x} + \frac{1}{4}$$

(2)

(*c*) Use the iterative formula

$$x_{n+1} = \frac{1}{\sin x_n} + \frac{1}{4}, \qquad x_0 = 1.25,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(*d*) By considering the change of sign of $\text{f}(x)$ in a suitable interval, verify that $\alpha = 1.291$ correct to 3 decimal places.

(2)

PMT

4. The function f is defined by

$$f : x \mapsto |2x - 5|, \quad x \in \mathbb{R}.$$

(*a*) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(2)

(*b*) Solve $f(x) = 15 + x$.

(3)

The function g is defined by

$$g : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leq x \leq 5.$$

(*c*) Find fg(2).

(2)

(*d*) Find the range of g.

(3)

5.

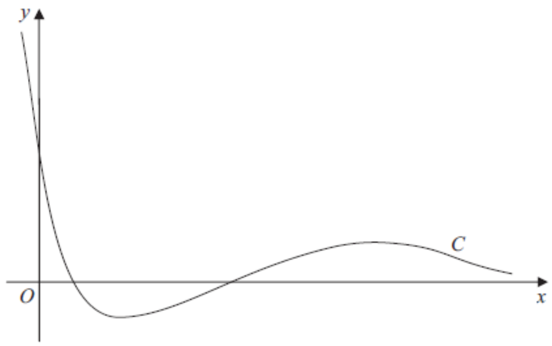

Figure 1

Figure 1 shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(*a*) Find the coordinates of the point where C crosses the y-axis.

(1)

(*b*) Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis.

(3)

(*c*) Find $\dfrac{dy}{dx}$.

(3)

(*d*) Hence find the exact coordinates of the turning points of C.

(5)

PMT

6.

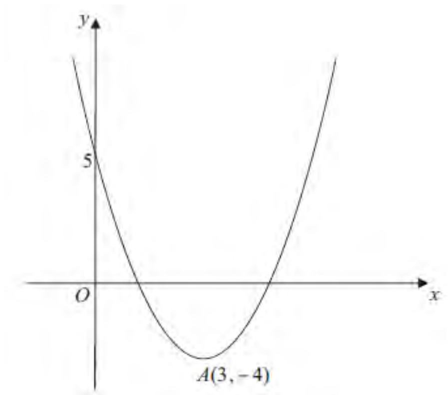

Figure 2

Figure 2 shows a sketch of the curve with the equation $y = f(x)$, $x \in \mathbb{R}$.

The curve has a turning point at $A(3, -4)$ and also passes through the point $(0, 5)$.

(*a*) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |f(x)|$,

(ii) $y = 2f(\frac{1}{2}x)$.

(4)

(*b*) Sketch the curve with equation $y = f(|x|)$.

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y-axis.

(3)

The curve with equation $y = f(x)$ is a translation of the curve with equation $y = x^2$.

(*c*) Find $f(x)$.

(2)

(*d*) Explain why the function f does not have an inverse.

(1)

7. (*a*) Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

(*b*) (i) Find the maximum value of $2\sin\theta - 1.5\cos\theta$.

(ii) Find the value of θ, for $0 \leq \theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), \quad 0 \leq t < 12,$$

where t hours is the number of hours after midday.

(*c*) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs.

(3)

(*d*) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)

8. (*a*) Simplify fully

$$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}.$$

(3)

Given that

$$\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15), \quad x \neq -5,$$

(*b*) find x in terms of e.

(4)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Monday 24 January 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

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Answers without working may not gain full credit.

H35404A

1. (*a*) Express $7\cos x - 24\sin x$ in the form $R\cos(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(3)

(*b*) Hence write down the minimum value of $7\cos x - 24\sin x$.

(1)

(*c*) Solve, for $0 \leq x < 2\pi$, the equation

$$7\cos x - 24\sin x = 10,$$

giving your answers to 2 decimal places.

(5)

---

2. (*a*) Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form.

(4)

Given that

$$\text{f}(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \qquad x > 1,$$

(*b*) show that

$$\text{f}(x) = \frac{3}{2x-1}.$$

(2)

(*c*) Hence differentiate f(x) and find f′(2).

(3)

---

3. Find all the solutions of

$$2\cos 2\theta = 1 - 2\sin\theta$$

in the interval $0 \leq \theta < 360°$.

(6)

PMT

4. Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ °C, of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt},$$

where A and k are positive constants.

Given that the initial temperature of the tea was 90 °C,

(*a*) find the value of A.

(2)

The tea takes 5 minutes to decrease in temperature from 90 °C to 55 °C.

(*b*) Show that $k = \frac{1}{5}\ln 2$.

(3)

(*c*) Find the rate at which the temperature of the tea is decreasing at the instant when $t = 10$. Give your answer, in °C per minute, to 3 decimal places.

(3)

5.

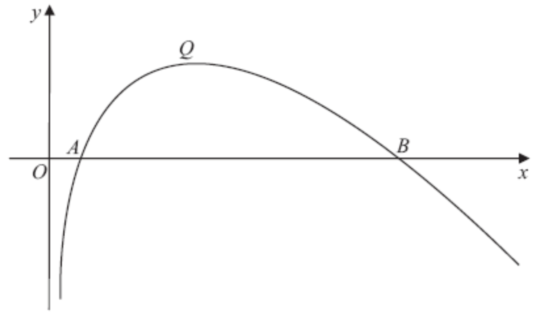

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (8 - x)\ln x, \quad x > 0.$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 1.

(*a*) Write down the coordinates of A and the coordinates of B.

(2)

(*b*) Find $f'(x)$.

(3)

(*c*) Show that the x-coordinate of Q lies between 3.5 and 3.6

(2)

(*d*) Show that the x-coordinate of Q is the solution of

$$x = \frac{8}{1 + \ln x}.$$

(3)

To find an approximation for the x-coordinate of Q, the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(*e*) Taking $x_0 = 3.55$, find the values of x_1, x_2 and x_3.
Give your answers to 3 decimal places.

(3)

PMT

6. The function f is defined by

$$f: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R}, \quad x \neq 5.$$

(*a*) Find $f^{-1}(x)$.

