

Mark Scheme (Results)

January 2008

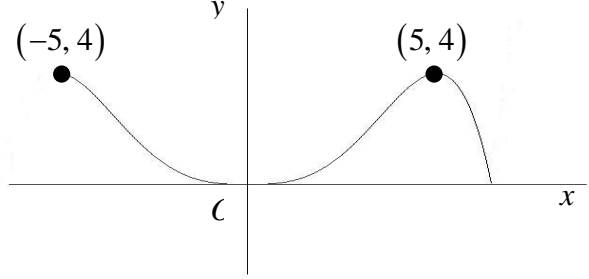
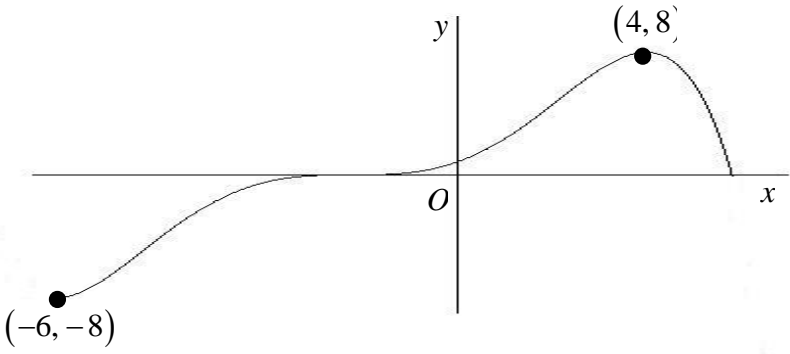
GCE

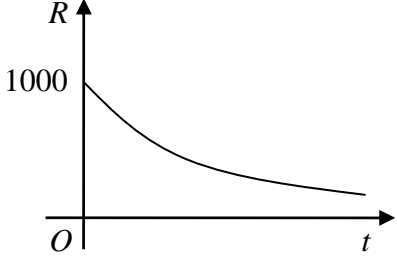
GCE Mathematics (6665/01)

January 2008
 6665 Core Mathematics C3
 Mark Scheme

Question Number	Scheme	Marks
<p>1.</p>	$x^2 - 1 \begin{array}{r} \frac{2x^2}{2x^4} \quad \frac{-1}{-3x^2 + x + 1} \\ \hline 2x^4 \quad -2x^2 \\ \hline -x^2 + x + 1 \\ \hline -x^2 \quad +1 \\ \hline x \end{array}$ <p>$a = 2$ stated or implied $c = -1$ stated or implied</p> $2x^2 - 1 + \frac{x}{x^2 - 1}$ <p>$a = 2, b = 0, c = -1, d = 1, e = 0$ $d = 1$ and $b = 0, e = 0$ stated or implied</p>	<p>M1 A1 A1</p> <p>A1</p> <p>[4]</p>
<p>2.</p>	<p>(a)</p> $\frac{dy}{dx} = 2e^{2x} \tan x + e^{2x} \sec^2 x$ $\frac{dy}{dx} = 0 \Rightarrow 2e^{2x} \tan x + e^{2x} \sec^2 x = 0$ $2 \tan x + 1 + \tan^2 x = 0$ $(\tan x + 1)^2 = 0$ $\tan x = -1 \quad *$ <p style="text-align: right;">cso</p> <p>(b)</p> $\left(\frac{dy}{dx} \right)_0 = 1$ <p>Equation of tangent at $(0, 0)$ is $y = x$</p>	<p>M1 A1+A1</p> <p>M1</p> <p>A1</p> <p>A1 (6)</p> <p>M1</p> <p>A1 (2)</p> <p>[8]</p>

Question Number	Scheme	Marks
3.	<p>(a) $f(2) = 0.38 \dots$ $f(3) = -0.39 \dots$ Change of sign (and continuity) \Rightarrow root in $(2, 3)$ *</p> <p>(b) $x_1 = \ln 4.5 + 1 \approx 2.50408$ $x_2 \approx 2.50498$ $x_3 \approx 2.50518$</p> <p>(c) Selecting $[2.5045, 2.5055]$, or appropriate tighter range, and evaluating at both ends. $f(2.5045) \approx 6 \times 10^{-4}$ $f(2.5055) \approx -2 \times 10^{-4}$ Change of sign (and continuity) \Rightarrow root $\in (2.5045, 2.5055)$ \Rightarrow root = 2.505 to 3 dp *</p> <p>Note: The root, correct to 5 dp, is 2.50524</p>	<p>M1 A1 (2)</p> <p>M1 A1 A1 (3)</p> <p>M1</p> <p>A1 (2) [7]</p>

