(1)

C3 June 06 - Solutions

1. a) $\dfrac{3x^2-x-2}{x^2-1} = \dfrac{(3x+2)(x-1)}{(x-1)(x+1)} = \dfrac{3x+2}{x+1}$

b) $\dfrac{3x^2-x-2}{(x^2-1)} - \dfrac{1}{x(x+1)} = \dfrac{3x+2}{x+1} - \dfrac{1}{x(x+1)}$

$= \dfrac{3x^2+2x-1}{x(x+1)}$

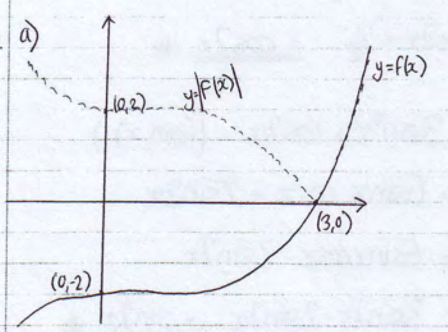
$= \dfrac{(3x-1)(x+1)}{x(x+1)} = \dfrac{3x-1}{x}$

2. a) $y = e^{3x} + \ln(2x)$ $\quad \dfrac{dy}{dx} = 3e^{3x} + \dfrac{2}{2x} = 3e^{3x} + \dfrac{1}{x}$

b) $y = (5+x^2)^{3/2}$ $\quad \dfrac{dy}{dx} = \dfrac{3}{2}(5+x^2)^{1/2} \times 2x$

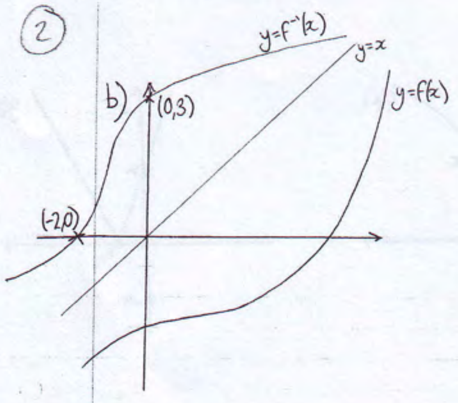
$= 3x(5+x^2)^{1/2}$

3. a)

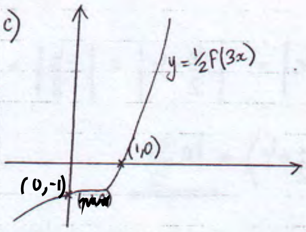


(2)

b)



c)



4 a) when $t=0$ $T = 400+25 = 425°C$

b) $300 = 400e^{-0.05t} + 25$

$\dfrac{275}{400} = e^{-0.05t}$

$-0.05t = \ln\left(\dfrac{275}{400}\right)$ $\qquad t = \dfrac{1}{-0.05}\ln\left(\dfrac{275}{400}\right) = 7.49 \text{ min}$ (3

c) Rate of change $\dfrac{dT}{dt} = -0.05 \times 400e^{-0.05t}$

$= -20e^{-0.05t}$

(3)

$\left.\dfrac{dT}{dt}\right|_{t=50} = -20e^{-0.05\times 50} = -1.64\ °C \text{ per min}$ (3sf).

d) The min value of $400e^{-0.05t}$ is >0 $\therefore$ T is always > 25.

5. a) $y = (\overset{u}{2x-1})\overset{v}{\tan 2x}$ $\qquad u = (2x-1)$ $\quad u' = 2$

$v = \tan 2x$ $\quad v' = 2\sec^2 2x$

$\dfrac{dy}{dx} = 2(2x-1)\sec^2 2x + 2\tan 2x$

minimum point when $\dfrac{dy}{dx} = 0$

$\therefore$ $(4x-2)\sec^2 2x + 2\tan 2x = 0$

$\dfrac{(4x-2)}{\cos^2 2x} + \dfrac{2\sin 2x}{\cos 2x} = 0$

$\dfrac{4x-2}{\cos^2 2x} + \dfrac{2\sin 2x\cos 2x}{\cos^2 2x} = 0$

$\therefore$ $4x-2 + 2\sin 2x\cos 2x = 0$

$4x-2 + \sin 4x = 0$

at P $x=k$ $\therefore$ $4k-2+\sin 4k = 0$

(4)