(3)

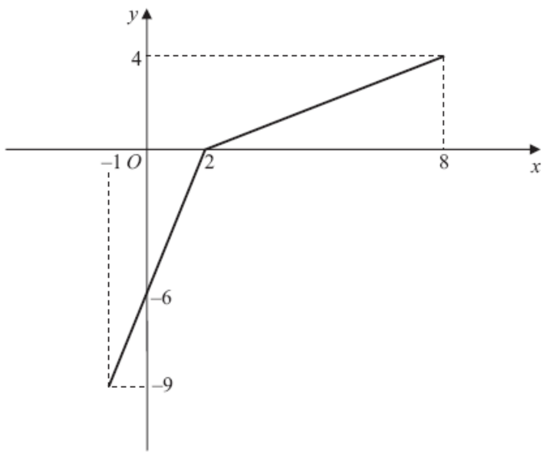

Figure 2

The function g has domain $-1 \leq x \leq 8$, and is linear from (–1, –9) to (2, 0) and from (2, 0) to (8, 4). Figure 2 shows a sketch of the graph of $y = g(x)$.

(*b*) Write down the range of g.

(1)

(*c*) Find gg(2).

(2)

(*d*) Find fg(8).

(2)

(*e*) On separate diagrams, sketch the graph with equation

(i) $y = |g(x)|$,

(ii) $y = g^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(*f*) State the domain of the inverse function g^{-1}.

(1)

7. The curve *C* has equation

$$y = \frac{3+\sin 2x}{2+\cos 2x}.$$

(*a*) Show that

$$\frac{dy}{dx} = \frac{6\sin 2x + 4\cos 2x + 2}{(2+\cos 2x)^2}$$

(4)

(*b*) Find an equation of the tangent to *C* at the point on *C* where $x = \frac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where *a* and *b* are exact constants.

(4)

8. Given that

$$\frac{d}{dx}(\cos x) = -\sin x,$$

(*a*) show that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

(3)

Given that

$$x = \sec 2y,$$

(*b*) find $\frac{dx}{dy}$ in terms of *y*.

(2)

(*c*) Hence find $\frac{dy}{dx}$ in terms of *x*.

(4)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Thursday 16 June 2011 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Calculators may NOT be used in this examination.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

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Answers without working may not gain full credit.

P38159A

1. Differentiate with respect to x

(*a*) $\ln(x^2 + 3x + 5)$,

(2)

(*b*) $\dfrac{\cos x}{x^2}$.

(3)

---

2. $$f(x) = 2\sin(x^2) + x - 2, \qquad 0 \le x < 2\pi.$$

(*a*) Show that $f(x) = 0$ has a root α between $x = 0.75$ and $x = 0.85$.

(2)

The equation $f(x) = 0$ can be written as $x = [\arcsin(1 - 0.5x)]^{\frac{1}{2}}$.

(*b*) Use the iterative formula

$$x_{n+1} = [\arcsin(1 - 0.5x_n)]^{\frac{1}{2}}, \qquad x_0 = 0.8,$$

to find the values of x_1, x_2 and x_3, giving your answers to 5 decimal places.

(3)

(*c*) Show that $\alpha = 0.80157$ is correct to 5 decimal places.

(3)

3.

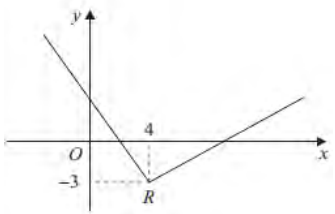

Figure 1

Figure 1 shows part of the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point R (4, – 3), as shown in Figure 1.

Sketch, on separate diagrams, the graphs of

(*a*) $y = 2\mathrm{f}(x + 4)$, **(3)**

(*b*) $y = |\mathrm{f}(-x)|$. **(3)**

On each diagram, show the coordinates of the point corresponding to R.

---

4. The function f is defined by

$$\mathrm{f}: x \mapsto 4 - \ln(x + 2), \quad x \in \mathbb{R}, \quad x \geq -1.$$

(*a*) Find $\mathrm{f}^{-1}(x)$. **(3)**

(*b*) Find the domain of f^{-1}. **(1)**

The function g is defined by

$$\mathrm{g}: x \mapsto \mathrm{e}^{x^2} - 2, \quad x \in \mathbb{R}.$$

(*c*) Find fg(x), giving your answer in its simplest form. **(3)**

(*d*) Find the range of fg. **(1)**

---

5. The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = p\mathrm{e}^{-kt},$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(*a*) Write down the value of p. **(1)**

(*b*) Show that $k = \frac{1}{4}\ln 3$. **(4)**

(*c*) Find the value of t when $\frac{\mathrm{d}m}{\mathrm{d}t} = -0.6\ln 3$. **(6)**

---

6. (*a*) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan\theta, \qquad \theta \neq 90n°, \quad n \in \mathbb{Z}.$$

(4)

(*b*) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$, **(3)**

(ii) solve, for $0 < x < 360°$,

$$\operatorname{cosec} 4x - \cot 4x = 1.$$

(5)

PMT

7. $$f(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9}, \qquad x \neq \pm 3,\ x \neq -\frac{1}{2}.$$

(*a*) Show that

$$f(x) = \frac{5}{(2x+1)(x+3)}.$$

(5)

The curve C has equation $y = f(x)$. The point $P\left(-1, -\frac{5}{2}\right)$ lies on C.

(*b*) Find an equation of the normal to C at P.

(8)

---

8. (*a*) Express $2\cos 3x - 3\sin 3x$ in the form $R\cos(3x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures.

(4)

$$f(x) = e^{2x}\cos 3x.$$

(*b*) Show that $f'(x)$ can be written in the form

$$f'(x) = Re^{2x}\cos(3x + \alpha),$$

where R and α are the constants found in part (*a*).

(5)

(*c*) Hence, or otherwise, find the smallest positive value of x for which the curve with equation $y = f(x)$ has a turning point.

(3)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Monday 23 January 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

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Information for Candidates

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Advice to Candidates

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P40084A

PMT

1. Differentiate with respect to x, giving your answer in its simplest form,

(*a*) $x^2 \ln(3x)$,

(4)

(*b*) $\dfrac{\sin 4x}{x^3}$.

(5)

2.

