

C3 June 07

1) $\ln a + \ln b = \ln ab$

$\ln x + \ln 3 = \ln 3x = \ln 6 \Rightarrow 3x = 6 \Rightarrow \underline{x = 2}$

b) $e^x + 3e^{-x} = 4$

$(xe^x)(e^x)^2 + 3 = 4(e^x) \Rightarrow (e^x)^2 - 4(e^x) + 3 = 0$

let $y = e^x \Rightarrow y^2 - 4y + 3 = 0 \Rightarrow (y-3)(y-1) = 0 \Rightarrow y = 3, y = 1$

$\Rightarrow e^x = 3 \Rightarrow \ln e^x = \ln 3 \Rightarrow \underline{x = \ln 3}$

$\Rightarrow e^x = 1 \Rightarrow \underline{x = 0}$

2) $\frac{2x+3}{x+2} - \frac{9+2x}{(2x-1)(x+2)} \Rightarrow \frac{(2x+3)(2x-1)}{(x+2)(2x-1)} - \frac{9+2x}{(x+2)(2x-1)}$

$\Rightarrow \frac{4x^2 + 4x - 3 - 9 - 2x}{(x+2)(2x-1)} \Rightarrow \frac{4x^2 + 2x - 12}{(x+2)(2x-1)} = \frac{(x+2)(4x-6)}{(x+2)(2x-1)}$

$\Rightarrow f(x) = \frac{4x-6}{2x-1}$

b) $u = 4x-6 \quad v = 2x-1 \quad f'(x) = \frac{vu' - uv'}{v^2} = \frac{8x-4-(8x-12)}{(2x-1)^2}$

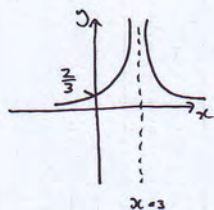
$f'(x) = \frac{8}{(2x-1)^2}$

3) $y = x^2 e^x \quad u = x^2 \quad v = e^x \quad y' = vu' + uv' \quad y' = 2xe^x + x^2 e^x \quad y' = x(x+2)e^x$

b) TP when $y' = 0$ hence $x = 0, x = -2, e^x = 0 \rightarrow$ impossible.

$x = 0 \Rightarrow y = 0 \quad (0, 0) \quad x = -2 \Rightarrow y = 4e^{-2} \quad (-2, 4e^{-2})$

4) $|g(x)|$



asymptotes $x = 3, y = \frac{2}{3}$

d) $|\frac{2}{x-3}| = 3$ crosses both parts of graph

$\frac{2}{x-3} = 3 \Rightarrow 2 = 3x-9 \Rightarrow 3x = 11 \Rightarrow \underline{x = \frac{11}{3}}$

$-\frac{2}{x-3} = 3 \Rightarrow -2 = 3x-9 \Rightarrow 3x = 7 \Rightarrow \underline{x = \frac{7}{3}}$

6) $R \sin(\theta + \alpha) = a \sin \theta + b \cos \theta$

$b = 2 = R \sin \alpha \quad \tan \alpha = \frac{2}{3}$
 $a = 3 = R \cos \alpha \quad \alpha = 0.588$
 $R = \sqrt{2^2 + 3^2} = \sqrt{13}$

$\sqrt{13} \sin(x + 0.588)$

b) max value of $\sin(\) = 1$

max value of $\sqrt{13} \times \sin(\) = \sqrt{13}$

max value of $(3 \sin x + 2 \cos x)^2 = (\sqrt{13})^4 = 13^2 = \underline{169}$

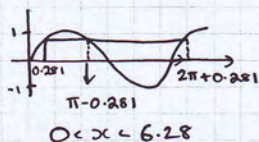
c) $\sqrt{13} \sin(x + 0.588) = 1 \Rightarrow \sin(x + 0.588) = \frac{1}{\sqrt{13}}$

$\Rightarrow x + 0.588 = \sin^{-1}(\frac{1}{\sqrt{13}}) = 0.281$

$x + 0.588 = 0.281 \Rightarrow x = -0.307 \times$

$x + 0.588 = \pi - 0.281 \Rightarrow x = 2.713 \checkmark$

$x + 0.588 = 2\pi + 0.281 \Rightarrow x = 5.976 \checkmark$



3) $y' = (x^2 + 2x)e^x \quad u = x^2 + 2x \quad v = e^x$
 $u' = 2x + 2 \quad v' = e^x$

