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Mark Scheme (Results)
Summer 2008

GCE

GCE Mathematics (6665/01)

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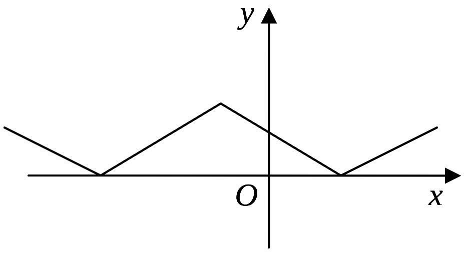
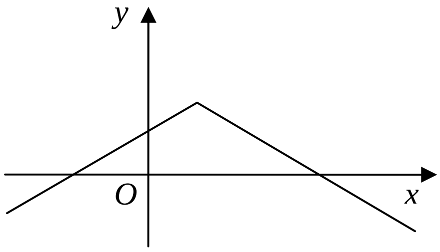
June 2008
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $e^{2x+1} = 2$	
	$2x+1 = \ln 2$	M1
	$x = \frac{1}{2}(\ln 2 - 1)$	A1 (2)
	(b) $\frac{dy}{dx} = 8e^{2x+1}$	B1
	$x = \frac{1}{2}(\ln 2 - 1) \Rightarrow \frac{dy}{dx} = 16$	B1
	$y - 8 = 16\left(x - \frac{1}{2}(\ln 2 - 1)\right)$	M1
	$y = 16x + 16 - 8\ln 2$	A1 (4)
		[6]

Question Number	Scheme		Marks	
2.	(a) $R^2 = 5^2 + 12^2$		M1	
	$R = 13$		A1	
	$\tan\alpha = \frac{12}{5}$		M1	
	$\alpha \approx 1.176$	cao	A1	(4)
	(b) $\cos(x-\alpha) = \frac{6}{13}$		M1	
	$x-\alpha = \arccos\frac{6}{13} = 1.091\ \ldots$		A1	
	$x = 1.091\ \ldots + 1.176\ \ldots \approx 2.267\ldots$	awrt 2.3	A1	
	$x-\alpha = -1.091\ \ldots$	accept … = 5.19 … for M	M1	
	$x = -1.091\ \ldots + 1.176\ \ldots \approx 0.0849\ldots$	awrt 0.084 or 0.085	A1	(5)
	(c)(i) $R_{\max} = 13$	ft their R	B1 ft	
	(ii) At the maximum, $\cos(x-\alpha) = 1$ *or* $x-\alpha = 0$		M1	
	$x = \alpha = 1.176\ \ldots$	awrt 1.2, ft their α	A1ft	(3)
				[12]

Question Number	Scheme		Marks	
3.	(a) 	 shape	B1	
		Vertices correctly placed	B1	(2)
	(b) 	 shape	B1	
		Vertex and intersections with axes correctly placed	B1	(2)
	(c) $P:(-1, 2)$		B1	
	$Q:(0, 1)$		B1	
	$R:(1, 0)$		B1	(3)
	(d) $x > -1$; $2 - x - 1 = \frac{1}{2}x$		M1 A1	
	Leading to $x = \frac{2}{3}$		A1	
	$x < -1$; $2 + x + 1 = \frac{1}{2}x$		M1	
	Leading to $x = -6$		A1	(5)
				[12]

Question Number	Scheme	Marks
4.	(a) $x^2-2x-3=(x-3)(x+1)$	B1
	$\mathrm{f}(x)=\dfrac{2(x-1)-(x+1)}{(x-3)(x+1)}$ $\left(\textit{or } \dfrac{2(x-1)}{(x-3)(x+1)}-\dfrac{x+1}{(x-3)(x+1)}\right)$	M1 A1
	$=\dfrac{x-3}{(x-3)(x+1)}=\dfrac{1}{x+1}$ * cso	A1 (4)
	(b) $\left(0,\dfrac{1}{4}\right)$ Accept $0<y<\dfrac{1}{4},\ 0<\mathrm{f}(x)<\dfrac{1}{4}$ etc.	B1 B1 (2)
	(c) Let $y=\mathrm{f}(x)$ $y=\dfrac{1}{x+1}$	
	$x=\dfrac{1}{y+1}$	
	$yx+x=1$	
	$y=\dfrac{1-x}{x}$ or $\dfrac{1}{x}-1$	M1 A1
	$\mathrm{f}^{-1}(x)=\dfrac{1-x}{x}$	
	Domain of f^{-1} is $\left(0,\dfrac{1}{4}\right)$ ft their part (b)	B1 ft (3)
	(d) $\mathrm{fg}(x)=\dfrac{1}{2x^2-3+1}$	
	$\dfrac{1}{2x^2-2}=\dfrac{1}{8}$	M1
	$x^2=5$	A1
	$x=\pm\sqrt{5}$ both	A1 (3)
		[12]