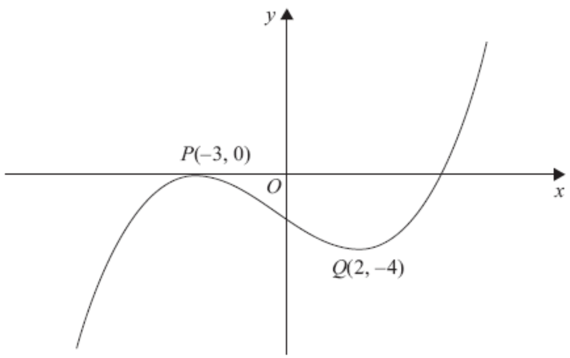

Figure 1

Figure 1 shows the graph of equation $y = f(x)$.

The points $P(-3, 0)$ and $Q(2, -4)$ are stationary points on the graph.

Sketch, on separate diagrams, the graphs of

(*a*) $y = 3f(x + 2)$,

(3)

(*b*) $y = |f(x)|$.

(3)

On each diagram, show the coordinates of any stationary points.

---

3. The area, A mm^2, of a bacterial culture growing in milk, t hours after midday, is given by

$$A = 20e^{1.5t}, \qquad t \geq 0.$$

(*a*) Write down the area of the culture at midday.

(1)

(*b*) Find the time at which the area of the culture is twice its area at midday. Give your answer to the nearest minute.

(5)

---

4. The point P is the point on the curve $x = 2\tan\left(y + \dfrac{\pi}{12}\right)$ with y-coordinate $\dfrac{\pi}{4}$.

Find an equation of the normal to the curve at P.

(7)

---

5. Solve, for $0 \leq \theta \leq 180°$,

$$2\cot^2 3\theta = 7\operatorname{cosec} 3\theta - 5.$$

Give your answers in degrees to 1 decimal place.

(10)

---

6. $$f(x) = x^2 - 3x + 2\cos\left(\tfrac{1}{2}x\right), \qquad 0 \le x \le \pi.$$

(*a*) Show that the equation $f(x) = 0$ has a solution in the interval $0.8 < x < 0.9$.

(2)

The curve with equation $y = f(x)$ has a minimum point P.

(*b*) Show that the x-coordinate of P is the solution of the equation

$$x = \frac{3 + \sin\left(\tfrac{1}{2}x\right)}{2}.$$

(4)

(*c*) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin\left(\tfrac{1}{2}x_n\right)}{2}, \qquad x_0 = 2,$$

find the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(*d*) By choosing a suitable interval, show that the x-coordinate of P is 1.9078 correct to 4 decimal places.

(3)

---

7. The function f is defined by

$$f : x \mapsto \frac{3(x+1)}{2x^2 + 7x - 4} - \frac{1}{x+4}, \qquad x \in \mathbb{R},\ x > \frac{1}{2}.$$

(*a*) Show that $f(x) = \dfrac{1}{2x-1}$.

(4)

(*b*) Find $f^{-1}(x)$.

(3)

(*c*) Find the domain of f^{-1}.

(1)

$$g(x) = \ln(x+1).$$

(*d*) Find the solution of $fg(x) = \dfrac{1}{7}$, giving your answer in terms of e.

(4)

---

8. (*a*) Starting from the formulae for $\sin(A+B)$ and $\cos(A+B)$, prove that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

(4)

(*b*) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3}\tan\theta}{\sqrt{3} - \tan\theta}.$$

(3)

(*c*) Hence, or otherwise, solve, for $0 \le \theta \le \pi$,

$$1 + \sqrt{3}\tan\theta = (\sqrt{3} - \tan\theta)\tan(\pi - \theta).$$

Give your answers as multiples of π.

(6)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Thursday 14 June 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

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Advice to Candidates

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P40686A

1. Express

$$\frac{2(3x+2)}{9x^2-4} - \frac{2}{3x+1}$$

as a single fraction in its simplest form.

(4)

2.

$$f(x) = x^3 + 3x^2 + 4x - 12$$

(*a*) Show that the equation f(x) = 0 can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \quad x \neq -3.$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2.

(*b*) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, \quad n \geq 0,$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3.

(3)

The root of f(x) = 0 is α.

(*c*) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(3)

PMT

3.

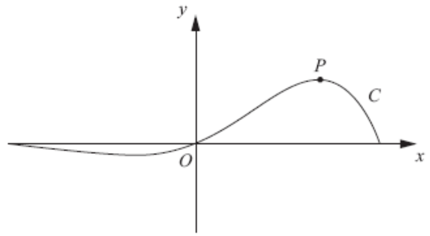

Figure 1

Figure 1 shows a sketch of the curve C which has equation

$$y = e^{x\sqrt{3}} \sin 3x, \quad -\frac{\pi}{3} \leq x \leq \frac{\pi}{3}.$$

(*a*) Find the x-coordinate of the turning point P on C, for which $x > 0$.
Give your answer as a multiple of π.

(6)

(*b*) Find an equation of the normal to C at the point where $x = 0$.

(3)

4.

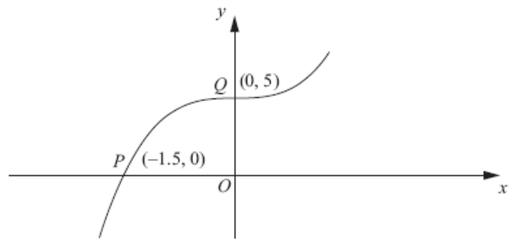

Figure 2

Figure 2 shows part of the curve with equation $y = f(x)$.
The curve passes through the points $P(-1.5, 0)$and $Q(0, 5)$ as shown.

On separate diagrams, sketch the curve with equation

(*a*) $y = |f(x)|$

(2)

(*b*) $y = f(|x|)$

(2)

(*c*) $y = 2f(3x)$

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

---

5. (*a*) Express $4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta$ in terms of $\sin \theta$ and $\cos \theta$.

(2)

(*b*) Hence show that

$$4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta = \sec^2 \theta .$$

(4)

(*c*) Hence or otherwise solve, for $0 < \theta < \pi$,

$$4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta = 4$$

giving your answers in terms of π.