$y'' = (2x+2)e^x + (x^2+2x)e^x$

$y'' = (x^2 + 4x + 2)e^x$

d) at $(0, 0) \quad y'' = 2e^0 = 2$ +ve \cup minima

at $(-2, 4e^{-2}) \quad y'' = (4-8+2)e^{-2} = -2e^{-2}$ -ve \cap maxima

4) $f(x) = -x^3 + 3x^2 - 1 = 0 \Rightarrow -x^3 + 3x^2 = 1$

$\Rightarrow 3x^2 - x^3 = 1 \Rightarrow x^2(3-x) = 1 \Rightarrow x^2 = \frac{1}{3-x} \Rightarrow x = \sqrt{\frac{1}{3-x}}$

$x_1 = 0.6, x_2 = 0.6455, x_3 = 0.6517, x_4 = 0.6526$

c) Show $x = 0.653$ is a root test 0.6525 and 0.6535 .

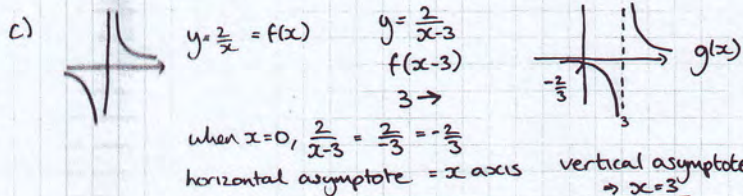
$f(0.6525) = -0.00054$ -ve $f(0.6535) = 0.002$ +ve

from change of sign rule 0.653 (3dp) is a root.

5) $f(g(4)) = f(\frac{2}{4-3}) = f(2) = \ln(4-1) = \underline{\ln 3}$

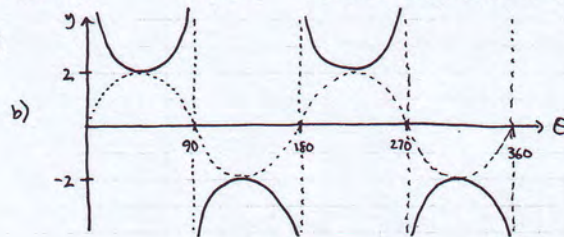
b) $y = \ln(2x-1) \Rightarrow x = \ln(2y-1) \Rightarrow e^x = 2y-1 \Rightarrow y = \frac{e^x + 1}{2}$

domain $x \in \mathbb{R}$.



7) $\frac{\sin \theta \times \sin \theta}{\cos \theta \times \sin \theta} + \frac{\cos \theta \times \cos \theta}{\sin \theta \times \cos \theta} \Rightarrow \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta} = \frac{2}{2 \sin \theta \cos \theta} = \frac{2}{\sin 2\theta}$

$= 2 \operatorname{cosec} 2\theta$

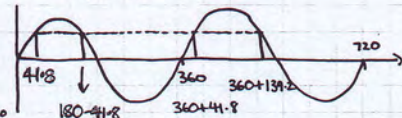


c) $2 \operatorname{cosec} 2\theta = 3 \Rightarrow \operatorname{cosec} 2\theta = \frac{3}{2} \Rightarrow \frac{1}{\sin 2\theta} = \frac{3}{2} \Rightarrow \sin 2\theta = \frac{2}{3}$

$\Rightarrow 2\theta = \sin^{-1}(\frac{2}{3}) = 41.8^\circ$

$2\theta = 41.8, 139.2, 401.8, 499.2$

$\theta = 20.9^\circ, 69.1^\circ, 200.9^\circ, 249.1^\circ$



8) $x = De^{-\frac{t}{8}}$ a) $D = 10, t = 5 \quad x = 10e^{-\frac{5}{8}} = 5.353 \text{ mg}$

b) $D = 15.353, t = 1 \text{ hr} \quad x = 15.353e^{-\frac{1}{8}} = 13.549 \text{ m}$

c) $3 = 15.353e^{-\frac{1}{8}t} \Rightarrow e^{-\frac{1}{8}t} = 0.195 \Rightarrow -\frac{1}{8}t = \ln 0.195$

$-\frac{1}{8}t = -1.633 \Rightarrow t = 13.06 \text{ hrs} \quad \underline{13 \text{ hrs } 3 \text{ min } 42 \text{ sec}}$