

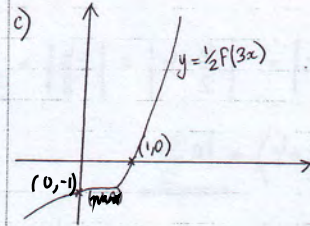
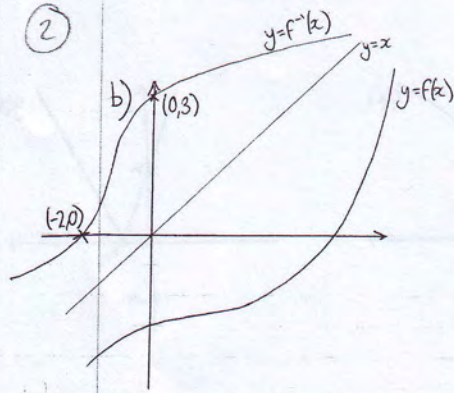
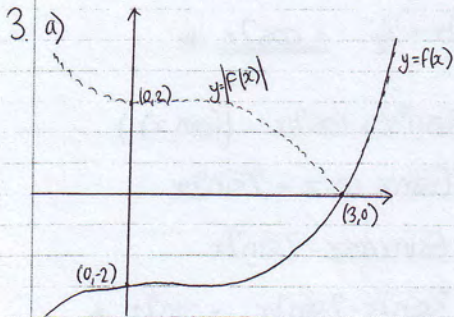
C3 June 06 - Solutions

1. a) $\frac{3x^2-x-2}{x^2-1} = \frac{(3x+2)(x-1)}{(x-1)(x+1)} = \frac{3x+2}{x+1}$

b) $\frac{3x^2-x-2}{(x^2-1)} - \frac{1}{x(x+1)} = \frac{3x+2}{x+1} - \frac{1}{x(x+1)}$
 $= \frac{3x^2+2x-1}{x(x+1)}$
 $= \frac{(3x-1)(x+1)}{x(x+1)} = \frac{3x-1}{x}$

2. a) $y = e^{3x} + \ln(2x) \quad \frac{dy}{dx} = 3e^{3x} + \frac{2}{2x} = 3e^{3x} + \frac{1}{x}$

b) $y = (5+x^2)^{3/2} \quad \frac{dy}{dx} = \frac{3}{2}(5+x^2)^{1/2} \times 2x$
 $= 3x(5+x^2)^{1/2}$



4. a) when $t=0 \quad T = 400 + 25 = 425^\circ\text{C}$

b) $300 = 400e^{-0.05t} + 25$

$\frac{275}{400} = e^{-0.05t}$

$-0.05t = \ln\left(\frac{275}{400}\right) \quad t = \frac{1}{-0.05} \ln\left(\frac{275}{400}\right) = \frac{7.49 \text{ min}}{3}$

c) Rate of change $\frac{dT}{dt} = -0.05 \times 400e^{-0.05t}$
 $= -20e^{-0.05t}$

3) $\frac{dT}{dt} \Big|_{t=50} = -20e^{-0.05 \times 50} = -1.64^\circ\text{C per min (3sf)}$

d) The min value of $400e^{-0.05t}$ is $> 0 \therefore T$ is always > 25 .

5. a) $y = (2x-1)\tan 2x \quad u = (2x-1) \quad u' = 2$
 $v = \tan 2x \quad v' = 2\sec^2 2x$
 $\frac{dy}{dx} = 2(2x-1)\sec^2 2x + 2\tan 2x$

minimum point when $\frac{dy}{dx} = 0$

$(4x-2)\sec^2 2x + 2\tan 2x = 0$

$\frac{(4x-2)}{\cos^2 2x} + \frac{2\sin 2x}{\cos 2x} = 0$

$\frac{4x-2}{\cos^2 2x} + \frac{2\sin 2x \cos 2x}{\cos^2 2x} = 0$

$4x-2 + 2\sin 2x \cos 2x = 0$

$4x-2 + \sin 4x = 0$

at $P \quad x=k \therefore \underline{4k-2 + \sin 4k = 0}$

4)