(3)

PMT

6. The functions f and g are defined by

$$\text{f}: x \mapsto e^x + 2, \qquad x \in \mathbb{R},$$

$$\text{g}: x \mapsto \ln x, \qquad x > 0.$$

(*a*) State the range of f. **(1)**

(*b*) Find fg(x), giving your answer in its simplest form. **(2)**

(*c*) Find the exact value of x for which f($2x + 3$) = 6. **(4)**

(*d*) Find f^{-1}, the inverse function of f, stating its domain. **(3)**

(*e*) On the same axes sketch the curves with equation $y = \text{f}(x)$ and $y = \text{f}^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes. **(4)**

---

7. (*a*) Differentiate with respect to x,

(i) $x^{\frac{1}{2}} \ln(3x)$,

(ii) $\dfrac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form. **(6)**

(*b*) Given that $x = 3\tan 2y$ find $\dfrac{dy}{dx}$ in terms of x. **(5)**

---

8. $$\text{f}(x) = 7\cos 2x - 24\sin 2x.$$

Given that $\text{f}(x) = R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(*a*) find the value of R and the value of α. **(3)**

(*b*) Hence solve the equation

$$7\cos 2x - 24\sin 2x = 12.5$$

for $0 \leq x < 180°$, giving your answers to 1 decimal place. **(5)**

(*c*) Express $14\cos^2 x - 48\sin x\cos x$ in the form $a\cos 2x + b\sin 2x + c$, where a, b, and c are constants to be found. **(2)**

(*d*) Hence, using your answers to parts (*a*) and (*c*), deduce the maximum value of

$$14\cos^2 x - 48\sin x\cos x.$$

(2)

TOTAL FOR PAPER: 75 MARKS

END

PMT

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Friday 25 January 2013 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P41486A

1. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(*a*) the value of w,

(2)

(*b*) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

2.

$$\mathrm{g}(x) = \mathrm{e}^{x-1} + x - 6$$

(*a*) Show that the equation $\mathrm{g}(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \qquad x < 6.$$

(2)

The root of $\mathrm{g}(x) = 0$ is α.

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \qquad x_0 = 2.$$

is used to find an approximate value for α.

(*b*) Calculate the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(*c*) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places.

(3)

3.

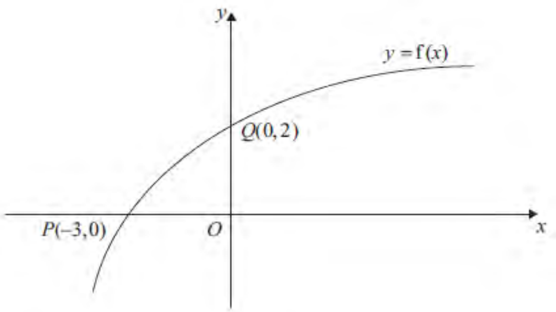

Figure 1

Figure 1 shows part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve passes through the points $Q(0, 2)$ and $P(-3, 0)$ as shown.

(*a*) Find the value of ff (-3).

(2)

On separate diagrams, sketch the curve with equation

(*b*) $y = \mathrm{f}^{-1}(x)$,

(2)

(*c*) $y = \mathrm{f}(|x|) - 2$,

(2)

(*d*) $y = 2\mathrm{f}\left(\frac{1}{2}x\right)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

4. (*a*) Express $6 \cos \theta + 8 \sin \theta$ in the form $R \cos (\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

(*b*)
$$\mathrm{p}(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, \quad 0 \leq \theta \leq 2\pi.$$

Calculate

(i) the maximum value of $\mathrm{p}(\theta)$,

(ii) the value of θ at which the maximum occurs.

(4)

5. (i) Differentiate with respect to x

(*a*) $y = x^3 \ln 2x$,

(*b*) $y = (x + \sin 2x)^3$.

(6)

Given that $x = \cot y$,

(ii) show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-1}{1 + x^2}$.

(5)

6. (i) Without using a calculator, find the exact value of

$$(\sin 22.5° + \cos 22.5°)^2.$$

You must show each stage of your working.

(5)

(ii) (*a*) Show that $\cos 2\theta + \sin \theta = 1$ may be written in the form

$$k \sin^2 \theta - \sin \theta = 0, \text{ stating the value of } k.$$

(2)

(*b*) Hence solve, for $0 \leq \theta < 360°$, the equation

$$\cos 2\theta + \sin \theta = 1.$$

(4)

PMT

7. $$\mathrm{h}(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \quad x \geq 0.$$

(*a*) Show that $\mathrm{h}(x) = \dfrac{2x}{x^2+5}$.

(4)

(*b*) Hence, or otherwise, find $\mathrm{h}'(x)$ in its simplest form.

(3)

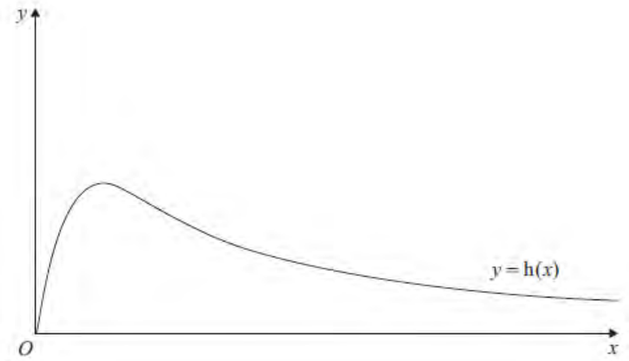

Figure 2

Figure 2 shows a graph of the curve with equation $y = \mathrm{h}(x)$.

(*c*) Calculate the range of $\mathrm{h}(x)$.

(5)

---

8. The value of Bob's car can be calculated from the formula

$$V = 17000\mathrm{e}^{-0.25t} + 2000\mathrm{e}^{-0.5t} + 500.$$

where V is the value of the car in pounds (£) and t is the age in years.

(*a*) Find the value of the car when $t = 0$.

(1)

(*b*) Calculate the exact value of t when $V = 9500$.

(4)

(*c*) Find the rate at which the value of the car is decreasing at the instant when $t = 8$.
Give your answer in pounds per year to the nearest pound.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6665/01R

Edexcel GCE

Core Mathematics C3 (R)

Advanced Subsidiary

Thursday 13 June 2013 – Morning

Time: 1 hour 30 minutes

<u>Materials required for examination</u>	<u>Items included with question papers</u>
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

This paper is strictly for students outside the UK.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P42953A

PMT

1. Express

$$\frac{3x+5}{x^2+x-12}-\frac{2}{x-3}$$

as a single fraction in its simplest form.

(4)

2.