6. a) $\sin^2\theta + \cos^2\theta \equiv 1$ $\qquad \div \sin^2\theta$

$1 + \cot^2\theta \equiv \text{cosec}^2\theta$

$\cot^2\theta - \text{cosec}^2\theta \equiv -1$

$\text{cosec}^2\theta - \cot^2\theta \equiv 1$ #

b) LHS $= \text{cosec}^4\theta - \cot^4\theta = (\text{cosec}^2\theta - \cot^2\theta)(\text{cosec}^2\theta +$

$= \text{cosec}^2\theta + \cot^2\theta$ #

c) $\text{cosec}^2\theta + \cot^2\theta = 2 - \cot\theta$ (from b))

$1 + \cot^2\theta + \cot^2\theta = 2 - \cot\theta$ (from a))

$2\cot^2\theta + \cot\theta - 1 = 0$

$(2\cot\theta - 1)(\cot\theta + 1) = 0$

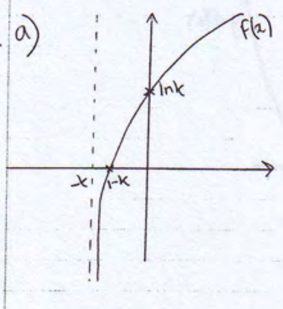
$\cot\theta = \frac{1}{2}$ $\quad \cot\theta = -1$

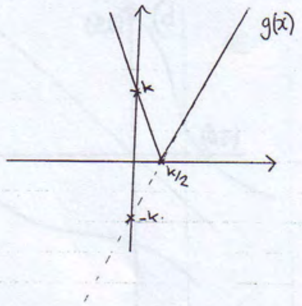
$\tan\theta = 2$ $\quad \tan\theta = -1$

$\theta = 63.4°$ $\quad \theta = -45°, 135°, \ldots$

$\theta = 135°$ $\quad$ since $90° < \theta < 180°$

7. a)





b) $x \in \mathbb{R}$.

c) $g\left(\frac{k}{4}\right) = \left|\frac{2k}{4} - k\right| = \left|\frac{k}{2} - k\right| = \left|-\frac{k}{2}\right| = \frac{k}{2}$

$fg\left(\frac{k}{4}\right) = \ln\left(\frac{k}{2} + k\right) = \underline{\underline{\ln\frac{3k}{2}}}$

d). $9y = 2x + 1$

$y = \frac{2}{9}x + \frac{1}{9}$ Grad of tangent $= \frac{2}{9}$

$y = \ln(x + k)$

$\frac{dy}{dx} = \frac{1}{x+k}$

$\left.\frac{dy}{dx}\right|_{x=3} = \frac{1}{3+k} = \frac{2}{9}$

$9 = 2(3+k)$ $\underline{k = 1.5}$

8 a) $\cos A = \frac{3}{4}$

$\cos^2 A + \sin^2 A = 1$

$\frac{9}{16} + \sin^2 A = 1$

$\sin A = \pm\sqrt{1 - \frac{9}{16}} = \pm\frac{\sqrt{7}}{4}$

$\therefore \sin 2A = 2\sin A\cos A$

$\sin 2A = 2 \times \pm\frac{\sqrt{7}}{4} \times \frac{3}{4} = \pm\frac{6\sqrt{7}}{16} = \underline{\underline{\pm\frac{3\sqrt{7}}{8}}}$

b) I) $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right)$

$= \cos 2x\cos\pi/3 - \sin 2x\sin\pi/3 + \cos 2x\cos\pi/3 + \sin 2x\sin$

$= 2\cos 2x\cos\pi/3$

$= 2\cos 2x \times 1/2 = \underline{\cos 2x}$ #

II) $y = 3\sin^2 x + \cos 2x$ (from I))

$\frac{dy}{dx} = 6\sin x.\cos x - 2\sin 2x$

$= 6\sin x\cos x - 2\sin 2x$

$= 3\sin 2x - 2\sin 2x = \underline{\sin 2x}$ #