

03 Jan 2008

① $\frac{2x^4 - 3x^2 + x + 1}{x^2 - 1} = 2x^2 - 1 + \frac{x}{x^2 - 1}$

$$\begin{array}{r} 2x^2 - 1 \\ x^2 + 0x - 1 \overline{) 2x^4 + 0x^3 - 3x^2 + x + 1} \\ \underline{2x^4 + 0x^3 - 2x^2} \\ -x^2 + x + 1 \\ \underline{-x^2 + 0x + 1} \\ x \end{array}$$

$a = 2, b = 0, c = -1, d = 1, e = 0$

② $y = e^{2x} \tan x$

a) Turning points at $\frac{dy}{dx} = 0$:

$$\begin{aligned} \frac{dy}{dx} &= e^{2x} \sec^2 x + 2e^{2x} \tan x \\ &= e^{2x} (\sec^2 x + 2 \tan x) \end{aligned}$$

$$\begin{aligned} u &= e^{2x} & v &= \tan x \\ u' &= 2e^{2x} & v' &= \sec^2 x \end{aligned}$$

$$\begin{aligned} e^{2x} (\sec^2 x + 2 \tan x) &= 0 \Rightarrow \sec^2 x + 2 \tan x = 0 \\ 1 + \tan^2 x + 2 \tan x &= 0 \\ (\tan x + 1)^2 &= 0 \\ \Rightarrow \tan x &= -1 \quad \text{QED} \end{aligned}$$

b) When $x = 0$,

$$\frac{dy}{dx} = e^0 (\sec^2 0 + 2 \tan 0) = 1 \times (1 + 0) = 1$$

$$y = e^0 \tan 0 = 1 \times 0 = 0$$

$y - 0 = 1(x - 0) \Rightarrow y = x$

③ $f(x) = \ln(x+2) - x + 1 \quad (x > -2)$

a) $f(2) = \ln 4 - 2 + 1 > 0$
 $f(3) = \ln 5 - 2 < 0$ } change of sign \Rightarrow root in interval

b) $x_0 = 2.5$
 $x_1 = 2.50408$
 $x_2 = 2.50498$
 $x_3 = 2.50518$

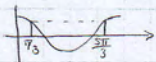
c) $f(2.5045) = 0.00058 > 0$
 $f(2.5055) = -0.00020 < 0$ } change of sign to 3dp \Rightarrow root = 2.505

⑥

a) $\cos(A+B) = \cos(2x+x) = \cos 2x \cos x - \sin 2x \sin x$
 $\Rightarrow \cos 3x = (2\cos^2 x - 1)\cos x - 2\sin x \cos x \sin x$
 $= 2\cos^3 x - \cos x - 2\cos x \sin^2 x$
 $= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x)$
 $= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$
 $= 4\cos^3 x - 3\cos x$

b) $\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} = \frac{\cos^2 x + (1 + \sin x)^2}{\cos x(1 + \sin x)}$
 $= \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{\cos x(1 + \sin x)}$
 $= \frac{2(1 + \sin x)}{\cos x(1 + \sin x)} = 2 \sec x \quad \text{QED}$

ii) $2 \sec x = 4$
 $\sec x = 2 \Rightarrow \cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$



⑦ $y = 3\sin 2x + 4\cos 2x$

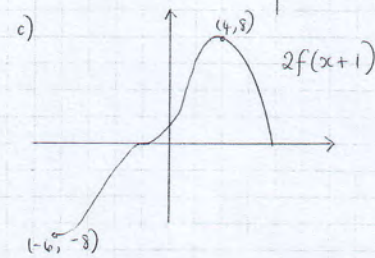
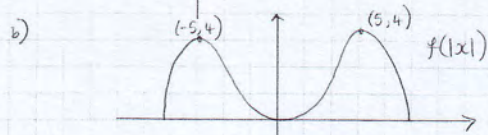
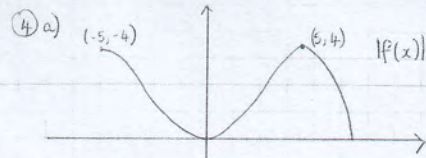
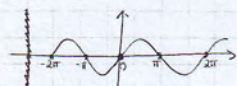
a) $\frac{dy}{dx} = 6\cos 2x - 8\sin 2x$