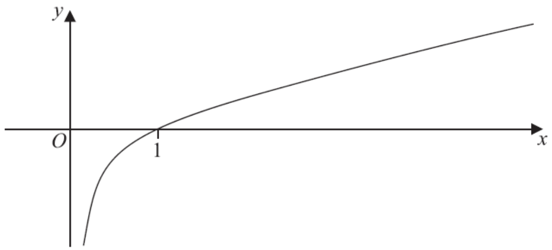

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, $x > 0$, where f is an increasing function of x. The curve crosses the x-axis at the point (1, 0) and the line $x = 0$ is an asymptote to the curve.

On separate diagrams, sketch the curve with equation

(*a*) $y = \mathrm{f}(2x)$, $x > 0$

(2)

(*b*) $y = |\mathrm{f}(x)|$, $x > 0$

(3)

Indicate clearly on each sketch the coordinates of the point at which the curve crosses or meets the x-axis.

3.

$$\mathrm{f}(x) = 7\cos x + \sin x$$

Given that $\mathrm{f}(x) = R\cos(x - a)$, where $R > 0$ and $0 < a < 90°$,

(*a*) find the exact value of R and the value of a to one decimal place.

(3)

(*b*) Hence solve the equation

$$7\cos x + \sin x = 5$$

for $0 \leq x < 360°$, giving your answers to one decimal place.

(5)

(*c*) State the values of k for which the equation

$$7\cos x + \sin x = k$$

has only one solution in the interval $0 \leq x < 360°$.

(2)

---

4. The functions f and g are defined by

$$\mathrm{f} : x \mapsto 2|x| + 3, \qquad x \in \mathbb{R}$$

$$\mathrm{g} : x \mapsto 3 - 4x, \qquad x \in \mathbb{R}$$

(*a*) State the range of f.

(2)

(*b*) Find fg(1).

(2)

(*c*) Find g^{-1}, the inverse function of g.

(2)

(*d*) Solve the equation

$$\mathrm{gg}(x) + [\mathrm{g}(x)]^2 = 0$$

(5)

---

5. (*a*) Differentiate

$$\frac{\cos 2x}{\sqrt{x}}$$

with respect to x.

(3)

(*b*) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(\sec^2 3x)$ can be written in the form

$$\mu(\tan 3x + \tan^3 3x)$$

where μ is a constant.

(3)

(*c*) Given $x = 2\sin\left(\frac{y}{3}\right)$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x, simplifying your answer.

(4)

---

6. (i) Use an appropriate double angle formula to show that

$$\operatorname{cosec} 2x = \lambda \operatorname{cosec} x \sec x,$$

and state the value of the constant λ.

(3)

(ii) Solve, for $0 \leq \theta < 2\pi$, the equation

$$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta$$

You must show all your working. Give your answers in terms of π.

(6)

7.

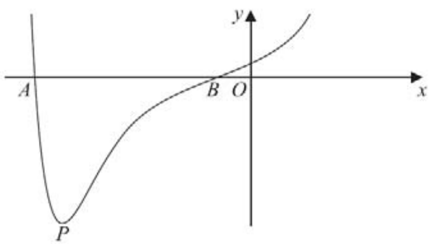

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (x^2 + 3x + 1)e^{x^2}$$

The curve cuts the x-axis at points A and B as shown in Figure 2.

(*a*) Calculate the x-coordinate of A and the x-coordinate of B, giving your answers to 3 decimal places.

(2)

(*b*) Find $\mathrm{f}'(x)$.

(3)

The curve has a minimum turning point P as shown in Figure 2.

(*c*) Show that the x-coordinate of P is the solution of

$$x = -\frac{3(2x^2+1)}{2(x^2+2)}$$

(3)

(*d*) Use the iteration formula

$$x_{n+1} = -\frac{3(2x_n^2+1)}{2(x_n^2+2)}, \quad \text{with } x_0 = -2.4,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

The x-coordinate of P is α.

(*e*) By choosing a suitable interval, prove that $\alpha = -2.43$ to 2 decimal places.

(2)

PMT

8.

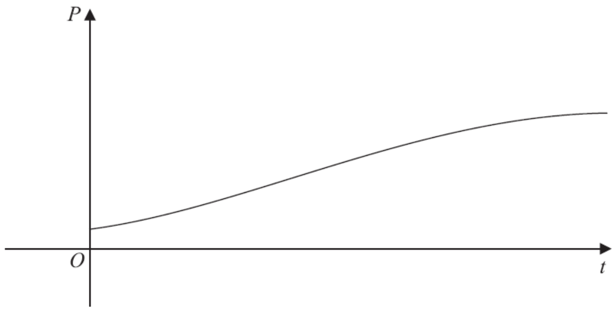

Figure 3

The population of a town is being studied. The population *P*, at time *t* years from the start of the study, is assumed to be

$$P = \frac{8000}{1+7e^{-kt}}, \qquad t \geq 0$$

where *k* is a positive constant.

The graph of *P* against *t* is shown in Figure 3.

Use the given equation to

(*a*) find the population at the start of the study,
(2)

(*b*) find a value for the expected upper limit of the population.
(1)

Given also that the population reaches 2500 at 3 years from the start of the study,

(*c*) calculate the value of *k* to 3 decimal places.
(5)

Using this value for *k*,

(*d*) find the population at 10 years from the start of the study, giving your answer to 3 significant figures.
(2)

(*e*) Find, using $\frac{dP}{dt}$, the rate at which the population is growing at 10 years from the start of the study.
(3)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Subsidiary

Thursday 13 June 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

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Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

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The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43016A

1. Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \qquad x \neq \pm 2$$

find the values of the constants a, b, c, d and e.

(4)

---

2. Given that

$$f(x) = \ln x, \qquad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$,

(ii) $y = |f(x)|$,

(iii) $y = -f(x - 4)$.

Show, on each diagram, the point where the graph meets or crosses the x-axis.
In each case, state the equation of the asymptote.

(7)

---

3. Given that

$$2\cos(x + 50)° = \sin(x + 40)°.$$

(*a*) Show, without using a calculator, that

$$\tan x° = \frac{1}{3}\tan 40°.$$

(4)

(*b*) Hence solve, for $0 \leq \theta < 360$,

$$2\cos(2\theta + 50)° = \sin(2\theta + 40)°,$$

giving your answers to 1 decimal place.

(4)

4.

$$f(x) = 25x^2 e^{2x} - 16, \qquad x \in \mathbb{R}.$$

(*a*) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$.

(5)

(*b*) Show that the equation $f(x) = 0$ can be written as $x = \pm\frac{4}{5}e^{-x}$.