Question Number	Scheme		Marks	
5.	(a) $\sin^2\theta + \cos^2\theta = 1$			
	$\div \sin^2\theta$ $\quad \dfrac{\sin^2\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$		M1	
	$1 + \cot^2\theta = \operatorname{cosec}^2\theta$ $\ast$	cso	A1	(2)
	Alternative for (a)			
	$1 + \cot^2\theta = 1 + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$		M1	
	$= \operatorname{cosec}^2\theta$ $\ast$	cso	A1	
	(b) $2\left(\operatorname{cosec}^2\theta - 1\right) - 9\operatorname{cosec}\theta = 3$		M1	
	$2\operatorname{cosec}^2\theta - 9\operatorname{cosec}\theta - 5 = 0$ *or* $5\sin^2\theta + 9\sin\theta - 2 = 0$		M1	
	$(2\operatorname{cosec}\theta + 1)(\operatorname{cosec}\theta - 5) = 0$ *or* $(5\sin\theta - 1)(\sin\theta + 2) = 0$		M1	
	$\operatorname{cosec}\theta = 5$ *or* $\sin\theta = \dfrac{1}{5}$		A1	
	$\theta = 11.5°, 168.5°$		A1 A1	(6)
				[8]

Question Number	Scheme	Marks
6.	(a)(i) $\dfrac{d}{dx}\left(e^{3x}(\sin x + 2\cos x)\right) = 3e^{3x}(\sin x + 2\cos x) + e^{3x}(\cos x - 2\sin x)$	M1 A1 A1 (3)
	$\left(= e^{3x}(\sin x + 7\cos x)\right)$	
	(ii) $\dfrac{d}{dx}\left(x^3 \ln(5x+2)\right) = 3x^2 \ln(5x+2) + \dfrac{5x^3}{5x+2}$	M1 A1 A1 (3)
	(b) $\dfrac{dy}{dx} = \dfrac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$	M1 $\dfrac{\text{A1}}{\text{A1}}$
	$= \dfrac{(x+1)(6x^2+12x+6-6x^2-12x+14)}{(x+1)^4}$	M1
	$= \dfrac{20}{(x+1)^3}$ * cso	A1 (5)
	(c) $\dfrac{d^2y}{dx^2} = -\dfrac{60}{(x+1)^4} = -\dfrac{15}{4}$	M1
	$(x+1)^4 = 16$	M1
	$x = 1, -3$ both	A1 (3)
		[14]
	Note: The simplification in part (b) can be carried out as follows	
	$\dfrac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$	
	$= \dfrac{(6x^3+18x^2+18x+6) - (6x^3+18x^2-2x-14)}{(x+1)^4}$	
	$= \dfrac{20x+20}{(x+1)^4} = \dfrac{20(x+1)}{(x+1)^4} = \dfrac{20}{(x+1)^3}$	M1 A1

Question Number	Scheme	Marks
7.	(a) $\mathrm{f}(1.4) = -0.568 \ \ldots < 0$	
	$\mathrm{f}(1.45) = 0.245 \ \ldots > 0$	M1
	Change of sign (and continuity) $\Rightarrow \alpha \in (1.4, 1.45)$	A1 (2)
	(b) $3x^3 = 2x + 6$	
	$x^3 = \dfrac{2x}{3} + 2$	
	$x^2 = \dfrac{2}{3} + \dfrac{2}{x}$	M1 A1
	$x = \sqrt{\left(\dfrac{2}{x} + \dfrac{2}{3}\right)}$ ∗ cso	A1 (3)
	(c) $x_1 = 1.4371$	B1
	$x_2 = 1.4347$	B1
	$x_3 = 1.4355$	B1 (3)
	(d) Choosing the interval $(1.4345, 1.4355)$ or appropriate tighter interval.	M1
	$\mathrm{f}(1.4345) = -0.01$ ...	
	$\mathrm{f}(1.4355) = 0.003 \ \ldots$	M1
	Change of sign (and continuity) $\Rightarrow \alpha \in (1.4345, 1.4355)$	
	$\Rightarrow \alpha = 1.435$, correct to 3 decimal places ∗ cso	A1 (3) **[11]**
	Note: $\alpha = 1.435\,304\,553 \ \ldots$	