When $x = 0$, $\frac{dy}{dx} = 6\cos 0 - 8\sin 0 = 6$
 $M_{\text{normal}} = -\frac{1}{6}$

$y - 4 = -\frac{1}{6}(x - 0)$
 $y = -\frac{1}{6}x + 4$

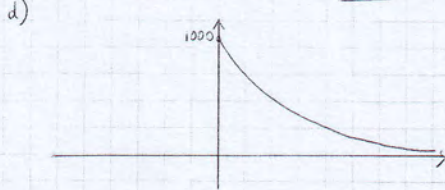
b) $R\sin(2x + \alpha) = R\sin 2x \cos \alpha + R\cos 2x \sin \alpha$
 $\Rightarrow 3\sin 2x + 4\cos 2x$
 $\Rightarrow R\cos \alpha = 3$
 $R\sin \alpha = 4$
 $\therefore R = 5, \tan \alpha = \frac{4}{3} \Rightarrow \alpha = 0.927$
 $\therefore y = 5\sin(2x + 0.927)$

c) On x-axis, $y = 0 \Rightarrow \sin(2x + 0.927) = 0 \quad -\pi \leq x \leq \pi$
 $2x + 0.927 = 0, \pi, 2\pi, -\pi, -2\pi \quad -5.356 \leq 2x + 0.927 < 7.21$
 $x = -3.61, -2.03, -0.46, 1.11, 5.36$



⑤ $R = 1000e^{-ct}$
 a) When $t = 0, R = 1000e^0 = 1000$
 b) $tR = 5730$
 $1000e^{-5730c} = 500$
 $e^{-5730c} = 0.5$
 $c = \frac{\ln 0.5}{-5730} = -0.0001210$

c) $t = 22920$
 $1000e^{-c \times 22920} = 6.25$



⑧ $f(x) = 1 - 2x^3$

a) $g(x) = \frac{3}{2}x - 4$
 $x = 1 - 2y^3$
 $2y^3 = 1 - x$
 $y^3 = \frac{1-x}{2}$
 $y = \sqrt[3]{\frac{1-x}{2}} \Rightarrow f^{-1}(x) = \sqrt[3]{\frac{1-x}{2}}$

b) $gf(x) = \frac{3}{1-2x^3} - 4$
 $= \frac{3}{1-2x^3} - \frac{4(1-2x^3)}{1-2x^3} = \frac{3-4+8x^3}{1-2x^3} = \frac{8x^3-1}{1-2x^3}$

c) $gf(x) = 0$
 $\Rightarrow 8x^3 - 1 = 0$
 $8x^3 = 1$
 $x^3 = \frac{1}{8} \Rightarrow x = \frac{1}{2}$

d) S.P. at $\frac{dy}{dx} = 0$

$$\begin{aligned} u &= 8x^3 - 1 & v &= 1 - 2x^3 & v^2 &= (1 - 2x^3)^2 \\ u' &= 24x^2 & v' &= -6x^2 \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{24x^2(1-2x^3) + 6x^2(8x^3-1)}{(1-2x^3)^2} \\ &= \frac{24x^2 - 48x^5 + 48x^5 - 6x^2}{(1-2x^3)^2} \\ &= \frac{18x^2}{(1-2x^3)^2} \end{aligned}$$

At $\frac{dy}{dx} = 0, \frac{18x^2}{(1-2x^3)^2} = 0 \Rightarrow 18x^2 = 0$
 $\Rightarrow x = 0$

When $x = 0, y = \frac{8 \times 0^3 - 1}{1 - 2 \times 0^3} = \frac{-1}{1} = -1$

S.P. = $(0, -1)$