(1)

The equation $f(x) = 0$ has a root α, where $\alpha = 0.5$ to 1 decimal place.

(*c*) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5}e^{-x_n}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(*d*) Give an accurate estimate for α to 2 decimal places, and justify your answer.

(2)

---

5. Given that

$$x = \sec^2 3y, \quad 0 < y < \frac{\pi}{6}$$

(*a*) find $\frac{dx}{dy}$ in terms of y.

(2)

(*b*) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

(4)

(*c*) Find an expression for $\frac{d^2y}{dx^2}$ in terms of x. Give your answer in its simplest form.

(4)

---

6. Find algebraically the exact solutions to the equations

(*a*) $\ln(4 - 2x) + \ln(9 - 3x) = 2\ln(x + 1), \quad -1 < x < 2,$

(5)

(*b*) $2^x e^{3x+1} = 10.$

Give your answer to (*b*) in the form $\frac{a + \ln b}{c + \ln d}$ where a, b, c and d are integers.

(5)

---

7. The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = f(x)$ is shown in Figure 1.

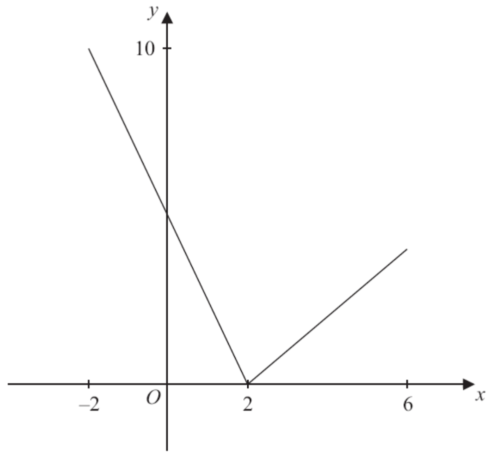

Figure 1

(*a*) Write down the range of f.

(1)

(*b*) Find ff(0).

(2)

The function g is defined by

$$g : x \rightarrow \frac{4 + 3x}{5 - x}, \qquad x \in \mathbb{R}, \qquad x \neq 5.$$

(*c*) Find $g^{-1}(x)$.

(3)

(*d*) Solve the equation $gf(x) = 16$.

(5)

8.

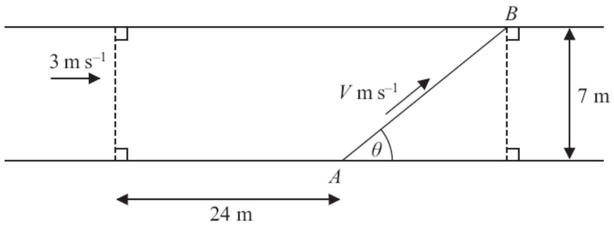

Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1}.

Kate is 24 m ahead of John when she starts to cross the road from the fixed point A.
John passes her as she reaches the other side of the road at a variable point B, as shown in Figure 2.

Kate's speed is V m s^{-1} and she moves in a straight line, which makes an angle θ, $0 < \theta < 150°$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24\sin\theta + 7\cos\theta}, \qquad 0 < \theta < 150°$$

(*a*) Express $24\sin\theta + 7\cos\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.
(3)

Given that θ varies,

(*b*) find the minimum value of V.
(2)

Given that Kate's speed has the value found in part (*b*),

(*c*) find the distance AB.
(3)

Given instead that Kate's speed is 1.68 m s^{-1},

(*d*) find the two possible values of the angle θ, given that $0 < \theta < 150°$.
(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)

6665A/01

Pearson Edexcel International Advanced Level

Core Mathematics C3

Advanced

Monday 27 January 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

P43136A

1.

$$f(x) = \sec x + 3x - 2, \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(*a*) Show that there is a root of $f(x) = 0$ in the interval [0.2, 0.4].

(2)

(*b*) Show that the equation $f(x) = 0$ can be written in the form

$$x = \frac{2}{3} - \frac{1}{3\cos x}$$

(1)

The solution of $f(x) = 0$ is α, where $\alpha = 0.3$ to 1 decimal place.

(*c*) Starting with $x_0 = 0.3$, use the iterative formula

$$x_{n+1} = \frac{2}{3} - \frac{1}{3\cos x_n}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(*d*) State the value of α correct to 3 decimal places.

(1)

2.

$$f(x) = \frac{15}{3x+4} - \frac{2x}{x-1} + \frac{14}{(3x+4)(x-1)}, \qquad x > 1$$

(*a*) Express $f(x)$ as a single fraction in its simplest form.

(4)

(*b*) Hence, or otherwise, find $f'(x)$, giving your answer as a single fraction in its simplest form.

(3)

---

3. (*a*) By writing $\operatorname{cosec} x$ as $\frac{1}{\sin x}$, show that

$$\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cot x$$

(3)

Given that $y = e^{3x} \operatorname{cosec} 2x$, $0 < x < \frac{\pi}{2}$,

(*b*) find an expression for $\frac{dy}{dx}$.

(3)

The curve with equation $y = e^{3x} \operatorname{cosec} 2x$, $0 < x < \frac{\pi}{2}$, has a single turning point.

(*c*) Show that the x-coordinate of this turning point is at $x = \frac{1}{2}\arctan k$ where the value of the constant k should be found.

(2)

---

4. A pot of coffee is delivered to a meeting room at 11am. At a time t minutes after 11am the temperature, θ°C, of the coffee in the pot is given by the equation

$$\theta = A + 60e^{-kt}$$

where A and k are positive constants.

Given also that the temperature of the coffee at 11am is 85°C and that 15 minutes later it is 58°C,

(*a*) find the value of A.

(1)

(*b*) Show that $k = \frac{1}{15}\ln\left(\frac{20}{11}\right)$.

(3)

(*c*) Find, to the nearest minute, the time at which the temperature of the coffee reaches 50°C.

(4)

5.

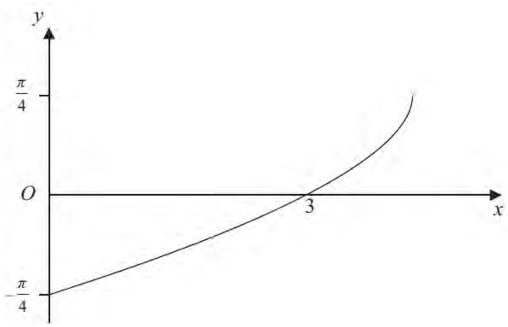

Figure 1

The curve shown in Figure 1 has equation

$$x = 3\sin y + 3\cos y, \qquad -\frac{\pi}{4} < y < \frac{\pi}{4}$$

(*a*) Express the equation of the curve in the form

$x = R\sin(y + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

(3)

(*b*) Find the coordinates of the point on the curve where the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ is $\frac{1}{2}$.

