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Mark Scheme (Results)
January 2010

GCE

Core Mathematics C3 (6665)

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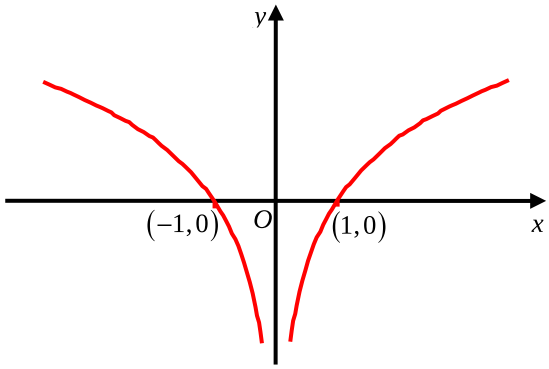
Question Number	Scheme		Marks
Q1	$\dfrac{x+1}{3x^2-3} - \dfrac{1}{3x+1}$		
	$= \dfrac{x+1}{3(x^2-1)} - \dfrac{1}{3x+1}$		
	$= \dfrac{x+1}{3(x+1)(x-1)} - \dfrac{1}{3x+1}$	$x^2-1 \to (x+1)(x-1)$ or $3x^2-3 \to (x+1)(3x-3)$ or $3x^2-3 \to (3x+3)(x-1)$ seen or implied anywhere in candidate's working.	**Award below**
	$= \dfrac{1}{3(x-1)} - \dfrac{1}{3x+1}$		
	$= \dfrac{3x+1-3(x-1)}{3(x-1)(3x+1)}$	Attempt to combine.	M1
	or $\dfrac{3x+1}{3(x-1)(3x+1)} - \dfrac{3(x-1)}{3(x-1)(3x+1)}$	Correct result.	A1
		Decide to award M1 here!!	**M1**
	$= \dfrac{4}{3(x-1)(3x+1)}$	Either $\dfrac{4}{3(x-1)(3x+1)}$ or $\dfrac{\frac{4}{3}}{(x-1)(3x+1)}$ or $\dfrac{4}{(3x-3)(3x+1)}$ or $\dfrac{4}{9x^2-6x-3}$	A1 **aef**
			[4]

