Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 5 / 0 1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Thursday 11 June 2009 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
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6	
7	
8	
Total	

Materials required for examination
Mathematical Formulae (Orange or Green)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1.

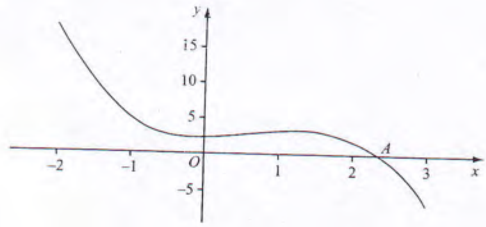

Figure 1

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x-axis at the point A where $x = \alpha$.

To find an approximation to α, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4.
Give your answers to 3 decimal places where appropriate.
(3)

(b) Show that $\alpha = 2.359$ correct to 3 decimal places.
(3)

a) $x_0 = 2.5$
$x_1 = 2.32$
$x_2 = 2.372$
$x_3 = 2.356$
$x_4 = 2.360$

b) $f(2.3595) = -1.42 \times 10^{-3}$
$f(2.3585) = 5.84 \times 10^{-3}$

by change of sign rule root must be 2.359 (3dp).

H 3 4 2 6 4 A 0 2 2 8

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2. (a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$.
(2)

(b) Solve, for $0 \leqslant \theta < 360°$, the equation

$$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2$$
(6)

a) $\sin^2\theta = 1 - \cos^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta}$

$\tan^2\theta = \sec^2\theta - 1$ qed.

b) $2(\sec^2\theta - 1) + 4\sec\theta + \sec^2\theta = 2$

$2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$

$3\sec^2\theta + 4\sec\theta - 4 = 0$

$(3\sec\theta - 2)(\sec\theta + 2) = 0$

$\sec\theta = \frac{2}{3}$ $\sec\theta = -2$

$\cos\theta = \frac{3}{2}$ $\cos\theta = -\frac{1}{2}$

(not possible) $\theta = 120°, 240°$

3. Rabbits were introduced onto an island. The number of rabbits, P, t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \quad t \in \mathbb{R}, t \geqslant 0$$

(a) Write down the number of rabbits that were introduced to the island.
(1)

(b) Find the number of years it would take for the number of rabbits to first exceed 1000.
(2)

(c) Find $\frac{dP}{dt}$.
(2)

(d) Find P when $\frac{dP}{dt} = 50$.
(3)

a) $t = 0 \Rightarrow P = 80e^0 = 80$

b) $P > 1000 \Rightarrow 80e^{\frac{1}{5}t} > 1000$
$\Rightarrow e^{\frac{1}{5}t} > 12.5$
$\Rightarrow \ln e^{\frac{1}{5}t} > \ln(12.5)$
$\Rightarrow \frac{1}{5}t > \ln(12.5)$
$\Rightarrow t > 5\ln(12.5)$ $t > 12.62..$

13 years

c) $\frac{dP}{dt} = 16e^{\frac{1}{5}t}$

d) $16e^{\frac{1}{5}t} = 50 \Rightarrow e^{\frac{1}{5}t} = 3.125$
$\Rightarrow \frac{1}{5}t = \ln(3.125)$
$\Rightarrow t = 5\ln(3.125) = 5.697$
$P = 80e^{\frac{1}{5}(5.697..)}$
$\Rightarrow P = 250$

4. (i) Differentiate with respect to x

(a) $x^2 \cos 3x$ **(3)**

(b) $\dfrac{\ln(x^2+1)}{x^2+1}$ **(4)**

(ii) A curve C has the equation

$$y=\sqrt{(4x+1)}, \quad x>-\tfrac{1}{4},\ y>0$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax+by+c=0$, where a, b and c are integers. **(6)**

a) $u=x^2 \qquad v=\cos 3x$
$u'=2x \qquad v'=-3\sin 3x$

$\Rightarrow 2x\cos 3x + -3x^2\sin 3x$

$= x(2\cos 3x - 3x\sin 3x)$

b) $u=\ln(x^2+1) \qquad v=x^2+1$

$u'=\dfrac{2x}{x^2+1} \qquad v'=2x$

$\Rightarrow \dfrac{(\cancel{x^2+1})\left(\dfrac{2x}{\cancel{x^2+1}}\right)-2x\ln(x^2+1)}{(x^2+1)^2}$

$= \dfrac{2x(1-\ln(x^2+1))}{(x^2+1)^2}$

4(ii) $y=(4x+1)^{\frac{1}{2}}$

$\dfrac{dy}{dx}=\frac{1}{2}(4x+1)^{-\frac{1}{2}}\times 4 = \dfrac{2}{\sqrt{4x+1}}$

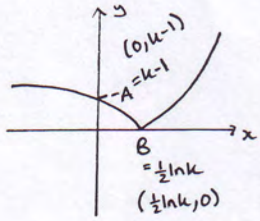
when $x=2$ $\quad m_t=\dfrac{2}{\sqrt{4(2)+1}}=\dfrac{2}{\sqrt{9}}=\dfrac{2}{3}$