Give your answers to 3 decimal places.

(6)

6. Given that a and b are constants and that $0 < a < b$,

(*a*) on separate diagrams, sketch the graph with equation

(i) $y = |2x + a|$,

(ii) $y = |2x + a| - b$.

Show on each sketch the coordinates of each point at which the graph crosses or meets the axes.

(6)

(*b*) Solve, for x, the equation

$$|2x + a| - b = \frac{1}{3}x$$

giving any answers in terms of a and b.

(4)

7. (i) (*a*) Prove that

$$\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$$

(You may use the double angle formulae and the identity
$\cos(A+B) \equiv \cos A\cos B - \sin A\sin B$)

(4)

(*b*) Hence solve the equation

$$2\cos 3\theta + \cos 2\theta + 1 = 0$$

giving answers in the interval $0 < \theta < \pi$.
Solutions based entirely on graphical or numerical methods are not acceptable.

(6)

(ii) Given that $\theta = \arcsin x$ and that $0 < \theta < \frac{\pi}{2}$, show that

$$\cot\theta = \frac{\sqrt{(1 - x^2)}}{x}, \qquad 0 < x < 1$$

(3)

8. The function f is defined by

$$\mathrm{f} : x \rightarrow 3 - 2\mathrm{e}^{-x}, \qquad x \in \mathbb{R}$$

(*a*) Find the inverse function, $\mathrm{f}^{-1}(x)$, and give its domain.

(5)

(*b*) Solve the equation $\mathrm{f}^{-1}(x) = \ln x$.

(4)

The equation $\mathrm{f}(t) = k\mathrm{e}^{t}$, where k is a positive constant, has exactly one real solution.

(*c*) Find the value of k.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6665/01R

Edexcel GCE

Core Mathematics C3 (R)

Advanced Subsidiary

Thursday 13 June 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

This paper is strictly for students outside the UK.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43163A

PMT

1. Express

$$\frac{3}{2x+3} - \frac{1}{2x-3} + \frac{6}{4x^2-9}$$

as a single fraction in its simplest form.

(4)

---

2. A curve C has equation $y = e^{4x} + x^4 + 8x + 5$.

(*a*) Show that the x coordinate of any turning point of C satisfies the equation

$$x^3 = -2 - e^{4x}$$

(3)

(*b*) On a pair of axes, sketch, on a single diagram, the curves with equations

(i) $y = x^3$,

(ii) $y = -2 - e^{4x}$

On your diagram give the coordinates of the points where each curve crosses the y-axis and state the equation of any asymptotes.

(4)

(*c*) Explain how your diagram illustrates that the equation $x^3 = -2 - e^{4x}$ has only one root.

(1)

The iteration formula

$$x_{n+1} = \left(-2 - e^{4x_n}\right)^{\frac{1}{3}}, \quad x_0 = -1$$

can be used to find an approximate value for this root.

(*d*) Calculate the values of x_1 and x_2, giving your answers to 5 decimal places.

(2)

(*e*) Hence deduce the coordinates, to 2 decimal places, of the turning point of the curve C.

(2)

---

3. (i) (*a*) Show that $2 \tan x - \cot x = 5 \operatorname{cosec} x$ may be written in the form

$$a \cos^2 x + b \cos x + c = 0$$

stating the values of the constants a, b and c.

(4)

(*b*) Hence solve, for $0 \leq x < 2\pi$, the equation

$$2 \tan x - \cot x = 5 \operatorname{cosec} x$$

giving your answers to 3 significant figures.

(4)

(ii) Show that

$$\tan \theta + \cot \theta \equiv \lambda \operatorname{cosec} 2\theta, \quad \theta \neq \frac{n\pi}{2}, \quad n \in \mathbb{Z}$$

stating the value of the constant λ.

(4)

PMT

4. (i) Given that

$$x = \sec^2 2y, \quad 0 < y < \frac{\pi}{4}$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4x\sqrt{(x-1)}}$$

(4)

(ii) Given that

$$y = \left(x^2 + x^3\right)\ln 2x$$

find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = \frac{\mathrm{e}}{2}$, giving your answer in its simplest form.

(5)

(iii) Given that

$$\mathrm{f}(x) = \frac{3\cos x}{(x+1)^{\frac{1}{3}}}, \qquad x \neq -1$$

show that

$$\mathrm{f}'(x) = \frac{\mathrm{g}(x)}{(x+1)^{\frac{4}{3}}}, \qquad x \neq -1$$

where $\mathrm{g}(x)$ is an expression to be found.

(3)

5. (*a*) Sketch the graph with equation

$$y = |4x - 3|$$

stating the coordinates of any points where the graph cuts or meets the axes.

(2)

Find the complete set of values of x for which

(*b*)

$$|4x - 3| > 2 - 2x$$

(4)

(*c*)

$$|4x - 3| > \frac{3}{2} - 2x$$

(2)

6. The function f is defined by

$$\mathrm{f}: x \rightarrow \mathrm{e}^{2x} + k^2, \qquad x \in \mathbb{R}, \qquad k \text{ is a positive constant.}$$

(*a*) State the range of f.

(1)

(*b*) Find f^{-1} and state its domain.

(3)

The function g is defined by

$$\mathrm{g}: x \rightarrow \ln(2x), \qquad x > 0$$

(*c*) Solve the equation

$$\mathrm{g}(x) + \mathrm{g}(x^2) + \mathrm{g}(x^3) = 6$$

giving your answer in its simplest form.

(4)

(*d*) Find $\mathrm{fg}(x)$, giving your answer in its simplest form.

(2)

(*e*) Find, in terms of the constant k, the solution of the equation

$$\mathrm{fg}(x) = 2k^2$$

(2)

7.