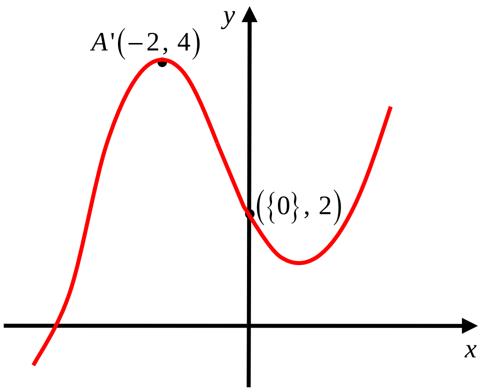
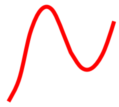
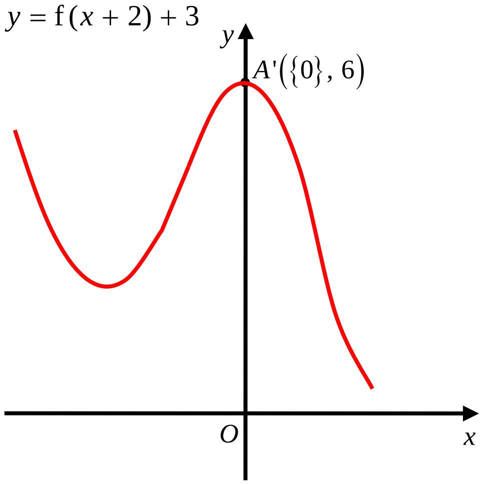
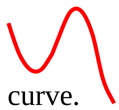
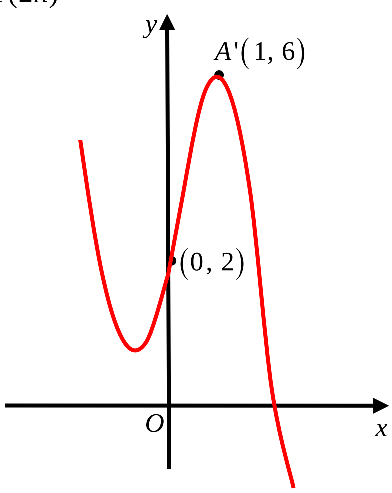
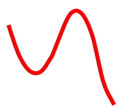
Question Number	Scheme		Marks
Q2	$\mathrm{f}(x) = x^3 + 2x^2 - 3x - 11$		
(a)	$\mathrm{f}(x) = 0 \;\Rightarrow\; x^3 + 2x^2 - 3x - 11 = 0$ $\Rightarrow\; x^2(x+2) - 3x - 11 = 0$	Sets $\mathrm{f}(x) = 0$ (can be implied) and takes out a factor of x^2 from $x^3 + 2x^2$, or x from $x^3 + 2x$ (slip).	M1
	$\Rightarrow\; x^2(x+2) = 3x + 11$ $\Rightarrow\; x^2 = \dfrac{3x+11}{x+2}$ $\Rightarrow\; x = \sqrt{\left(\dfrac{3x+11}{x+2}\right)}$	then rearranges to give the quoted result on the question paper.	A1 **AG**
			(2)
(b)	Iterative formula: $x_{n+1} = \sqrt{\left(\dfrac{3x_n + 11}{x_n + 2}\right)}$, $x_1 = 0$		
	$x_2 = \sqrt{\left(\dfrac{3(0) + 11}{(0) + 2}\right)}$	An attempt to substitute $x_1 = 0$ into the iterative formula. Can be implied by $x_2 = \sqrt{5.5}$ ***or 2.35*** or awrt 2.345	M1
	$x_2 = 2.34520788...$ $x_3 = 2.037324945...$	Both $x_2 =$ awrt 2.345 and $x_3 =$ awrt 2.037	A1
	$x_4 = 2.058748112...$	$x_4 =$ awrt 2.059	A1
			(3)
(c)	Let $\mathrm{f}(x) = x^3 + 2x^2 - 3x - 11 = 0$		
	$\mathrm{f}(2.0565) = -0.013781637...$	Choose suitable interval for x, e.g. [2.0565, 2.0575] or tighter	M1
	$\mathrm{f}(2.0575) = 0.0041401094...$	any one value awrt 1 sf	dM1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α is such that $\alpha \in (2.0565, 2.0575) \Rightarrow \alpha = 2.057$ (3 dp)	both values correct awrt 1sf, sign change and conclusion	A1
			(3)
		As a minimum, both values must be correct to 1 sf, candidate states "change of sign, hence root".	
			[8]

Question Number	Scheme		Marks
Q3 (a)	$5\cos x - 3\sin x = R\cos(x+\alpha)$, $R>0$, $0<x<\frac{\pi}{2}$		
	$5\cos x - 3\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$		
	Equate $\cos x$: $5 = R\cos\alpha$ Equate $\sin x$: $3 = R\sin\alpha$		
	$R = \sqrt{5^2+3^2}$; $= \sqrt{34}$ $\{= 5.83095..\}$	$R^2 = 5^2 + 3^2$ $\sqrt{34}$ or awrt 5.8	M1; A1
	$\tan\alpha = \frac{3}{5} \Rightarrow \alpha = 0.5404195003...^c$	$\tan\alpha = \pm\frac{3}{5}$ or $\tan\alpha = \pm\frac{5}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{5}{\text{their } R}$	M1
		$\alpha =$ awrt 0.54 or $\alpha =$ awrt 0.17π or $\alpha = \frac{\pi}{\text{awrt } 5.8}$	A1
	Hence, $5\cos x - 3\sin x = \sqrt{34}\cos(x + 0.5404)$		
			(4)
(b)	$5\cos x - 3\sin x = 4$		
	$\sqrt{34}\cos(x+0.5404) = 4$		
	$\cos(x+0.5404) = \frac{4}{\sqrt{34}}$ $\{= 0.68599...\}$	$\cos(x \pm \text{their } \alpha) = \frac{4}{\text{their } R}$	M1
	$(x+0.5404) = 0.814826916...^c$	For applying $\cos^{-1}\left(\frac{4}{\text{their } R}\right)$	M1
	$x = 0.2744...^c$	awrt 0.27^c	A1
	$(x+0.5404) = 2\pi - 0.814826916...^c$ $\{= 5.468358...^c\}$	$2\pi -$ their 0.8148	ddM1
	$x = 4.9279...^c$	awrt 4.93^c	A1
	Hence, $x = \{0.27, 4.93\}$		
			(5)
			[9]