6. a) $\sin^2 \theta + \cos^2 \theta = 1 \quad \div \sin^2 \theta$

$1 + \cot^2 \theta = \text{cosec}^2 \theta$

$\cot^2 \theta - \text{cosec}^2 \theta = -1$

$\underline{\text{cosec}^2 \theta - \cot^2 \theta = 1} \quad \#$

b) $\text{LHS} = \text{cosec}^4 \theta - \cot^4 \theta = (\text{cosec}^2 \theta - \cot^2 \theta)(\text{cosec}^2 \theta + \cot^2 \theta)$
 $= \underline{\text{cosec}^2 \theta + \cot^2 \theta} \quad \#$

c) $\text{cosec}^2 \theta + \cot^2 \theta = 2 - \cot \theta \quad (\text{from b})$

$1 + \cot^2 \theta + \cot^2 \theta = 2 - \cot \theta \quad (\text{from a})$

$2\cot^2 \theta + \cot \theta - 1 = 0$

$(2\cot \theta - 1)(\cot \theta + 1) = 0$

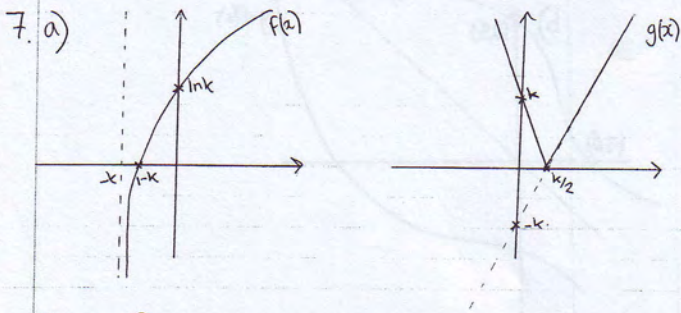
$\cot \theta = \frac{1}{2} \quad \cot \theta = -1$

$\tan \theta = 2 \quad \tan \theta = -1$

$\theta = 63.4^\circ \quad \theta = -45^\circ, 135^\circ, \dots$

$\underline{\theta = 135^\circ}$ since $90^\circ < \theta < 180^\circ$

5



b) $x \in \mathbb{R}$.

c) $g\left(\frac{k}{4}\right) = \left|2\frac{k}{4} - k\right| = \left|\frac{k}{2} - k\right| = \left|-\frac{k}{2}\right| = \frac{k}{2}$

$$fg\left(\frac{k}{4}\right) = \ln\left(\frac{k}{2} + k\right) = \ln\frac{3k}{2}$$

d) $qy = 2x + 1$

$$y = \frac{2x+1}{q} \quad \text{Grad of tangent} = \frac{2}{q}$$

$$y = \ln(x+k)$$

$$\frac{dy}{dx} = \frac{1}{x+k}$$

$$\left.\frac{dy}{dx}\right|_{x=3} = \frac{1}{3+k} = \frac{2}{q}$$

$$q = 2(3+k) \quad \underline{k=1.5}$$

6

8a) $\cos A = \frac{3}{4}$

$$\cos^2 A + \sin^2 A = 1$$

$$\frac{9}{16} + \sin^2 A = 1$$

$$\sin A = \pm \sqrt{1 - \frac{9}{16}} = \pm \frac{\sqrt{7}}{4}$$

$$\therefore \sin 2A = 2 \sin A \cos A$$

$$\sin 2A = 2 \times \frac{\pm \sqrt{7}}{4} \times \frac{3}{4} = \pm \frac{6\sqrt{7}}{16} = \pm \frac{3\sqrt{7}}{8}$$

b) $\pm) \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right)$

$$= \cos 2x \cos \frac{\pi}{3} - \sin 2x \sin \frac{\pi}{3} + \cos 2x \cos \frac{\pi}{3} + \sin 2x \sin \frac{\pi}{3}$$

$$= 2 \cos 2x \cos \frac{\pi}{3}$$

$$= 2 \cos 2x \times \frac{1}{2} = \underline{\underline{\cos 2x}} \quad \#$$

II) $y = 3 \sin^2 x + \cos 2x$ (from \pm)

$$\frac{dy}{dx} = 6 \sin x \cdot \cos x - 2 \sin 2x$$

$$= 6 \sin x \cos x - 2 \sin 2x$$

$$= 3 \sin 2x - 2 \sin 2x = \underline{\underline{\sin 2x}} \quad \#$$