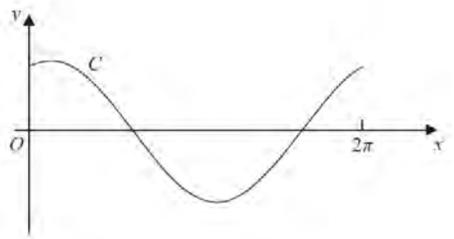

Figure 1

Figure 1 shows the curve C, with equation $y = 6\cos x + 2.5\sin x$ for $0 \leq x \leq 2\pi$.

(*a*) Express $6\cos x + 2.5\sin x$ in the form $R\cos(x - \alpha)$, where R and α are constants with $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your value of α to 3 decimal places.
(3)

(*b*) Find the coordinates of the points on the graph where the curve C crosses the coordinate axes.
(3)

A student records the number of hours of daylight each Sunday throughout the year. She starts on the last Sunday in May with a recording of 18 hours, and continues until her final recording 52 weeks later.

She models her results with the continuous function given by

$$H = 12 + 6\cos\left(\frac{2\pi t}{52}\right) + 2.5\sin\left(\frac{2\pi t}{52}\right), \qquad 0 \leq t \leq 52$$

where H is the number of hours of daylight and t is the number of weeks since her first recording.

Use this function to find

(*c*) the maximum and minimum values of H predicted by the model,
(3)

(*d*) the values for t when $H = 16$, giving your answers to the nearest whole number.

[*You must show your working. Answers based entirely on graphical or numerical methods are not acceptable.*]
(6)

TOTAL FOR PAPER: 75 MARKS

END

Paper Reference(s)
6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Monday 16 June 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43164A

PMT

1. The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{4x+1}{x-2}, \qquad x > 2$$

(*a*) Show that

$$\mathrm{f}'(x) = \frac{-9}{(x-2)^2}$$

(3)

Given that P is a point on C such that $\mathrm{f}'(x) = -1$,

(*b*) find the coordinates of P.

(3)

---

2. Find the exact solutions, in their simplest form, to the equations

(*a*) $2 \ln (2x + 1) - 10 = 0$

(2)

(*b*) $3^x e^{4x} = e^7$

(4)

---

3. The curve C has equation $x = 8y \tan 2y$.

The point P has coordinates $\left(\pi, \frac{\pi}{8}\right)$.

(*a*) Verify that P lies on C.

(1)

(*b*) Find the equation of the tangent to C at P in the form $ay = x + b$, where the constants a and b are to be found in terms of π.

(7)

4.

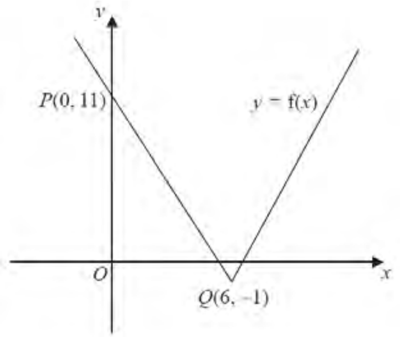

Figure 1

Figure 1 shows part of the graph with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $Q(6, -1)$.

The graph crosses the y-axis at the point $P(0, 11)$.

Sketch, on separate diagrams, the graphs of

(*a*) $y = |\mathrm{f}(x)|$

(2)

(*b*) $y = 2\mathrm{f}(-x) + 3$

(3)

On each diagram, show the coordinates of the points corresponding to P and Q.

Given that $\mathrm{f}(x) = a|x - b| - 1$, where a and b are constants,

(*c*) state the value of a and the value of b.

(2)

PMT

5. $$\mathrm{g}(x) = \frac{x}{x+3} + \frac{3(2x+1)}{x^2+x-6}, \qquad x > 3$$

(*a*) Show that $\mathrm{g}(x) = \dfrac{x+1}{x-2}$, $x > 3$

(4)

(*b*) Find the range of g.

(2)

(*c*) Find the exact value of a for which $\mathrm{g}(a) = \mathrm{g}^{-1}(a)$.

(4)

6.

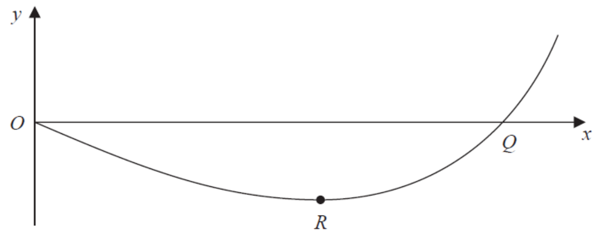

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 2\cos\left(\frac{1}{2}x^2\right) + x^3 - 3x - 2$$

The curve crosses the x-axis at the point Q and has a minimum turning point at R.

(*a*) Show that the x coordinate of Q lies between 2.1 and 2.2.

(2)

(*b*) Show that the x coordinate of R is a solution of the equation

$$x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$$

(4)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{2}{3}x_n\sin\left(\frac{1}{2}x_n^{\ 2}\right)}, \qquad x_0 = 1.3$$

(*c*) find the values of x_1 and x_2 to 3 decimal places.

(2)

---

7. (*a*) Show that

$$\operatorname{cosec} 2x + \cot 2x = \cot x, \qquad x \neq 90n°, \qquad n \in \mathbb{Z}$$

(5)

(*b*) Hence, or otherwise, solve, for $0 \leq \theta < 180°$,

$$\operatorname{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3}$$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

---

8. A rare species of primrose is being studied. The population, P, of primroses at time t years after the study started is modelled by the equation

$$P = \frac{800e^{0.1t}}{1+3e^{0.1t}}, \qquad t \geq 0, \quad t \in \mathbb{R}$$

(*a*) Calculate the number of primroses at the start of the study.

(2)

(*b*) Find the exact value of t when $P = 250$, giving your answer in the form $a\ln(b)$ where a and b are integers.

(4)

(*c*) Find the exact value of $\dfrac{dP}{dt}$ when $t = 10$. Give your answer in its simplest form.

(4)

(*d*) Explain why the population of primroses can never be 270.

(1)

---

9. (*a*) Express $2 \sin \theta - 4 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(3)

$$H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find

(*b*) (i) the maximum value of $H(\theta)$,

(ii) the smallest value of θ, for $0 \leq \theta \leq \pi$, at which this maximum value occurs.

(3)

Find

(*c*) (i) the minimum value of $H(\theta)$,

(ii) the largest value of θ, for $0 \leq \theta \leq \pi$, at which this minimum value occurs.

(3)

TOTAL FOR PAPER: 75 MARKS

END