Part (b): If there are any EXTRA solutions inside the range $0 \le x < 2\pi$, then withhold the final accuracy mark if the candidate would otherwise score all 5 marks. Also ignore EXTRA solutions outside the range $0 \le x < 2\pi$.

Question Number	Scheme		Marks
Q4 (i)	$y = \dfrac{\ln(x^2+1)}{x}$		
	$u = \ln(x^2+1) \Rightarrow \dfrac{du}{dx} = \dfrac{2x}{x^2+1}$	$\ln(x^2+1) \rightarrow \dfrac{\text{something}}{x^2+1}$	M1
		$\ln(x^2+1) \rightarrow \dfrac{2x}{x^2+1}$	A1
	Apply quotient rule: $\begin{Bmatrix} u = \ln(x^2+1) & v = x \\ \dfrac{du}{dx} = \dfrac{2x}{x^2+1} & \dfrac{dv}{dx} = 1 \end{Bmatrix}$		
	$\dfrac{dy}{dx} = \dfrac{\left(\dfrac{2x}{x^2+1}\right)(x) - \ln(x^2+1)}{x^2}$	Applying $\dfrac{xu' - \ln(x^2+1)v'}{x^2}$ correctly.	M1
		Correct differentiation with correct bracketing but allow recovery.	A1
			(4)
	$\left\{\dfrac{dy}{dx} = \dfrac{2}{(x^2+1)} - \dfrac{1}{x^2}\ln(x^2+1)\right\}$	{Ignore subsequent working.}	
(ii)	$x = \tan y$		
	$\dfrac{dx}{dy} = \sec^2 y$	$\tan y \rightarrow \sec^2 y$ or an attempt to differentiate $\dfrac{\sin y}{\cos y}$ using either the quotient rule or product rule.	M1*
		$\dfrac{dx}{dy} = \sec^2 y$	A1
	$\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y}$ $\{= \cos^2 y\}$	Finding $\dfrac{dy}{dx}$ by reciprocating $\dfrac{dx}{dy}$.	dM1*
	$\dfrac{dy}{dx} = \dfrac{1}{1+\tan^2 y}$	For writing down or applying the identity $\sec^2 y = 1 + \tan^2 y$, which must be applied/stated completely in y.	dM1*
	Hence, $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$, (as required)	For the correct proof, leading on from the previous line of working.	A1 **AG**
			(5)
			[9]

Question Number	Scheme		Marks
Q5	$y = \ln\|x\|$		
		Right-hand branch in quadrants 4 and 1. Correct shape.	B1
		Left-hand branch in quadrants 2 and 3. Correct shape.	B1
		Completely correct sketch and both $(-1,\{0\})$ and $(1,\{0\})$	B1
			(3)
			[3]