$y=\sqrt{4(2)+1}=\sqrt{9}=3$

$y-3=\frac{2}{3}(x-2) \Rightarrow 3y-9=2x-4$

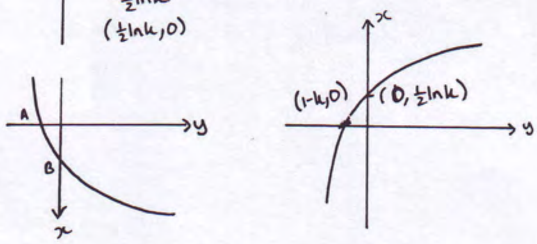
$2x-3y+5=0$

5a)



$|f(x)|$

b)



c) $f(x)=e^{2x}-k \qquad$ range $>-k$

d) $y=e^{2x}-k \Rightarrow x=e^{2y}-k$

$x+k=e^{2y}$

$\ln(x+k)=2y$

$y=\frac{1}{2}\ln(x+k)=f^{-1}(x)$

e) domain of $f^{-1}(x)$ = range $f(x)$ so $x>-k$

6. (a) Use the identity $\cos(A+B)=\cos A\cos B-\sin A\sin B$, to show that

$$\cos 2A = 1-2\sin^2 A$$

(2)

The curves C_1 and C_2 have equations

C_1: $y=3\sin 2x$

C_2: $y=4\sin^2 x - 2\cos 2x$

(b) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$

(3)

(c) Express $4\cos 2x+3\sin 2x$ in the form $R\cos(2x-\alpha)$, where $R>0$ and $0<\alpha<90°$, giving the value of α to 2 decimal places. **(3)**

(d) Hence find, for $0 \leqslant x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2$$

giving your answers to 1 decimal place. **(4)**

$\cos(A+A)=\cos A\cos A-\sin A\sin A=\cos^2 A-\sin^2 A$

$1-\sin^2 A=\cos^2 A \Rightarrow (1-\sin^2 A)-\sin^2 A$

$\Rightarrow \cos 2A = 1-2\sin^2 A$

b) $3\sin 2x = 4\sin^2 x - 2\cos 2x$

$2\sin^2 A = 1-\cos 2A \Rightarrow 4\sin^2 A = 2-2\cos 2A$

$\Rightarrow 3\sin 2x = (2-2\cos 2x)-2\cos 2x$

$\Rightarrow 3\sin 2x = 2-4\cos 2x$

$\Rightarrow 4\cos 2x+3\sin 2x=2 \qquad$ qed.

6c) $R^2 = 3^2 + 4^2 \Rightarrow R = 5$

$\tan\alpha = \frac{3}{4} \quad \alpha = 36.87$

$\Rightarrow 4\cos 2x + 3\sin 2x = 5\cos(2x - 36.87)$

d) $5\cos(2x - 36.87) = 2$

$2x - 36.87 = \cos^{-1}\left(\frac{2}{5}\right) = 66.42°, 293.58°$

$2x = 103.29, 330.45°$

$x = 51.65°, 165.23° \qquad x = 51.6°, 165.2°$

7. The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R}, x \neq -4, x \neq 2$$

(a) Show that $f(x) = \frac{x-3}{x-2}$ **(5)**

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R}, x \neq \ln 2$$

(b) Differentiate $g(x)$ to show that $g'(x) = \frac{e^x}{(e^x - 2)^2}$ **(3)**

(c) Find the exact values of x for which $g'(x) = 1$ **(4)**

a) $\frac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$

$= \frac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$

$= \frac{x^2 + x - 12}{(x-2)(x+4)} = \frac{\cancel{(x+4)}(x-3)}{(x-2)\cancel{(x+4)}} = \frac{x-3}{x-2}$

b) $u = e^x - 3 \qquad v = e^x - 2$

$u' = e^x \qquad v' = e^x$

$\Rightarrow \frac{e^x(e^x - 2) - e^x(e^x - 3)}{(e^x - 2)^2}$

$\Rightarrow \frac{e^{2x} - 2e^x - e^{2x} + 3e^x}{(e^x - 2)^2} = \frac{e^x}{(e^x - 2)^2}$

7c) $\frac{e^x}{(e^x - 2)^2} = 1 \Rightarrow e^x = (e^x - 2)^2$

$e^x = (e^x)^2 - 4e^x + 4$

$(e^x)^2 - 4e^x - e^x + 4 = 0$

$(e^x)^2 - 5e^x + 4 = 0$

$(e^x - 4)(e^x - 1) = 0$

$e^x = 4 \qquad e^x = 1$

$x = \ln 4 \qquad x = \ln 1$

$x = 0, x = \ln 4$

8. (a) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$. **(1)**

(b) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8\cos x = 0$$

giving your answers to 2 decimal places. **(5)**

a) $\sin 2x = 2\sin x\cos x$

b) $\frac{1}{\sin x} - 8\cos x = 0 \qquad (\times \sin x)$

$1 - 8\sin x\cos x = 0$

$1 - 4(2\sin x\cos x) = 0$

$1 - 4\sin 2x = 0 \Rightarrow \sin 2x = \frac{1}{4}$

$2x = \sin^{-1}\left(\frac{1}{4}\right) = 0.2526.., \pi - 0.2526..$

$x = 0.1263.., 1.4444..$

$x = 0.13, 1.44$