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4.	<p>(a)</p>  <p>(b) For the purpose of marking this paper, the graph is identical to (a)</p> <p>(c)</p>  <p>General shape – unchanged Translation to left</p>	<p>Shape (5, 4) B1 (-5, 4) B1 (3)</p> <p>Shape (5, 4) B1 (-5, 4) B1 (3)</p> <p>General shape – unchanged B1 Translation to left B1 (4, 8) B1 (-6, -8) B1 (4)</p> <p>In all parts of this question ignore any drawing outside the domains shown in the diagrams above. [10]</p>

Question Number	Scheme	Marks
<p>5.</p>	<p>(a) 1000</p>	<p>B1 (1)</p>
	<p>(b) $1000e^{-5730c} = 500$ $e^{-5730c} = \frac{1}{2}$ $-5730c = \ln \frac{1}{2}$ $c = 0.000121$</p>	<p>M1 A1 M1 cao A1 (4)</p>
	<p>(c) $R = 1000e^{-22920c} = 62.5$</p>	<p>Accept 62-63 M1 A1 (2)</p>
	<p>(d)</p> <div style="text-align: center;">  </div>	<p>Shape 1000 B1 B1 (2) [9]</p>

Question Number	Scheme	Marks
<p>6.</p>	<p>(a) $\cos(2x+x) = \cos 2x \cos x - \sin 2x \sin x$ $= (2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x$ $= (2\cos^2 x - 1)\cos x - 2(1 - \cos^2 x)\cos x$ any correct expression $= 4\cos^3 x - 3\cos x$</p> <p>(b)(i) $\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} = \frac{\cos^2 x + (1 + \sin x)^2}{(1 + \sin x)\cos x}$ $= \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1 + \sin x)\cos x}$ $= \frac{2(1 + \sin x)}{(1 + \sin x)\cos x}$ $= \frac{2}{\cos x} = 2 \sec x$ *</p> <p>(c) $\sec x = 2$ or $\cos x = \frac{1}{2}$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$</p> <p style="text-align: right;">accept awrt 1.05, 5.24</p>	<p>M1 M1 A1 A1 (4)</p> <p>M1 A1 M1 A1 (4) cso</p> <p>M1 A1, A1 (3)</p> <p style="text-align: right;">[11]</p>
<p>7.</p>	<p>(a) $\frac{dy}{dx} = 6\cos 2x - 8\sin 2x$ $\left(\frac{dy}{dx}\right)_0 = 6$ $y - 4 = -\frac{1}{6}x$</p> <p>(b) $R = \sqrt{(3^2 + 4^2)} = 5$ $\tan \alpha = \frac{4}{3}, \alpha \approx 0.927$</p> <p>(c) $\sin(2x + \text{their } \alpha) = 0$ $x = -2.03, -0.46, 1.11, 2.68$</p> <p>First A1 any correct solution; second A1 a second correct solution; third A1 all four correct and to the specified accuracy or better. Ignore the y-coordinate.</p>	<p>M1 A1 B1 M1 A1 (5) or equivalent</p> <p>M1 A1 M1 A1 (4) awrt 0.927</p> <p>M1 A1 A1 A1 (4)</p> <p style="text-align: right;">[13]</p>

Question Number	Scheme	Marks
<p>8.</p>	<p>(a) $x = 1 - 2y^3 \Rightarrow y = \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ or $\sqrt[3]{\frac{1-x}{2}}$</p> <p>$f^{-1} : x \mapsto \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$</p> <p style="text-align: right;">Ignore domain</p>	<p>M1 A1 (2)</p>
	<p>(b) $gf(x) = \frac{3}{1-2x^3} - 4$</p> <p style="margin-left: 40px;">$= \frac{3-4(1-2x^3)}{1-2x^3}$</p> <p style="margin-left: 40px;">$= \frac{8x^3-1}{1-2x^3}$ *</p> <p>$gf : x \mapsto \frac{8x^3-1}{1-2x^3}$</p> <p style="text-align: right;">Ignore domain</p>	<p>M1 A1</p> <p>M1</p> <p>cso A1 (4)</p>
	<p>(c) $8x^3 - 1 = 0$</p> <p style="text-align: right;">Attempting solution of numerator = 0</p>	<p>M1</p>
	<p>$x = \frac{1}{2}$</p> <p style="text-align: right;">Correct answer and no additional answers</p>	<p>A1 (2)</p>
	<p>(d) $\frac{dy}{dx} = \frac{(1-2x^3) \times 24x^2 + (8x^3-1) \times 6x^2}{(1-2x^3)^2}$</p> <p style="margin-left: 40px;">$= \frac{18x^2}{(1-2x^3)^2}$</p>	<p>M1 A1</p> <p>A1</p>
	<p>Solving their numerator = 0 and substituting to find y.</p>	<p>M1</p>
	<p>$x = 0, y = -1$</p>	<p>A1 (5)</p> <p>[13]</p>