Question Number	Scheme	Marks
Q6 (i)	$y = \mathrm{f}(-x) + 1$ 	
	Shape of  and must have a maximum in quadrant 2 and a minimum in quadrant 1 or on the positive y-axis.	B1
	Either $(\{0\}, 2)$ or $A'(-2, 4)$	B1
	Both $(\{0\}, 2)$ and $A'(-2, 4)$	B1
		(3)
(ii)	$y = \mathrm{f}(x+2) + 3$ 	
	Any translation  of the original curve.	B1
	The ***translated maximum*** has either x-coordinate of 0 (can be implied) or y-coordinate of 6.	B1
	The translated curve has maximum $(\{0\}, 6)$ and is in the correct position on the Cartesian axes.	B1
		(3)
(iii)	$y = 2\mathrm{f}(2x)$ 	
	Shape of  with a minimum in quadrant 2 and a maximum in quadrant 1.	B1
	Either $(\{0\}, 2)$ or $A'(1, 6)$	B1
	Both $(\{0\}, 2)$ and $A'(1, 6)$	B1
		(3)
		[9]

Question Number	Scheme		Marks
Q7 (a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$		
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$	$\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$	M1
		$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$	Convincing proof. Must see both underlined steps.	A1 **AG**
			(3)
(b)	$y = e^{2x}\sec 3x$		
	$\left\{\begin{array}{ll} u = e^{2x} & v = \sec 3x \\ \dfrac{du}{dx} = 2e^{2x} & \dfrac{dv}{dx} = 3\sec 3x \tan 3x \end{array}\right\}$	Seen or implied. Either $e^{2x} \to 2e^{2x}$ or $\sec 3x \to 3\sec 3x \tan 3x$	M1
		Both $e^{2x} \to 2e^{2x}$ and $\sec 3x \to 3\sec 3x \tan 3x$	A1
	$\dfrac{dy}{dx} = 2e^{2x}\sec 3x + 3e^{2x}\sec 3x \tan 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v'	M1
		$2e^{2x}\sec 3x + 3e^{2x}\sec 3x \tan 3x$	A1 **isw**
			(4)
(c)	Turning point $\Rightarrow \dfrac{dy}{dx} = 0$		
	Hence, $e^{2x}\sec 3x(2 + 3\tan 3x) = 0$	Sets their $\dfrac{dy}{dx} = 0$ and factorises (or cancels) out at least e^{2x} from at least two terms.	M1
	{Note $e^{2x} \neq 0$, $\sec 3x \neq 0$, so $2 + 3\tan 3x = 0$,}		
	giving $\tan 3x = -\frac{2}{3}$	$\tan 3x = \pm k$; $k \neq 0$	M1
	$\Rightarrow 3x = -0.58800 \Rightarrow x = \{a\} = -0.19600...$	Either awrt -0.196^c or awrt -11.2°	A1
	Hence, $y = \{b\} = e^{2(-0.196)}\sec(3 \times -0.196)$		
	$= 0.812093... = 0.812$ (3sf)	0.812	A1 **cao**
			(4)
			[11]

Part (c): If there are any EXTRA solutions for x (or a) inside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$ or ANY EXTRA solutions for y (or b)**,** (for these values of x) then withhold the final accuracy mark.
Also ignore EXTRA solutions outside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$.

Question Number	Scheme		Marks
Q8	$\operatorname{cosec}^2 2x - \cot 2x = 1, \quad (\text{eqn }*) \qquad 0 \le x \le 180^\circ$		
	Using $\operatorname{cosec}^2 2x = 1 + \cot^2 2x$ gives $1 + \cot^2 2x - \cot 2x = 1$	Writing down or using $\operatorname{cosec}^2 2x = \pm 1 \pm \cot^2 2x$ or $\operatorname{cosec}^2\theta = \pm 1 \pm \cot^2\theta$.	M1
	$\underline{\cot^2 2x - \cot 2x} = 0 \quad$ or $\quad \cot^2 2x = \cot 2x$	For either $\underline{\cot^2 2x - \cot 2x}\ \{= 0\}$ or $\cot^2 2x = \cot 2x$	A1
	$\cot 2x(\cot 2x - 1) = 0 \quad$ or $\quad \cot 2x = 1$	Attempt to factorise or solve a quadratic (See rules for factorising quadratics) or cancelling out $\cot 2x$ from both sides.	dM1
	$\cot 2x = 0 \quad$ or $\quad \cot 2x = 1$	Both $\cot 2x = 0$ and $\cot 2x = 1$.	A1
	$\cot 2x = 0 \Rightarrow (\tan 2x \to \infty) \Rightarrow 2x = 90, 270$ $\Rightarrow x = 45, 135$ $\cot 2x = 1 \Rightarrow \tan 2x = 1 \Rightarrow 2x = 45, 225$ $\Rightarrow x = 22.5, 112.5$	Candidate attempts to divide at least one of their principal angles by 2. This will be usually implied by seeing $x = 22.5$ resulting from $\cot 2x = 1$.	ddM1
	Overall, $x = \{22.5, 45, 112.5, 135\}$	***Both*** $x = 22.5$ and $x = 112.5$ ***Both*** $x = 45$ and $x = 135$	A1 B1
			[7]

If there are any EXTRA solutions inside the range $0 \le x \le 180^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final accuracy mark (the sixth mark in this question).
Also ignore EXTRA solutions outside the range $0 \le x \le 180^\circ$.

Question Number	Scheme		Marks
Q9 (i)(a)	$\ln(3x-7)=5$		
	$e^{\ln(3x-7)}=e^5$	Takes e of both sides of the equation. This can be implied by $3x-7=e^5$.	M1
	$3x-7=e^5 \Rightarrow x=\dfrac{e^5+7}{3}\ \{=51.804...\}$	Then rearranges to make x the subject.	dM1
		Exact answer of $\dfrac{e^5+7}{3}$.	A1
			(3)
(b)	$3^x e^{7x+2}=15$		
	$\ln\left(3^x e^{7x+2}\right)=\ln 15$	Takes ln (or logs) of both sides of the equation.	M1
	$\ln 3^x+\ln e^{7x+2}=\ln 15$	Applies the addition law of logarithms.	M1
	$x\ln 3+7x+2=\ln 15$	$x\ln 3+7x+2=\ln 15$	A1 oe
	$x(\ln 3+7)=-2+\ln 15$	Factorising out at least two x terms on one side and collecting number terms on the other side.	ddM1
	$x=\dfrac{-2+\ln 15}{7+\ln 3}\ \{=0.0874...\}$	***Exact answer*** of $\dfrac{-2+\ln 15}{7+\ln 3}$	A1 oe
			(5)
(ii) (a)	$f(x)=e^{2x}+3,\ x\in\mathbb{R}$		
	$y=e^{2x}+3 \Rightarrow y-3=e^{2x}$ $\Rightarrow \ln(y-3)=2x$ $\Rightarrow \frac{1}{2}\ln(y-3)=x$	Attempt to make x (or swapped y) the subject	M1
		Makes e^{2x} the subject and takes ln of both sides	M1
	Hence $f^{-1}(x)=\underline{\frac{1}{2}\ln(x-3)}$	$\underline{\frac{1}{2}\ln(x-3)}$ or $\underline{\ln\sqrt{(x-3)}}$ or $\underline{f^{-1}(y)=\frac{1}{2}\ln(y-3)}$ (see appendix)	<u>A1</u> **cao**
	$f^{-1}(x)$: Domain: $\underline{x>3}$ or $\underline{(3,\infty)}$	Either $\underline{x>3}$ or $\underline{(3,\infty)}$ or $\underline{\text{Domain}>3}$.	B1
			(4)
(b)	$g(x)=\ln(x-1),\ x\in\mathbb{R},\ x>1$		
	$fg(x)=e^{2\ln(x-1)}+3\ \{=(x-1)^2+3\}$	An attempt to put function g into function f.	M1
		$e^{2\ln(x-1)}+3$ or $(x-1)^2+3$ or x^2-2x+4.	A1 isw
	$fg(x)$: Range: $\underline{y>3}$ or $\underline{(3,\infty)}$	Either $\underline{y>3}$ or $\underline{(3,\infty)}$ or $\underline{\text{Range}>3}$ or $\underline{fg(x)>3}$.	B1
			(3)
			[15]

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